The tuning of M13 for TWIST, theory and some results

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Abstract

Beamline M13 was tuned in the fall of 2000. Since there is only diagnostics at the end of the beamline, the tuning was done in a fairly indirect way, making use of various slits. This paper aims to clarify the used tuning procedures. It shows theoretical distributions similar to measured distributions, and explains in practical terms some relevant theoretical concepts that are normally used to describe the beam optics and the transformation of the initial beam parameters as the beam travels through the beamline. A result of the tuning was that the experimental settings of the quadrupoles agreed within a few percent with the theoretical settings.

1 Introduction

The requirements for the M13 beam for TWIST have been described in two reports. The first report [1] deals with general optics considerations for injection of a beam into a solenoid. The second report [2] describes a new optimum tune.

The normal M13 tune has focus positions F1, F2 and F3. At F1, after bending magnet B1, are horizontal and vertical foci, which are imaged by a quadrupole triplet at F2, before B2. The final quadrupoles Q6 and Q7 give a double focus at F3, typically 1.15 m from the exit of Q7. There are momentum slits in the horizontal plane at F1 and F2. Formerly, there were also vertical slits at F1 and F2, which were removed a long time ago. It is essential that vertical slits are again installed at F2. The function of the vertical slits is to restrict and define the vertical source size that is seen by the channel. The angle acceptance can be varied by the horizontal and vertical jaws before B1.

The new tune differs from the normal tune in that it has no vertical focus at F1, but only at F2. This requires a lower strength of quadrupole Q1. Therefore, the size of the vertical beam at the jaws is larger for a given initial divergence.

This makes possible a more precise selection of the initial vertical divergence. A second point is a reduced strength of Q4, to make the vertical divergence at F3 independent of the initial vertical beam size. In other words, there is not only a vertical focus at F3 but also a vertical waist.

The purpose of the tuning was to establish the various foci in the beam by looking at intensity distributions, beam spots and divergences at F3 for various slit positions. The tuning was done for surface muons, and for 120 MeV/c pions. There were two x and y wire chambers, namely W1 at F3 and W2 at 30 cm downstream of F3. Therefore, in the case of pions it was possible to make $x-\theta$ and $y-\phi$ emittance plots.

Section 2 gives general optics considerations. It deals in particular with the concepts of image and waist, which are two entirely different entities. Section 3 explains the various strategies that were used to obtain the experimental settings of the quadrupoles.

2 General optics considerations, focus, image and waist

In the following, the terms focus and image have the same meaning. The optical properties of an optical system are described completely by the transfer matrix R, acting on the coordinates of the particles in the beam. The matrix R is used to define a focus. The beam itself is described by the beam matrix σ, which is used to define a waist.

2.1 The transfer matrix R and the definition of image

The horizontal particle coordinates at any longitudinal position can be obtained from the matrix equation:

$$\begin{pmatrix} X \end{pmatrix} = \begin{pmatrix} R \end{pmatrix} \begin{pmatrix} X_0 \end{pmatrix} \tag{1}$$

or

$$\begin{pmatrix} x \\ \theta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \end{pmatrix} \tag{2}$$

Similarly in the vertical plane:

$$\begin{pmatrix} Y \end{pmatrix} = \begin{pmatrix} R \end{pmatrix} \begin{pmatrix} Y_0 \end{pmatrix} \tag{3}$$

or

$$\begin{pmatrix} y \\ \phi \end{pmatrix} = \begin{pmatrix} R_{33} & R_{34} \\ R_{43} & R_{44} \end{pmatrix} \begin{pmatrix} y_0 \\ \phi_0 \end{pmatrix} \tag{4}$$

Here, the coordinates at the production target are x_0 and y_0 for the positions, and θ_0 and ϕ_0 for the angles. In addition, the horizontal (bend plane) coordinates depend on δ, which is the percentage difference of the momentum of the particle with the central momentum. It can be taken into account by making the horizontal transfer matrix 3-dimensional.

Writing out the equations gives:

$$x = R_{11}x_0 + R_{12}\theta_0 + R_{16}\delta \tag{5}$$

$$\theta = R_{21}x_0 + R_{22}\theta_0 + R_{26}\delta \tag{6}$$

$$y = R_{33}y_0 + R_{34}\phi_0 \tag{7}$$

$$\phi = R_{43}y_0 + R_{44}\phi_0 \tag{8}$$

Here, R_{16} and R_{26} are the position and angle momentum dispersion. $R_{16} = R_{26} = 0$ before B1, and also after B2 if the beam is doubly achromatic, a condition that is necessary to keep both the position and the angle small at the final focus F3.

