Systematic Uncertainties

Sun-Chong Wang

TRIUMF, Vancouver, B.C. V6T 2A3, Canada

Sept 3, 2001

Abstract

This note estimates systematical errors of the $\mathcal{TWIST}$ spectrometer due to temperature and pressure extremes. Also discussed are errors due to jitter in time zero and inhomogeneity in B. It concludes that the errors are in general tolerable.

1. Introduction

The E614 experiment is to measure the Michel spectrum with an unprecedented precision. The proposed accuracy of the $\mathcal{TWIST}$ spectrometer at a level of parts in 10^5 on the Michel parameters allows one to improve current limits on the weak coupling constants within the Standard model. If, however, the result suggests deviations from Standard model prescriptions, it has the potential to shed light on directions toward models beyond the Standard model.

To accomplish the goal, various systematic effects on the experimental accuracy have to be studied and thereby corrected for. This note estimates uncertainties in the Michel parameters due to changes in chamber pressure and temperature which may occur during data taking. In Section 2, measurement errors are estimated by Garfield simulations. Section 3 shows how errors propagate from spatial measurements to track parameters: e^+ momentum and the dip angle. Impact on the Michel parameters is shown in Section 4. Section 5 summaries.

2. STR's by GARFIELD

Robert Openshaw has observed that the atmospheric pressure in the meson hall can move away from 760 torr by at most 3% and chamber temperature from 24 by at most 10 degrees Celsius. It's known that gas properties vary as the ambient conditions change. To see the effect, we prepared the drift time-drift distance relationships of the DME gas at different pressures (or temperatures) using the Garfield. The mean, over drift distance within the cell, of the shift is calculated. Then the average, $\overline{\mathrm{STR}}$, of the means over angles is calculated. The results are shown in Table 1.

Table 1

	σ in μm
temperature (24 ± 10 C)	17.4
pressure (760 ± 23 torr)	15.5
time zero jitter (1 ns)	3.4
B inhomogeneity (10^{-4})	0.5

Table 1: First column lists the variables changed in the Garfield simulation. Second are the resulting changes in the drift distance averaged over distances to the sense wire and over track angles (from 0 to 60 per 5 degree). The default STR is the one with DME gas at 24 C, 760 torr, and in a uniform B field of 2.2 Tesla.

It's noted that shifts of the curve are in one way (or the other). For example, when the pressure is lowered, the whole STR curve moves up. And, when the temperature drops, the entire STR curve moves down. All the shifts are calculated relative to the standard STR at 760 torr and 297 K. Also note that the numbers were obtained under extreme conditions. For example, $|\overline{\text{STR}}$ @ 34C - $\overline{\text{STR}}$ @ 24C$|$ and $|\overline{\text{STR}}$ @ 14C - $\overline{\text{STR}}$ @ 24C$|$ were found and then averaged to get 17.4 μm in the table. In general, effect of one extreme is very symmetrical to that of the other extreme, except that 14 C is a bit more adverse than 34 C.

The next step is to relate the spatial uncertainties, σ, to the momentum and angular resolution of the spectrometer.

3. **Energy and Angular Resolution**

If our tracking program always looks up the default STR table (at 24 C and 760 torr) for translation from tdc to the distance of closest approach, bias incurs and propagates into the determination of e^+ momenta and $\cos\theta$'s. The resulting Michel spectrum is distorted and the extracted Michel parameters will be off. We derive the distortion in this section.

The projection of the helical trajectory on the $x-y$ is a circle, whose radius, R, determines the transverse momentum of the positron, p_t,

$$p_t = \kappa\, q\, B\, R, \tag{1}$$

where $\kappa = 299.792458$ (MeV$/c$)T^{-1}m^{-1}. q equals 1 for positron and the magnetic field strength B is 2.2 Tesla. The track parameters in Kalman fitting has the following relations,

$$x = R\cos(\phi_0+\phi) - R\cos\phi_0, \tag{2a}$$

$$y = R\sin(\phi_0+\phi) - R\sin\phi_0. \tag{2b}$$

Measurement uncertainties on various planes accumulate and contribute to the uncertainty in R,

$$\begin{aligned}\frac{1}{\sigma_R^2} &= \sum_{i=1}^{N}\left[\frac{1}{\sigma_x^2}\Big(\frac{\partial x_i}{\partial R}\Big)^2 + \frac{1}{\sigma_y^2}\Big(\frac{\partial y_i}{\partial R}\Big)^2\right] \\ &= \frac{2}{\sigma^2}\sum_{i=1}^{N}\Big[1-\cos(\phi_0+\phi_i)\cos\phi_0 - \sin(\phi_0+\phi_i)\sin\phi_0\Big] \\ &\approx \frac{2}{\sigma^2}\sum_{i=1}^{N}1 = \frac{2N}{\sigma^2},\end{aligned} \tag{3}$$

where N is the number of UV planes upstream or downstream and equal to 11 in our case. We have also assumed that the measurement error on the U plane is the same as that on the V plane: $\sigma_x = \sigma_y \equiv \sigma$. We get,

$$\sigma_{p_t} = \kappa\, q\, B\, \sigma_R \approx \frac{\kappa\, q\, B}{\sqrt{2N}} \sigma. \tag{4}$$

