TRIUMF Experiment E614
Technical Note No. 50

Muon polarisation in the $\pi_{\mu 2}$ decay

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1 Introduction

Formulae are given for the kinematics and the muon polarization in the muonic decays $\pi^+ \to \mu^+\nu_\mu$, $\pi^- \to \mu^-\nu_\mu$ and the corresponding kaon decays, at rest or in flight. The same formulae can be used for the electronic decays.

This work was undertaken in the framework of another experiment (KEK-E246 experiment, "Search for a transverse polarization of the muon in the $K^+ \to \pi^0\mu^+\nu$ decay"), following extensive discussions with Peter Gumplinger. It will also be useful for E614 since this experiment makes use of muons from $\pi^+ \to \mu^+\nu$ decays (either from rest or in flight).

As an application, the effect of thermal motion on the muon polarization is evaluated.

2 The kinematics

Since the discussion of the polarization of the muon involves the understanding of the kinematics, the latter will also be discussed in detail.

2.1 Notations

The notations used in this report are given in Table 1. The notations for the angles $\theta_{\pi\mu}$ and $\theta_{\mu\nu}$ are self-explanatory.

Particle	Pion	Muon	Neutrino
Mass	m_π	m_μ	0
Energy-momentum 4-vector	$\tilde{p}_\pi$	$\tilde{p}_\mu$	$\tilde{p}_\nu$
Total energy	E_π	E_μ	E_ν
Momentum 3-vector	$\vec{p}_\pi$	$\vec{p}_\mu$	$\vec{p}_\nu$
Velocity	$\vec{v}_\pi$	$\vec{v}_\mu$	$\vec{v}_\nu$
Momentum (modulus)	p_π	p_μ	p_ν
Velocity (modulus)	v_π	v_μ	c
Polarization 4-vector	n.a.	$\tilde{s}_\mu$	not used
Polarization 3-vector	n.a.	$\vec{\xi}$	not used

Table 1: Notations used for the kinematics and polarization

2.2 Angular distribution for the muon

Energy-momentum conservation:

$$\tilde{p}_\pi = \tilde{p}_\mu + \tilde{p}_\nu \tag{1}$$

leads to:

$$m_\pi^2 + m_\mu^2 - 2E_\pi E_\mu + 2\vec{p}_\pi \cdot \vec{p}_\mu = 0 \tag{2}$$

and to the second degree equation ($ap_\mu^2 + 2b'p_\mu + c = 0$):

$$4p_\mu^2[E_\pi^2 - p_\pi^2 \cos^2\theta_{\pi\mu}] - 4p_\mu(m_\pi^2 + m_\mu^2)p_\pi \cos\theta_{\pi\mu} + 4m_\mu^2 p_\pi^2 - (m_\pi^2 - m_\mu^2)^2 = 0 \tag{3}$$

with:

$$a = 4(E_\pi^2 - p_\pi^2 \cos^2\theta_{\pi\mu}) \tag{4}$$

$$b' = -4m_\mu E_0 p_\pi \cos\theta_{\pi\mu} \tag{5}$$

$$c = 4m_\mu^2 p_\pi^2 - (m_\pi^2 - m_\mu^2)^2 = 4m_\mu^2(p_\pi^2 - p_0^2) \tag{6}$$

where:

$$p_0 = \frac{m_\pi^2 - m_\mu^2}{2m_\mu} = 39.35 \text{ MeV/c} \tag{7}$$

The determinant is:

$$\Delta' = 16m_\mu^2 E_\pi^2(p_0^2 - p_\pi^2 \sin^2\theta_{\pi\mu}) \tag{8}$$

$\vec{p}_0$ is the transition momentum. The corresponding total energy is:

$$E_0 = \frac{m_\pi^2 + m_\mu^2}{2m_\mu} = 145.01 \text{ MeV} \tag{9}$$

and the corresponding kinetic energy:

$$T_0 = \frac{(m_\pi - m_\mu)^2}{2m_\mu} = 5.44 \text{ MeV} \tag{10}$$

If there are roots, their product has the sign of c/a, hence the sign of $p_\pi - p_0$, and their sum has the sign of $(-b'/a)$, hence the sign of $\cos\theta_{\pi\mu}$ (note that a is always positive).

There are three cases (note that $\sin\theta_{\pi\mu}$ is positive by definition):

- $p_\pi < p_0$

  There is no restriction on the angle $\theta_{\pi\mu}$ since:

  $$\sin\theta_{\pi\mu} \leq 1 < \frac{p_0}{p_\pi} \tag{11}$$

  There are two roots of opposite sign, one positive and one negative. Only the positive root has physical significance.

  $$p_\mu = m_\mu \frac{E_0 p_\pi \cos\theta_{\pi\mu} + E_\pi \sqrt{p_0^2 - p_\pi^2 \sin^2\theta_{\pi\mu}}}{E_\pi^2 - p_\pi^2 \cos^2\theta_{\pi\mu}} \tag{12}$$

  For a given value of $\theta_{\pi\mu}$ there is only one value for the muon momentum. The muon can be emitted backward as well as forward.

- $p_\pi > p_0$

  All muons are emitted forward ($\cos\theta_{\pi\mu}$ is always positive):

  $$2E_\pi E_\mu - m_\pi^2 - m_\mu^2 > 2m_\pi \frac{(m_\pi^2 + m_\mu^2)}{2m_\mu} - (m_\pi^2 + m_\mu^2) = (m_\pi^2 + m_\mu^2)\left(\frac{m_\pi}{m_\mu} - 1\right) > 0 \tag{13}$$

  There is a limitation on the angle $\theta_{\pi\mu}$:

  $$\sin\theta_{\pi\mu} < \frac{p_0}{p_\pi} \qquad \theta_{\pi\mu} < \theta_0 \tag{14}$$

  where:

  $$\sin\theta_0 = \frac{p_0}{p_\pi} \tag{15}$$

  There are two positive roots. Both have physical significance.

  $$p_\mu = m_\mu \frac{E_0 p_\pi \cos\theta_{\pi\mu} \pm E_\pi \sqrt{p_0^2 - p_\pi^2 \sin^2\theta_{\pi\mu}}}{E_\pi^2 - p_\pi^2 \cos^2\theta_{\pi\mu}} \tag{16}$$

  The muon is emitted in the forward direction and for a given value of $\theta_{\pi\mu}$ there are two values of the muon momentum.