A vertical focus or image occurs when $R_{34} = 0$ in equation 7. In that case all rays departing from a point with $y_0 = 0$ pass again through $y = 0$, independently of the initial angle ϕ_0. The magnification of the initial object is then R_{33}. In the horizontal plane R_{12} and R_{11} play the equivalent role. The fact of the existence of the image or focus is a characteristic of the optics system itself, as described by the transfer matrix R, and is independent of the specific characteristics, like size and divergence, of the beam itself.

2.2 Definition of waist, using the beam σ matrix

The beam entering an optics system can be quite complicated. In fact, this is the case for the M13 surface muon beam. However, when designing a system, the beam is usually described by a six-dimensional ellipsoid in six-dimensional phase space. If the beam emittance is small, there is no optics coupling between horizontal and vertical transverse phase space. The beam phase space can then in each plane be represented by a two-dimensional phase space ellipse, using the σ matrix, a positive definite symmetric matrix. Horizontally, the matrix is:

$$\begin{pmatrix}\sigma\end{pmatrix} = \begin{pmatrix}\sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22}\end{pmatrix} \tag{9}$$

The ellipse is described with the inverse matrix:

$$\begin{pmatrix}\sigma\end{pmatrix}^{-1} = \frac{1}{\epsilon^2}\begin{pmatrix}\sigma_{22} & -\sigma_{21} \\ -\sigma_{21} & \sigma_{11}\end{pmatrix} \tag{10}$$

Here, ϵ is the emittance of the phase space area contained inside the ellipse in units of π mm mr.

The equation of the ellipse is:

$$\begin{pmatrix}x, \theta\end{pmatrix}\begin{pmatrix}\sigma\end{pmatrix}^{-1}\begin{pmatrix}x \\ \theta\end{pmatrix} = 1 \tag{11}$$

or

$$\sigma_{22}x^2 - 2\sigma_{21}x\theta + \sigma_{11}\theta^2 = \epsilon^2 = \det(\sigma) \tag{12}$$

This is a slanted ellipse, the slant being determined by the value of σ_{21}. The diagonal σ matrix elements have a physical meaning, since they determine the maximum size of the projections of the ellipse on either the x-axis or the θ-axis. Namely:

$$x_{\text{max}} = \sqrt{\sigma_{11}} \tag{13}$$

and

$$\theta_{\text{max}} = \sqrt{\sigma_{22}} \tag{14}$$

When the off-diagonal terms of the matrix are zero, namely $\sigma_{21} = 0$, then the ellipse is upright. The beam has a waist. In that case the area of the ellipse is $\pi\epsilon = \pi x_{\text{max}}\theta_{\text{max}} = \pi\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}$.

It is usual to start the beam transport with an upright ellipse. During transport through the beamline, the beam ellipse is in general not upright. The transformation of the beam σ matrix is given by:

$$\Big(\sigma(\text{new})\Big) = \Big(R\Big)\Big(\sigma(\text{initial})\Big)\Big(R\Big)^T \tag{15}$$

where R^T is the transpose of the transfer matrix R.

If the initial ellipse is upright the new matrix element σ_{21} becomes:

$$\sigma_{21}(\text{new}) = \mathrm{R}_{11}\mathrm{R}_{21}\sigma_{11}(\text{initial}) + \mathrm{R}_{12}\mathrm{R}_{22}\sigma_{22}(\text{initial}) \tag{16}$$

There is a horizontal focus at some longitudinal position in the beamline if the R-matrix element $R_{12} = 0$. In that case, there is in general not a waist at that position. The only way that a waist and a focus can coincide is if both $R_{12} = 0$ and $R_{21} = 0$. Otherwise, there is a waist if

$$(\text{initial aspect ratio})^2 = (\frac{\theta_{\text{max,initial}}}{x_{\text{max,initial}}})^2 = \frac{\sigma_{22}(\text{initial})}{\sigma_{11}(\text{initial})} = -\frac{R_{11}R_{21}}{R_{22}R_{12}} \tag{17}$$

The left part of this equation is according to equations 13 and 14 just the square of the aspect ratio of the initial upright ellipse. The R-matrix elements are fixed for a given optics system, if the currents in the magnets stay constant. Therefore, there is a waist for only that particular aspect ratio, and not for an other. If, for instance, the apertures of slits in the beamline are changed so that the position or angle acceptance of the beamline is changed, then the aspect ratio will in general change, and there will not longer be a waist.