We turn to the angular uncertainty for a moment. It's noted that θ is related to the dip angle, λ, by, $\theta = \frac{\pi}{2} - \lambda$. Furthermore, the track parameter, t $(= \tan\lambda)$, equals $\frac{z}{R\phi}$. We have, using Equ (2) and similar to Equ (3),

$$\begin{aligned}
\frac{1}{\sigma_t^2} &= \sum_{i=1}^{N} \left[\frac{1}{\sigma_x^2} \left(\frac{\partial x_i}{\partial t} \right)^2 + \frac{1}{\sigma_y^2} \left(\frac{\partial y_i}{\partial t} \right)^2 \right] \\
&\approx \frac{1}{\sigma^2 t^4} \sum_{i=1}^{N} z_i^2 \\
&= \frac{(\Delta z)^2}{\sigma^2 t^4} \sum_{i=1}^{N} i^2 \\
&= \frac{(\Delta z)^2 N(N+1)(2N+1)}{\sigma^2 t^4\, 6},
\end{aligned} \tag{5}$$

where again $\sigma_x = \sigma_y \equiv \sigma$ is assumed, and Δz is the spacing between UV modules. Since they are not evenly placed in our spectrometer, we approximate an average Δz of 0.05 m. The angular uncertainty is therefore,

$$\sigma_\theta = \sin^2\theta\ \sigma_t = \frac{\cos^2\theta}{\Delta z} \sqrt{\frac{6}{N(N+1)(2N+1)}} \cdot \sigma \tag{6}$$

We now return to momentum. Since $p = (p_t^2 + p_z^2)^{\frac{1}{2}} = p_t(1+t^2)^{\frac{1}{2}}$, one has, ignoring the covariance between p_t and t,

$$\begin{aligned}
\sigma_p &= \left[(1+t^2)\sigma_{p_t}^2 + \left(\frac{t}{1+t^2} p \right)^2 \sigma_t^2 \right]^{\frac{1}{2}} \\
&= \left[\left(\frac{\kappa q B}{\sin\theta} \right)^2 \frac{1}{2N} + \frac{p^2}{(\Delta z)^2} \left(\frac{\cos^3\theta}{\sin\theta} \right)^2 \frac{6}{N(N+1)(2N+1)} \right]^{\frac{1}{2}} \cdot \sigma
\end{aligned} \tag{7}$$

4. **Michel Parameters**

Pressure (or temperature) fluctuations are to smear the Michel spectrum,

$$
\begin{aligned}
\frac{d^2N}{dxd(\cos\theta)} =&(x^2-x_0^2)^{\frac{1}{2}}\Big\{6x(1-x)+\frac{4}{3}\rho(4x^2-3x-x_0^2)+6\eta x_0(1-x)\\
&+P_\mu\xi\cos\theta(x^2-x_0^2)^{\frac{1}{2}}\big[2(1-x)+\frac{4}{3}\delta(4x-4+(1-x_0^2)^{\frac{1}{2}})\big]\Big\}
\end{aligned}
\tag{8}
$$

by,

$$
\begin{aligned}
E &\to E+G(0,\sigma_E) \quad \text{and}\\
\cos\theta &\to \cos\theta+G(0,\sigma_{\cos\theta}),
\end{aligned}
\tag{9}
$$

where $G(0,\sigma)$ stands for a Gaussian distribution with mean 0 and standard deviation σ. The extent of smearing, $\sigma_E=(p/E)\sigma_p$ and $\sigma_{\cos\theta}$, are readily found in Equs. (6) and (7).

We firstly generate a Michel spectrum by sampling 10^9 times from the distribution of Equ. (8) with $\rho=0.75, \eta=0, P_\mu\xi=1$, and $\delta=0.75$. The resulting high statistics spectrum is our (perfect) data. The ranges of x and θ in the sampling are $0.3<x<1$, where radiative correction is not important, and $10<\theta<70$ degrees, respectively. The bin size is around 100 keV in energy and around 1 degree in θ. Next, Equ. (8) with a running Michel parameter, say ρ, is fitted to the data to extract ρ. The results are shown in Table 2. It's noted that the errors in the table are purely statistical.