- $p_\pi = p_0$

  The roots are given by:

  $$p_\mu = 0 \tag{17}$$

  and:

  $$p_\mu = 2m_\mu \frac{E_0 p_0 \cos\theta_{\pi\mu}}{E_0^2 - p_0^2 \cos^2\theta_{\pi\mu}} = 2m_\mu \frac{(m_\pi^2 + m_\mu^2)(m_\pi^2 - m_\mu^2)\cos\theta_{\pi\mu}}{(m_\pi^2 + m_\mu^2)^2 - (m_\pi^2 - m_\mu^2)^2 \cos^2\theta_{\pi\mu}} \tag{18}$$

2.3 Angular distribution for the neutrino

Energy-momentum conservation gives:

$$2(E_\pi E_\nu - \vec{p}_\pi \cdot \vec{p}_\nu) = m_\pi^2 - m_\mu^2 \tag{19}$$

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos\theta_{\pi\nu})} \tag{20}$$

with the limiting values:

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi^2}(E_\pi + p_\pi) \quad \text{for} \quad \theta = 0 \tag{21}$$

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi^2}(E_\pi - p_\pi) \quad \text{for} \quad \theta = \pi \tag{22}$$

There is no restriction on the angle $\theta_{\pi\nu}$.

2.4 Figures

- Figure 1: muon momentum vs pion-muon angle for five values of the pion momentum, below the critical momentum.
- Figure 2: muon kinetic energy vs pion-muon angle for five values of the pion momentum, below the critical momentum.
- Figure 3: muon momentum vs pion-muon angle for five values of the pion momentum, above the critical momentum.
- Figure 4: muon kinetic energy vs pion-muon angle for five values of the pion momentum, above the critical momentum.
- Figure 5: muon momentum vs pion-muon angle. The pion momentum is equal to the critical momentum.
- Figure 6: muon kinetic energy vs pion-muon angle. The pion momentum is equal to the critical momentum.
- Figure 7: muon momentum vs pion-muon angle. The transition across the critical momentum.
- Figure 8: muon kinetic energy vs pion-muon angle. The transition across the critical momentum.

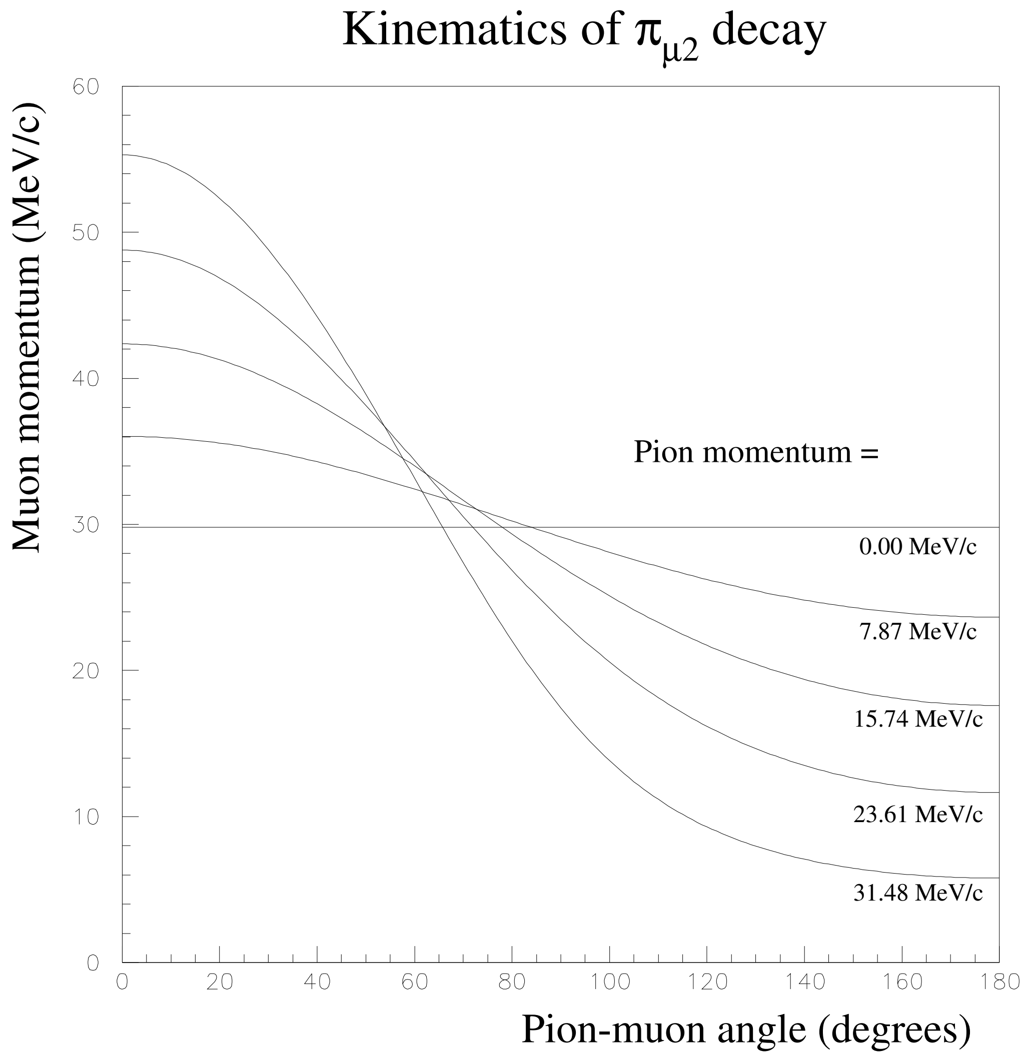

Figure 1: The muon momentum as a function of the pion-muon angle. The pion momentum is below the critical momentum. There is no limiting angle, the pion-muon angle ranging from 0 to 180 degrees. For every pion-muon angle there is only one muon momentum.