2.3 Special consideration of the focus or waist question for secondary beams

For primary beams, or for beams circulating in accelerators, the original beam size and divergence give frequently comparable contributions to the divergence at a given point in the beam. Therefore, the distinction between a focus and a waist is great. In secondary beams, the angle acceptance is usually very large in order to obtain a workable beam intensity. The initial source size is, however, often modest.In that case the divergence in the beam depends mostly on the initial divergence. In other words, the terms $R_{21}x_0$ and $R_{43}y_0$ in equations 6 and 8 are often small. In that case, it does in practice not make much difference whether there is a waist or a focus. In the case of the surface muon beam for TWIST, the angle acceptance is severely curtailed by closing the jaws. The distinction between focus and waist is then again important. Ultimately, what is needed for injection into the solenoid is a waist with the optimum aspect ratio.

2.4 Raytracing of a phase ellipse through M13

Calculations were done with ZGOUBI for an initial upright horizontal phase ellipse with half axes of 4.3 mm for x and 25 mr for θ. Similarly, in the vertical plane the half axes were 3.0 mm for y and 25 mr for ϕ. Figure 1 shows how the horizontal ellipse developes along the beamline at the indicated positions. The marked points on the ellipses show the values of x and θ for particular rays. Three aspects are important. Firstly, the ellipse as a whole is transformed according to equation 15. Secondly, since the area of $\pi\epsilon$, enclosed by the ellipse, is conserved, every point or ray on the circumference of the original ellipse lies on the circumference of the new ellipse. The point travels around the ellipse. It advances in phase. As a matter of fact, all points advance in phase by the same amount. Thirdly, the elliptic shape is distorted by higher order aberrations.

Figure 1a gives the initial ellipse. At the jaws the ellipse is long and thin, making possible an accurate angle selection. At F1, there is again almost a waist, even though the beam was tuned for a focus. All points have advanced by almost 180 degrees, because the image is inverted. A further 180 degrees rotation occurs between F1 and the focus, again nearly a waist, at F2. At both locations the ellipse is distorted by higher order aberrations, because

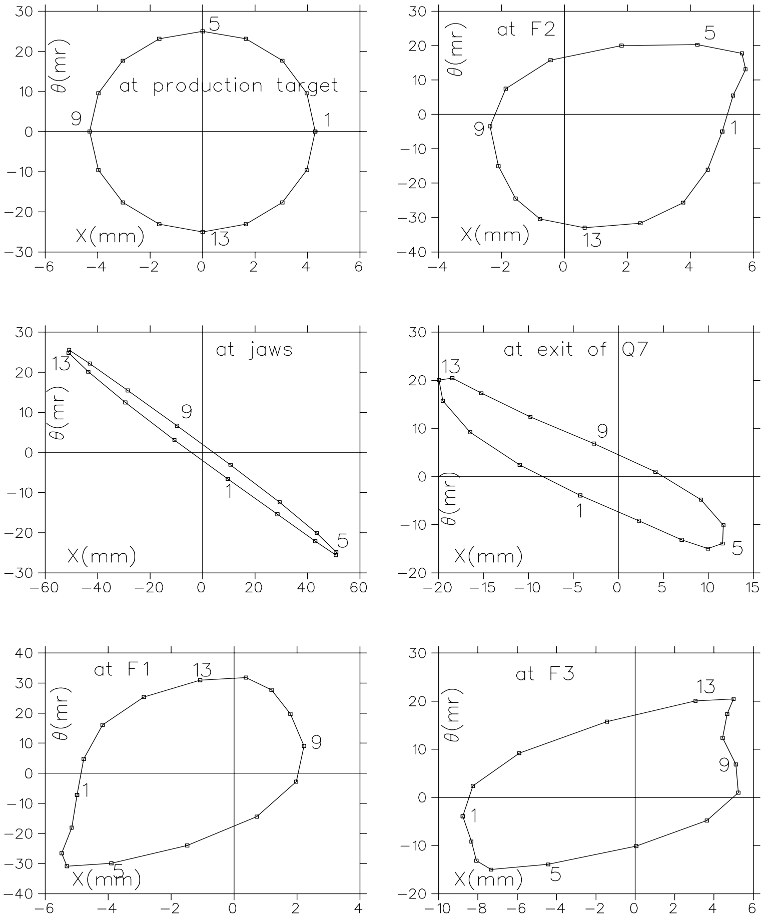

Fig. 1. The horizontal phase space ellipse at various locations along the beamline. The half axes of the initial ellipse are 4.3 mm for x and 25 mr for θ

the initial divergence is already quite large, namely almost half of the total horizontal angle acceptance that is obtained with fully open jaws. At the exit of Q7, the ellipse is again slanted. At F3 is a focus, but there is not quite a waist, as can be seen from the first order slant. In practice, it is not easy to determine whether there is a waist, because of the higher order distortions.