Table 2

parameters	best-fitted value	statistical error
ρ	0.74996 (1.00)	6×10^{-5}
η	-0.00149 (0.99)	176×10^{-5}
ξ	1.00005 (0.99)	13×10^{-5}
δ	0.74997 (1.00)	9×10^{-5}

Table 2: 10^9 pairs of energy and angle are generated by Monte Carlo according to the distribution of Equ. (8) given $\rho = 0.75, \eta = 0, P_\mu \xi = 1$, and $\delta = 0.75$. The resulting two-dimensional histogram is fitted to Equ. (8) with now freely adjusting parameters (one at a time). The 2nd column are the re-constructed parameters, the numbers in the parenthesis being χ^2/NDF. The last column are the errors.

We repeat the above exercise except that smearing of Equ. (9) are now introduced after sampling from Equ. (8) and before binning into histograms. Errors in the Michel parameters now contain both systematic (due to the smearing) and statistical. Smearing in Equs. (9) not only dramatically increase the chi-square of the fit, but also shift the best-fitted Michel parameters away from $\rho = 0.75, \eta = 0, P_\mu \xi = 1$, and $\delta = 0.75$. The shifts can however be restored by an application of Bayesian de-convolution. A one-dimensional example for $0.3 < x < 1$ and $\theta = 35$ degree has been demonstrated. The systematic errors can now be subtracted from the total by,

$$\sigma_{sys} = \sqrt{\sigma_{total}^2 - \sigma_{stat}^2}. \tag{10}$$

The results are shown in Table 3.

It is seen that shifts in Michel parameters increases with σ. Numbers in the parenthesis are again the ratios of the χ^2 to the number of degrees of freedom of the fit. The systematical errors in the last column do not change a lot with the σ and they are evaluated for a σ of 20 μm.

Table 3

parameters	best-fitted value			systematical
	100 μm	20 μm	10 μm	error
ρ	0.74608 (19.1)	0.74934 (2.14)	0.74968 (1.27)	0.5×10^{-5}
η	0.10296 (19.1)	0.01760 (2.15)	0.00691 (1.26)	179×10^{-5}
ξ	0.98112 (19.0)	0.99707 (2.14)	0.99873 (1.28)	17×10^{-5}
δ	0.74296 (18.9)	0.74883 (2.13)	0.74953 (1.27)	0.7×10^{-5}

Table 3: 10^9 pairs of energy and angle are generated by Monte Carlo according to distribution Equ. (8) with $\rho = 0.75, \eta = 0, P_\mu \xi = 1$, and $\delta = 0.75$. Energies and angles are then smeared according to Equ. (9) for $\sigma = 100$, 20, 10 μm before binning. The resulting two-dimensional histogram is fitted to Equ. (8) with now freely adjusting parameters (one at a time). The 2nd, 3rd, and 4th column are the re-constructed parameters, and the last column are the *systematical* errors evaluated according to Equ. (10).

5. **Summary**

Various sources of fluctuations and their effects on spatial measurements are simulated by Garfield and listed in Table 1. Equs. (6) and (7) provide the momentum and angular dispersion due to the measurement errors. The energy resolution,

σ_E, is about 4 keV and angular resolution, σ_θ, about 10^{-3} degree for $\sigma = 20\ \mu$m. Statistical errors on the Michel parameters are estimated, followed by convolving the Michel spectrum according to the dispersion. Table 4 conclude the estimates on the systematical errors. They are found to be an order of magnitude below the present publicized uncertainties. They are also smaller than the statistical errors at the proposed level of statistics. However, systematic errors due to temperatures in η and ξ can become close to their statistical counterparts in very extreme scenarios.

Table 4

source of error	ρ ($\times 10^{-5}$)	η ($\times 10^{-5}$)	ξ ($\times 10^{-5}$)	δ ($\times 10^{-5}$)
temperature $\pm$ 10 K	≤ 0.5	≤ 179	≤ 17	≤ 0.7
pressure $\pm$ 3%	≤ 0.5	≤ 179	≤ 17	≤ 0.7
time zero $\pm$ 1 ns	$\ll 0.5$	$\ll 179$	$\ll 17$	$\ll 0.7$
B inhomogeneity $\pm\ 10^{-4}$	$\ll 0.5$	$\ll 179$	$\ll 17$	$\ll 0.7$
statistical at 10^9	6	176	13	9

Table 4: Estimates of errors due to fluctuations in environmental and instrumental conditions.

REFERENCES

(1) R. Veenhof, "Garfield User Guide", version 7.04, CERN, Jan 25, 2001

(2) D.E. Groom et al., "Statistics", *European Physical Journal*, **C15** (2000) 1

(3) Particle Data Group. "Review of Particle Properties", *Phys. Rev.*, **D50** (1994) 1177