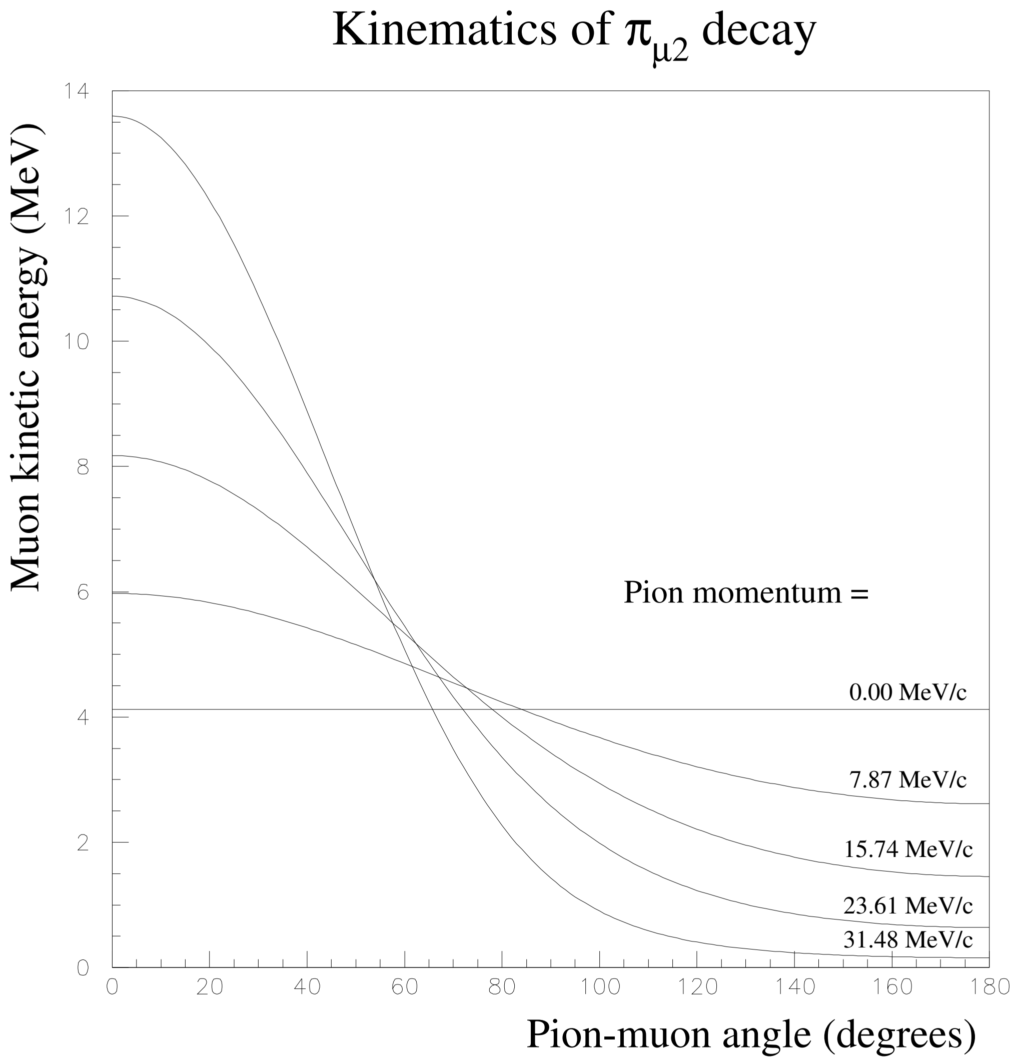

Figure 2: The muon kinetic energy as a function of the pion-muon angle. The pion momentum is below the critical momentum. There is no limiting angle, the pion-muon angle ranging from 0 to 180 degrees. For every pion-muon angle there is only one muon kinetic energy.