Figure 2a gives the initial ellipse in the vertical phase space. The higher order distortions are now much smaller, because the vertical angle acceptance is only about 15 percent of the totally possible angle acceptance. The ellipse at the jaws is slanted, but fairly fat. This means that the selection of the vertical angle acceptance is not as accurate as in the horizontal case. The ellipse is also slanted at F1, because there is neither a focus or a waist at that location. At F2, the phase advance is close to 180 degrees, because there is a focus here, which is almost a waist. At the exit of Q7, the ellipse is slanted. It is about twice as long and twice as thin as at the jaws. This suggests that it may be advantageous to do the vertical angle selection at the exit of Q7. At F3, there is again almost a waist. The matrix elements R_{34} andR_{34} are both close to

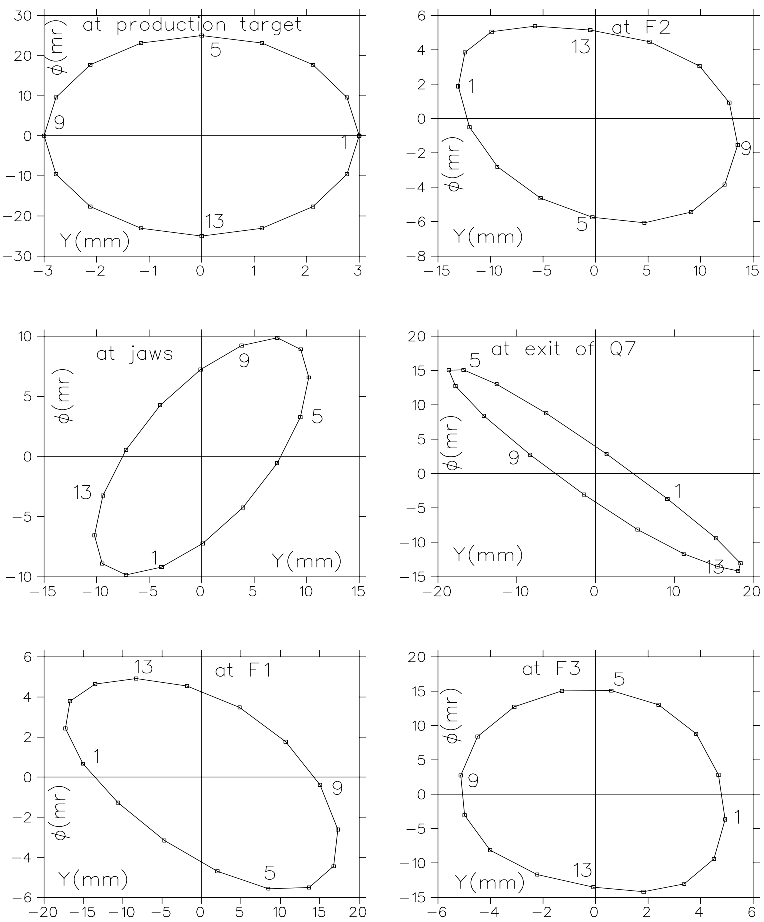

Fig. 2. The vertical phase space ellipse at various locations along the beamline. The half axes of the initial ellipse are 3.0 mm for y and 25 mr for ϕ

zero.

3 Tuning strategies

After setting the bending magnets, the quadrupoles are set to the theoretical values for the proposed M13 tune for TWIST. The jaws are set to accept initial angle divergences of about 25 mr. Then the values are established for the horizontally focussing quadrupoles Q2, the pair Q3 and Q5, and Q6. Next, a value for Q1 is determined. Finally, a range of values for Q4 and Q7 is found. The resulting quadrupole settings compared to the theoretical values are shown in table 1. Q2 was determined using surface muons. The settings of all other quadrupoles were found with 120 MeV/c pions. The agreement is good, except for Q7, something that is not understood. When the tune was scaled down to surface muon momentum, it was found that the vertical beam spot was very broad. An optimal beam spot was obtained for a value of Q7

Table 1
The difference in current for the experimental quadrupole settings compared to the theoretical settings for 120 MeV/c

quadrupole	difference in percent
Q1	-0.2
Q2	-2.0
Q3	-4.0
Q4	+1.0
Q5	-4.0
Q6	-4.4
Q7	-15.0

Table 2
Poletip fields versus currrent for the M13 magnets

quadrupole	effective length (cm)	poletip radius (cm)	poletip field (gauss per ampere)
Q1	29.286	10.319	6.66
Q2	35.76	15.5575	15.17
Q3	27.432	10.16	8.88
Q4	35.92	15.5575	15.10
Q5	27.432	10.16	8.88
Q6	35.92	15.5575	15.10
Q7	35.92	15.5575	15.10

close to the theoretical value.

Table 2 shows the characteristics of the quadrupoles that were used to calculate the magnet currents. Note that for Q3 and Q5 the conversion factor is 8.88 gauss per ampere. The value in the TRIUMF magnet book is 10.88. However, a crude measurement in 1980 showed that the actual value was closer to 8.0. Comparisons for several past theoretical and experimental tunes gave the value of 8.88 gauss per amp which is used here.

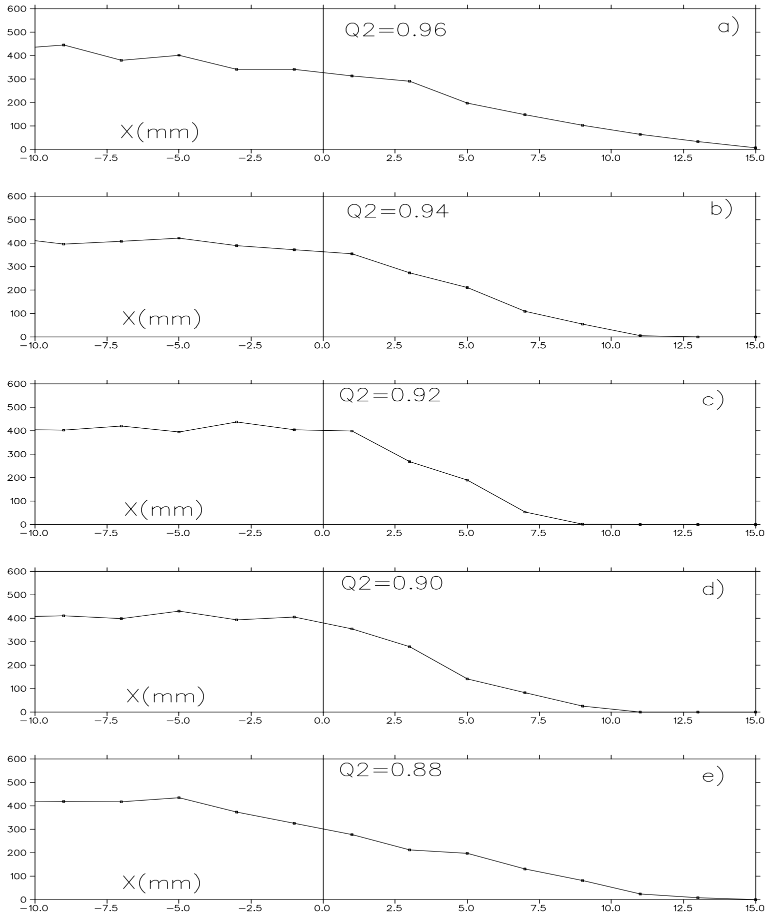

Fig. 3. Horizontal intensity distribution at F1 for 5 values of Q2. In case c the distribution falls sharpest. Therefore, Q2 should be set to 0.92

3.1 Determination of Q2

The momentum slit af F1 is closed to a small value, namely 5 mm full width. The intensity at F3 is measured as function of the slit position. This is done while the beam momentum is set close to the momentum of the surface muon edge at 29.8 MeV/c. The intensity falls about two orders of magnitude above 29.8 MeV/c, where there are only cloud muons. If there is a horizontal focus at F1, the intensity should drop sharply as function of the slit position, because of the precise momentum selection. If there is no focus, the momentum selection is less sharp, which causes the drop to be more gradual. In figure 3 theoretical horizontal intensity distributions are given for five indicated settings of Q2, roughly in steps of 2 percent of the clearly optimum value of Q2 of 0.92 kG. This method of determining Q2 worked well in practice.