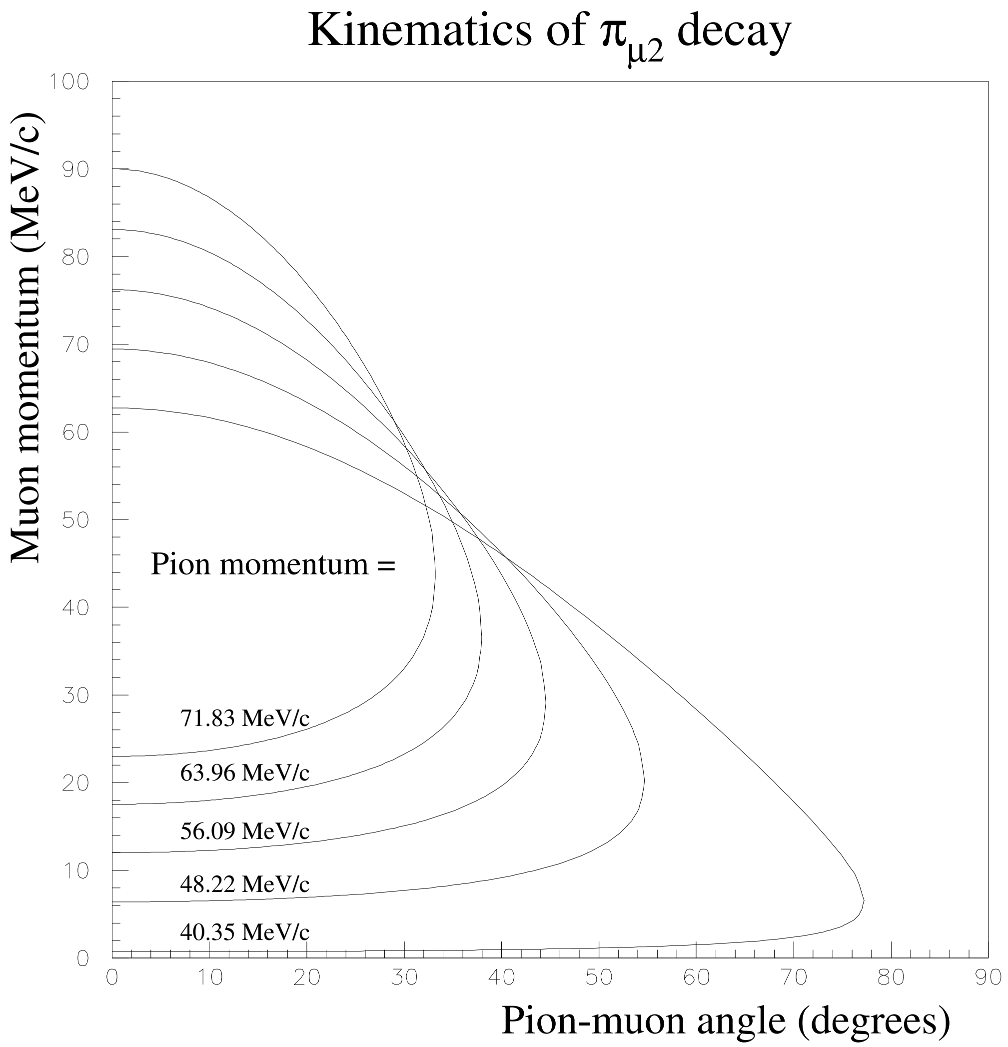

Figure 3: The muon momentum as a function of the pion-muon angle. The pion momentum is above the critical momentum. There is a limiting angle, the pion-muon angle ranging from 0 to a certain maximum. For every pion-muon angle there are two muon momenta.

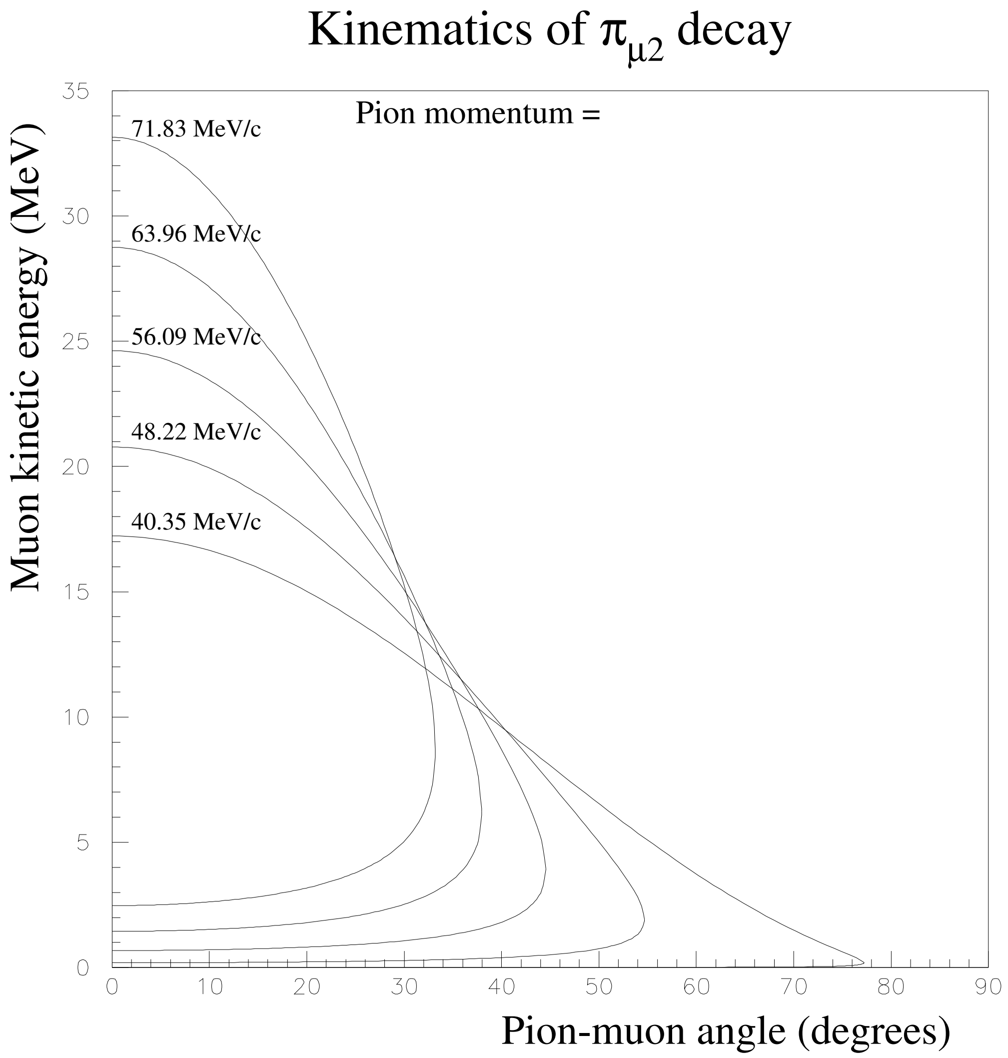

Figure 4: The muon kinetic energy as a function of the pion-muon angle. The pion momentum is above the critical momentum. There is a limiting angle, the pion-muon angle ranging from 0 to a certain maximum. For every pion-muon angle there are two muon kinetic energies.

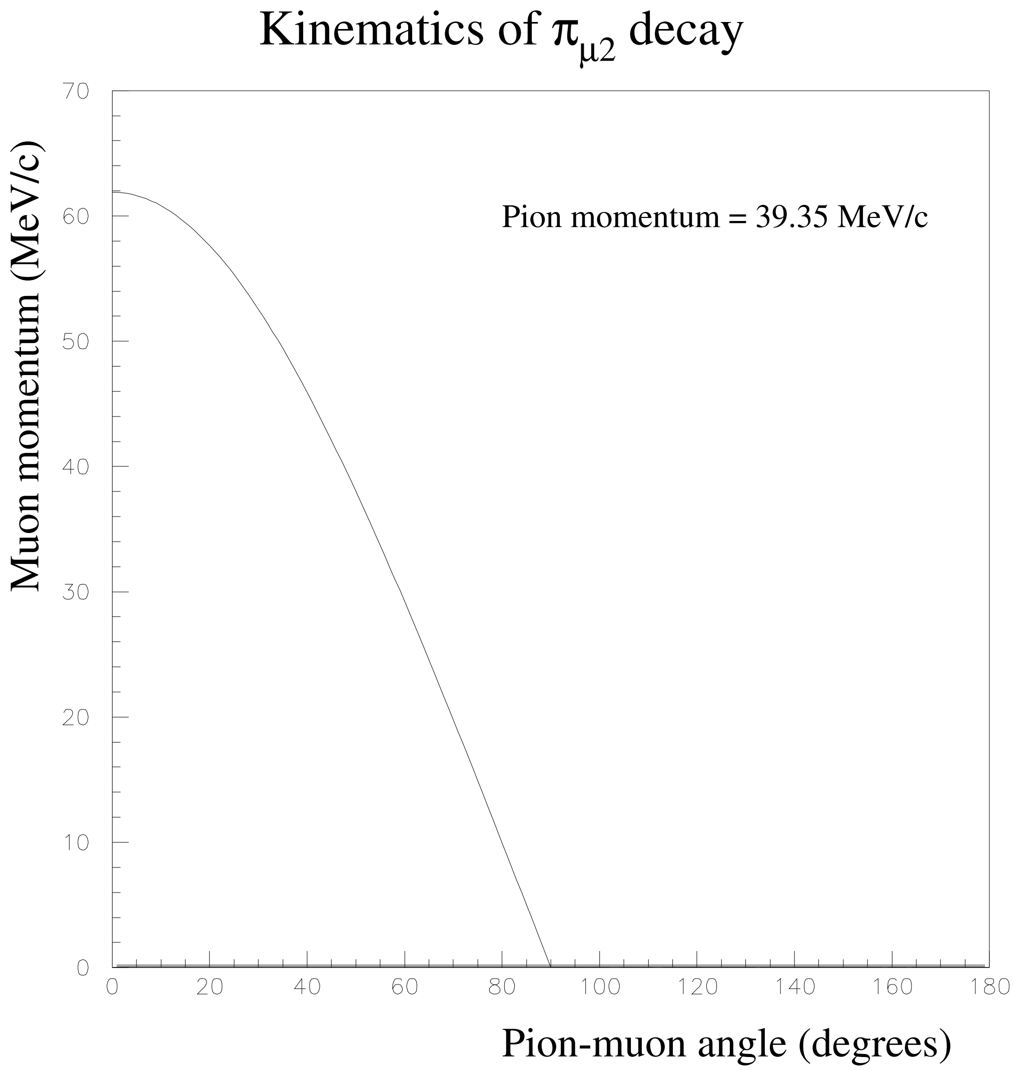

Figure 5: The muon momentum as a function of the pion-muon angle. The pion momentum is equal to the critical momentum. There is a limiting angle, the pion-muon angle ranging from 0 to 90 degrees. For every pion-muon angle there are two muon momenta; one of them is equal to zero: in this case the pion-muon angle is undetermined (any value from 0 to 180 degrees.

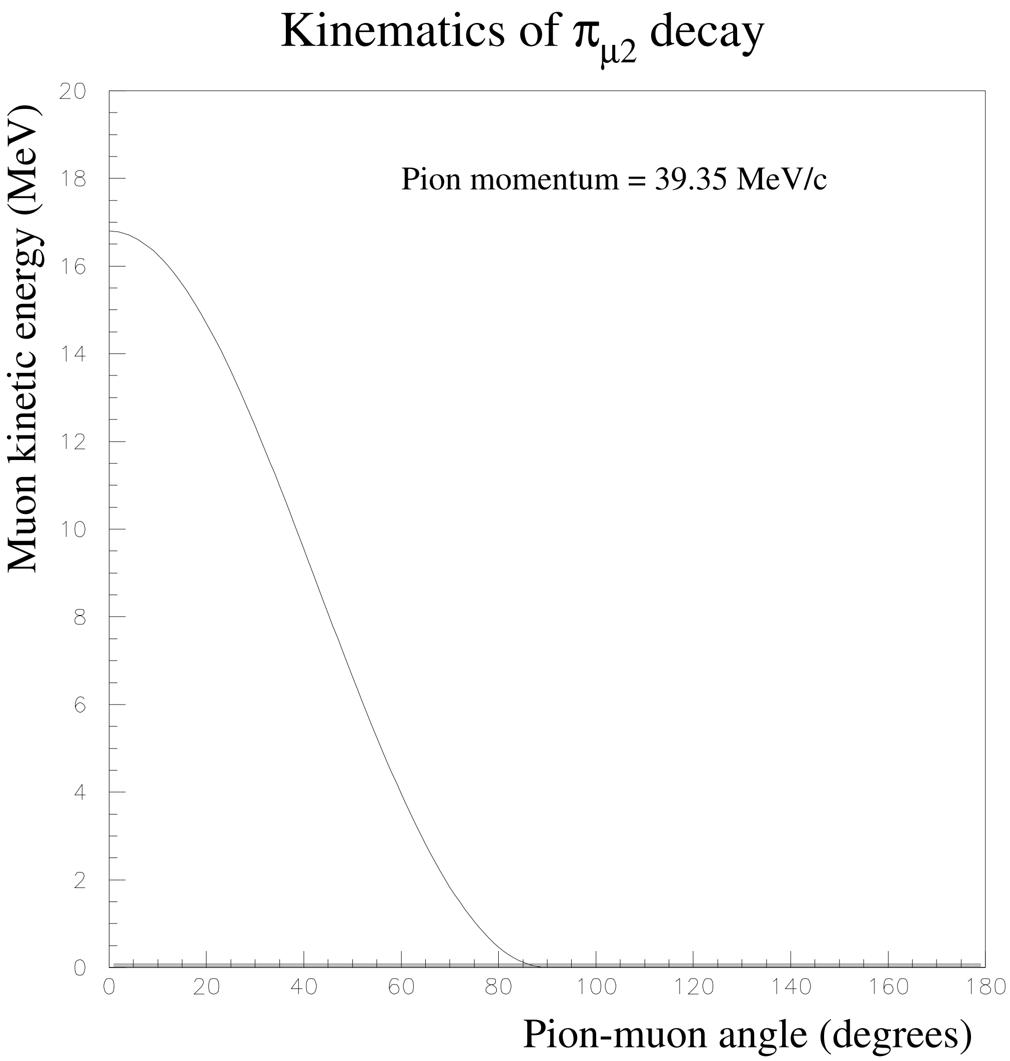

Figure 6: The muon kinetic energy as a function of the pion-muon angle. The pion momentum is equal to the critical momentum. There is a limiting angle, the pion-muon angle ranging from 0 to 90 degrees. For every pion-muon angle there are two muon kinetic energies; one of them is equal to zero: in this case the pion-muon angle is undetermined (any value from 0 to 180 degrees).