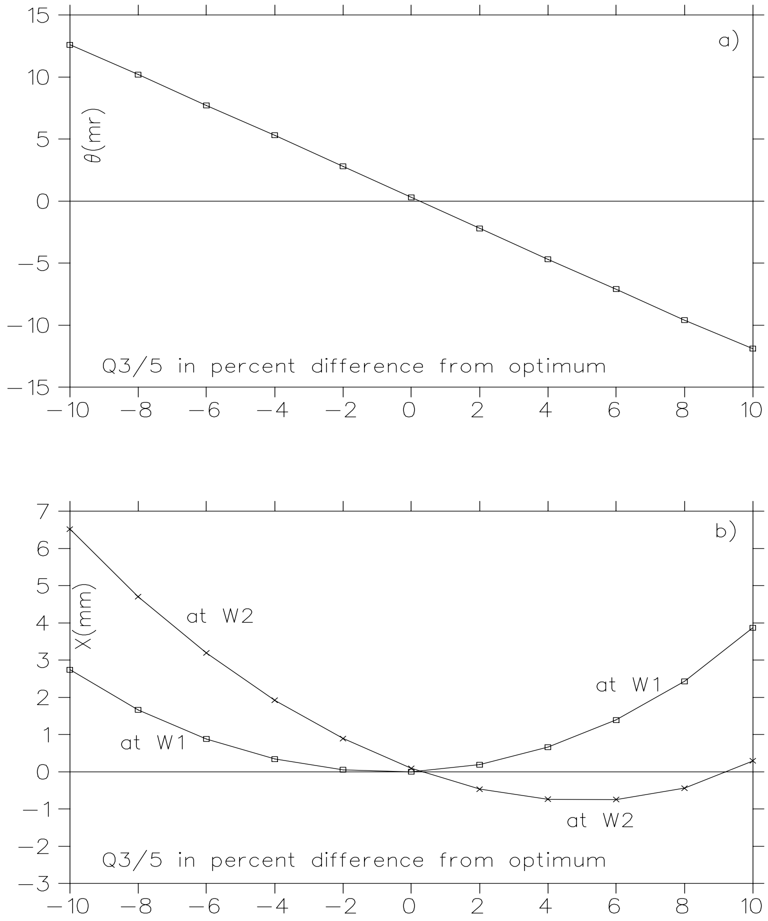

Fig. 4. θ and x at wire chambers 1 and 2 for different values of Q3 and Q5.

3.2 Tuning of the Q3 and Q5 pair

Due to the symmetry in the channel Q3 and Q5 have always the same strength. The purpose of these quadrupoles is to make the beam achromatic in position as well as angle after B2. The matrix elements R_{16} and R_{26} in equations 5 and 6 have to be zero. At the same time the F1 horizontal spot is imaged at F2 with -1 magnification. A F1 slit position of +20 mm from the beam axis corresponds to a momentum of +1.6 percent above the central momentum. If the beam is achromatic, the beam should still be on the axis and in the direction of the axis af F3. Figure 4 gives results for a ± 10 percent range of Q3 and Q5. In this case 120 MeV/c pions are used, because it is then possible to determine the direction of the beam at F3 by using both wire chambers. The top figure shows the change in direction, which is a linear function of Q3 and Q5. The bottom function shows the position of the beam centroid at wire chamber W1 at F3, and at W2, 30 cm downstream of F3. It is, in practice, not so easy to see a change in position, especially since at W2 the beam is close to the axis for the whole 10 percent above the optimum setting. It is better to look at change in direction which corresponds to the difference in position between W1 and W2. During the tuning, checks were done for only a few settings of Q3 and Q5. It is better to do a more systematic study as indicated in figure 4a. However, an other indication of the correct quadrupole settings is that there is a horizontal focus at F2. Therefore, the slit at F1 was positioned on the axis with a width of 10 mm. It turned out that 95 percent of the intensity passed through a 12 mm wide slit at F2, a good indication, that the beam is almost focussed at F2.

3.3 The value of Q1

The purpose of Q1 is to make a vertical focus at F2. However, there is no vertical slit at F2, and this made it impossible to determine Q1 in a direct way. Two indirect methods were tried. Firstly, an attempt was made to focus the beam vertically in Q4. In that case, changing Q4 would not greatly influence the beam at F3. In practice, it was not easy to determine this effect. Therefore, a second method was applied. There is an absorber wheel, 15 cm downstream of F1, containing a horizontally wide slot with a 1 cm high vertical aperture. There, the beam was focussed vertically, by determining the value of Q1 for which the transmission of intensity through the aperture was a maximum. This is a slightly tricky method, because the jaws were closed to accept only about ± 25 mr initial vertical angle. If Q1 is changed, the beam size at the jaws is also changed, resulting in a higher or lower intensity arriving at the hole. Calculations give a 1 percent downward correction for that effect. Next, the value of Q1 was reduced by about 8 percent on the basis of theoretical calculations in order to give a vertical focus at F2. The error in the determination of Q1 is, therefore, mainly the error in that extrapolation.