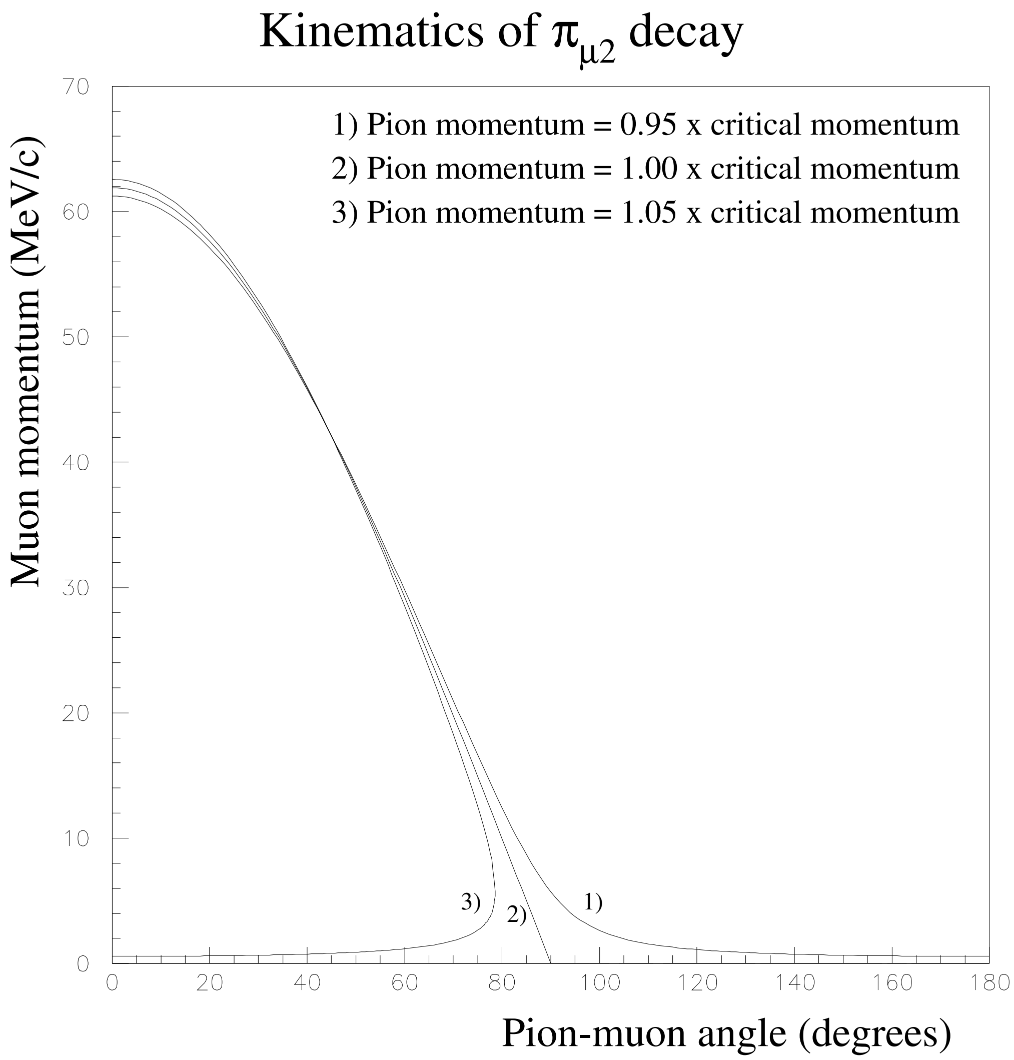

Figure 7: The muon momentum as a function of the pion-muon angle. This figure shows the transition across the critical momentum.

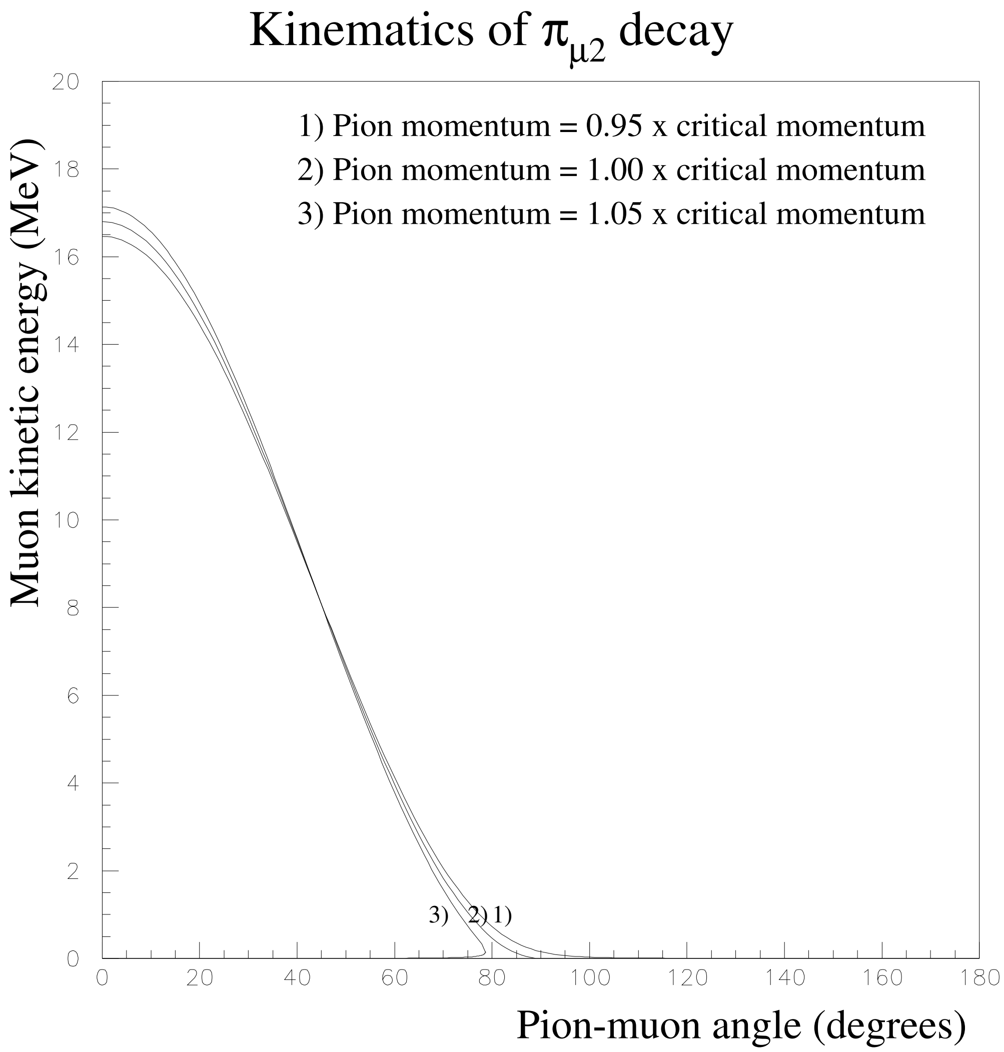

Figure 8: The muon kinetic energy as a function of the pion-muon angle. This figure shows the transition across the critical momentum.

2.5 The maximum π-μ angle vs. the pion momentum

Figure 9 shows the variation of the π-μ angle as a function of the pion momentum.

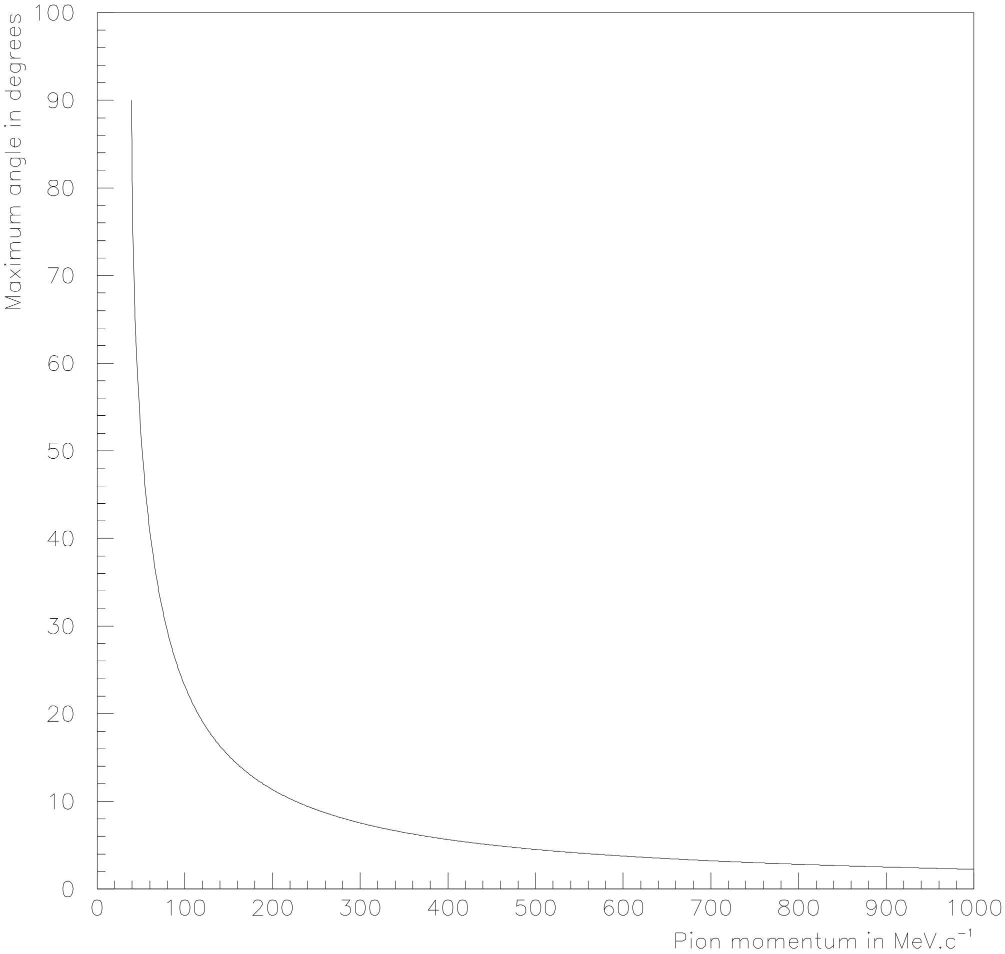

Figure 9: The maximum π-μ angle vs. the pion momentum

3 Distribution of the π-μ laboratory angle

An analytical calculation is possible, but tedious. It is simpler to use a Monte-Carlo calculation.

First the cosine of the C.M. π-μ angle is chosen at random between -1 and $+1$ (Figure 10). Then the C.M. π-μ angle is obtained (Figure 11). The components of the C.M. muon momentum are:

$$\begin{aligned} P_x^* &= P^* \cos\theta^* \\ P_y^* &= P^* \sin\theta^* \end{aligned} \tag{23}$$

where:

$$P^* = \frac{M_\pi^2 - M_\mu^2}{2M_\pi} = 29.8 \text{ MeV/c} \tag{24}$$

and the corresponding total energy:

$$E^* = \frac{M_\pi^2 + M_\mu^2}{2M_\pi} = 109.8 \text{ MeV/c} \tag{25}$$

P_x^* and P_y^* are shown in Figures 12 and 13.