3.4 Investigation of Q4, together with Q7

The purpose of Q4 is to make the vertical angle ϕ at F3 independent of the initial vertical source size. In other words, the matrix element R_{43} in equation 8 has to be small. There is no direct way of doing this. Instead, for about a ± 10 percent range of values for Q4 a waist, an upright ellipse in the $y - \phi$ plane, was established with Q7. Theoretical results are given in figure 5. The top graph gives the value of Q7 as function of Q4. The middle part gives the FWHM for ϕ, and the bottom part gives the FWHM for y at F3 in wire chamber W1, and in W2. The product of that y and ϕ is constant, because the phase space area of the beam is conserved, and for a waist that area is just equal to that product. The zero on the horizontal axis corresponds to the value of Q4 for which R_{43} is zero. This is close to the maximum in y and the

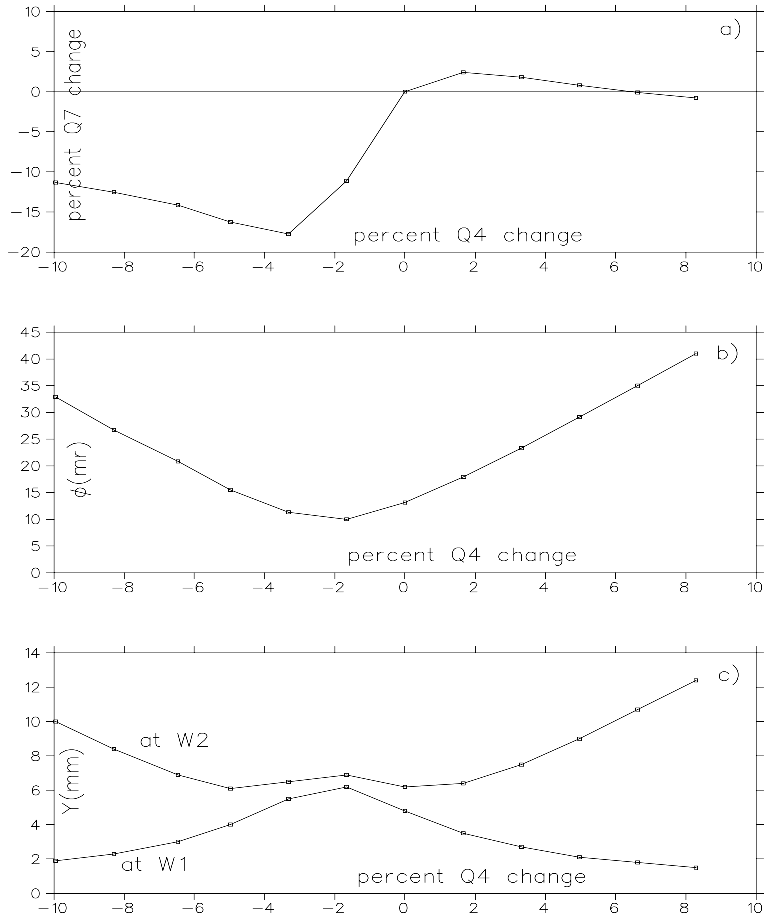

Fig. 5. ϕ and y at wire chambers W1 and W2 for different values of Q4, after adjusting Q7 to give a vertical waist at W1. Figure a shows how Q7 varies with Q4

minimum in ϕ. During the experiment the waist was found for various values of Q4, from which the experimentally optimum value for Q4 was interpolated. Examples for vertical waists are given in figure 6 for the optimum value of Q4 and for ± 5 percent. In agreement with figure 5 , the divergence is large and the beam size is small for the latter two cases. It is worthwhile to note, that it is not the aim to make the smallest beam size, but to establish the optimum aspect ratio for the waist that is injected into the solenoid. This will mainly been done by using the vertical jaws, and a vertical slit which will be installed at F2. However, varying Q4 together with Q7, helps also a lot. Therefore, the requirement that R_{43} is zero can be relaxed to some extent to obtain additional flexibility. Figure 7 shows the phase space plot for the same Q4 and Q7 settings as in figure 6 but for a six times larger initial vertical divergence, namely ± 150 mr, instead of ± 25 mr. As stated at the end of subsection 2.2 there is only a waist for one particular value of the initial divergence for given quadrupole settings. Figure 7 shows that indeed there is not longer a waist. All three phase space plots are slanted. It would be necessary to adjust Q7 to again obtain a waist. Furthermore, the ellipses are now distorted by third

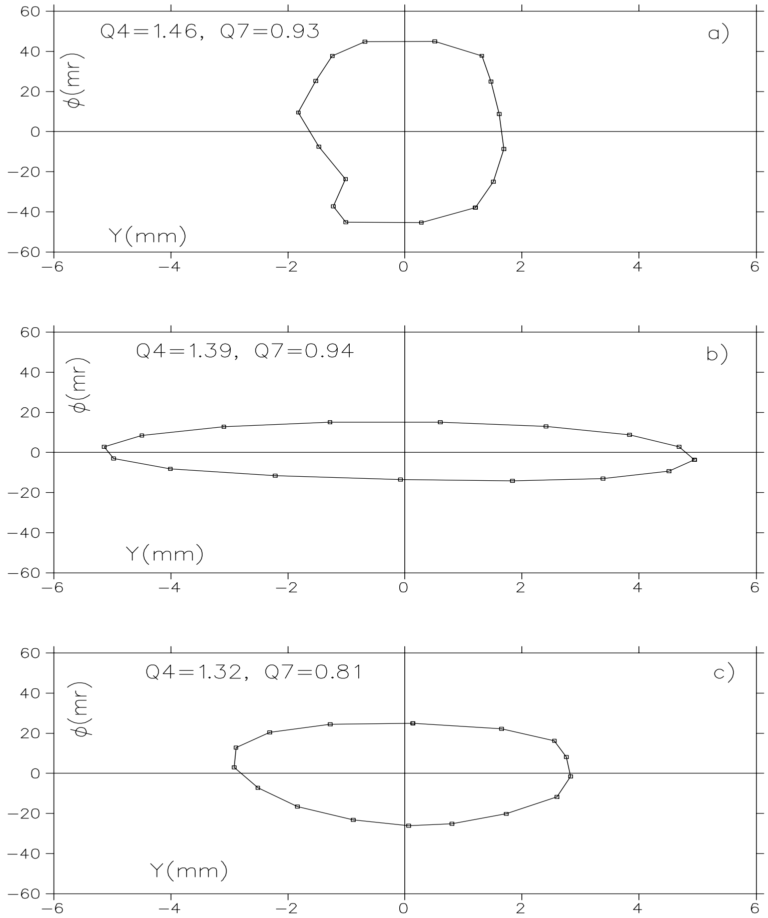

Fig. 6. The vertical phase space ellipse at F3 for 3 combinations of settings of Q4 and Q7. The half axes of the initial ellipse are 3.0 mm for y and 25 mr for ϕ

order aberrations. This is most clear in the top figure, and to a lesser extent in the bottom figure.

References

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[2] Optimization of the M13 Surface Muon Beam for Experiment TWIST, and its Injection onto the Solenoid, Jaap Doornbos, TRIUMF, August 3, 2000. http://www.triumf.ca/people/doornbos/homepage.html

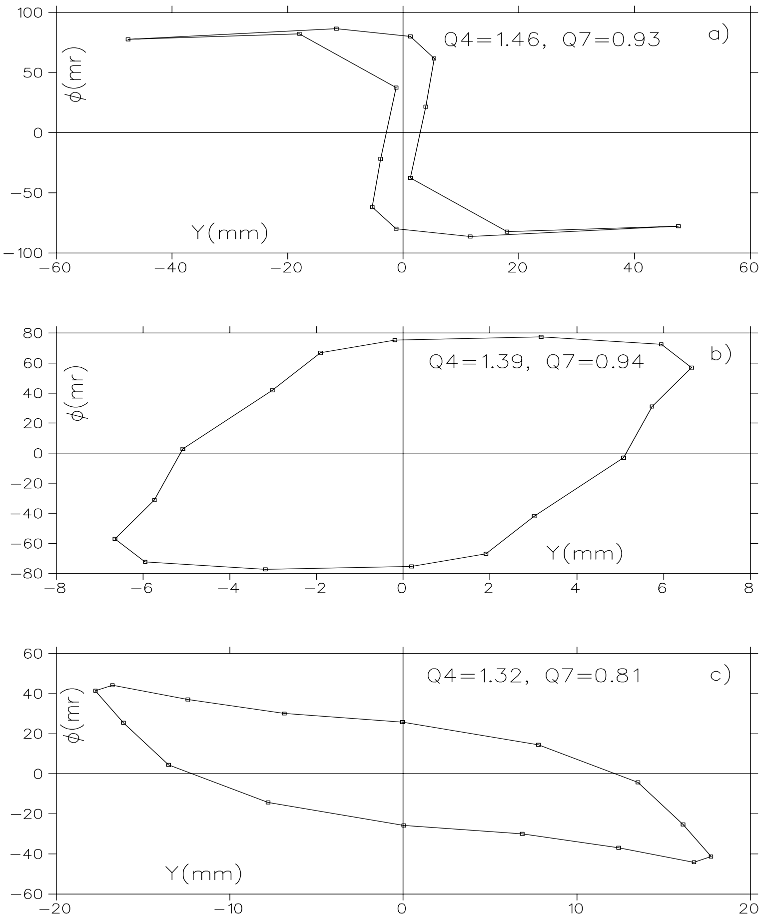

Fig. 7. The vertical phase space ellipse at F3 for 3 combinations of settings of Q4 and Q7. The half axes of the initial ellipse are 3.0 mm for y and 150 mr for ϕ