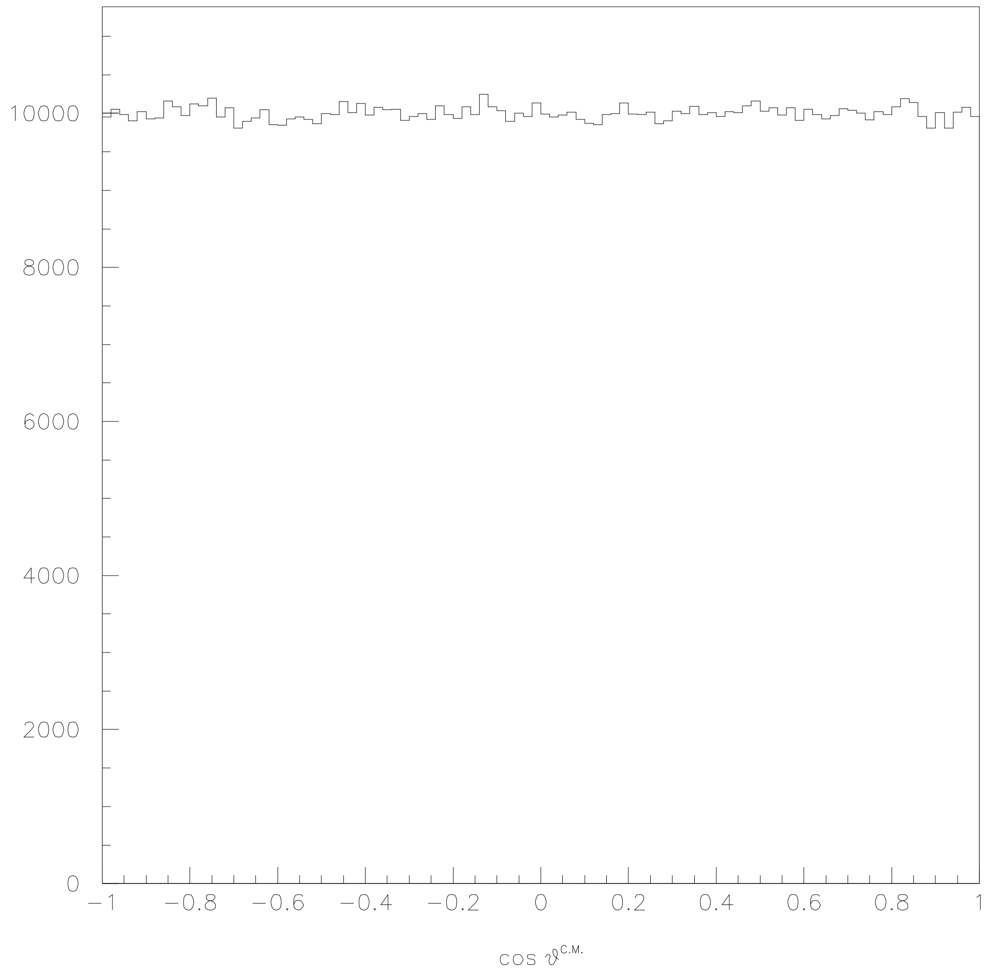

Figure 10: π C.M. Cosine of π-μ angle.

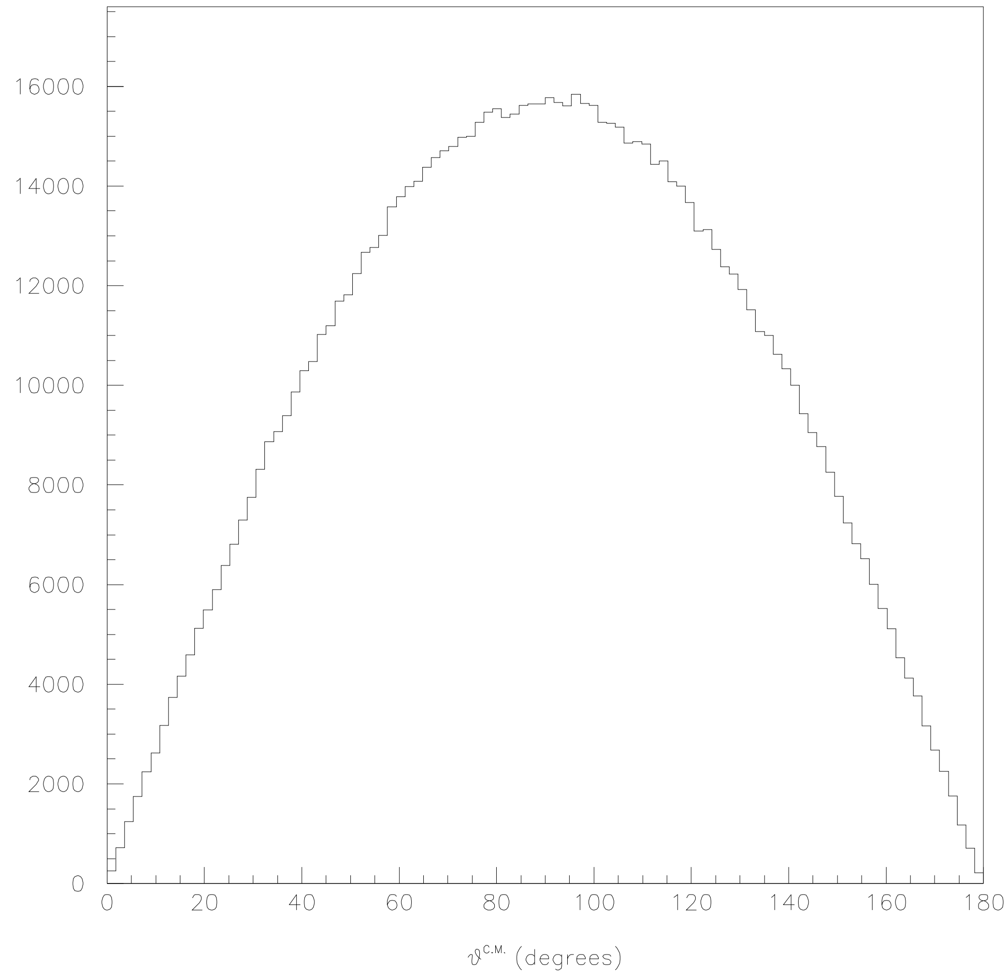

Figure 11: π C.M. π-μ angle (in degrees).

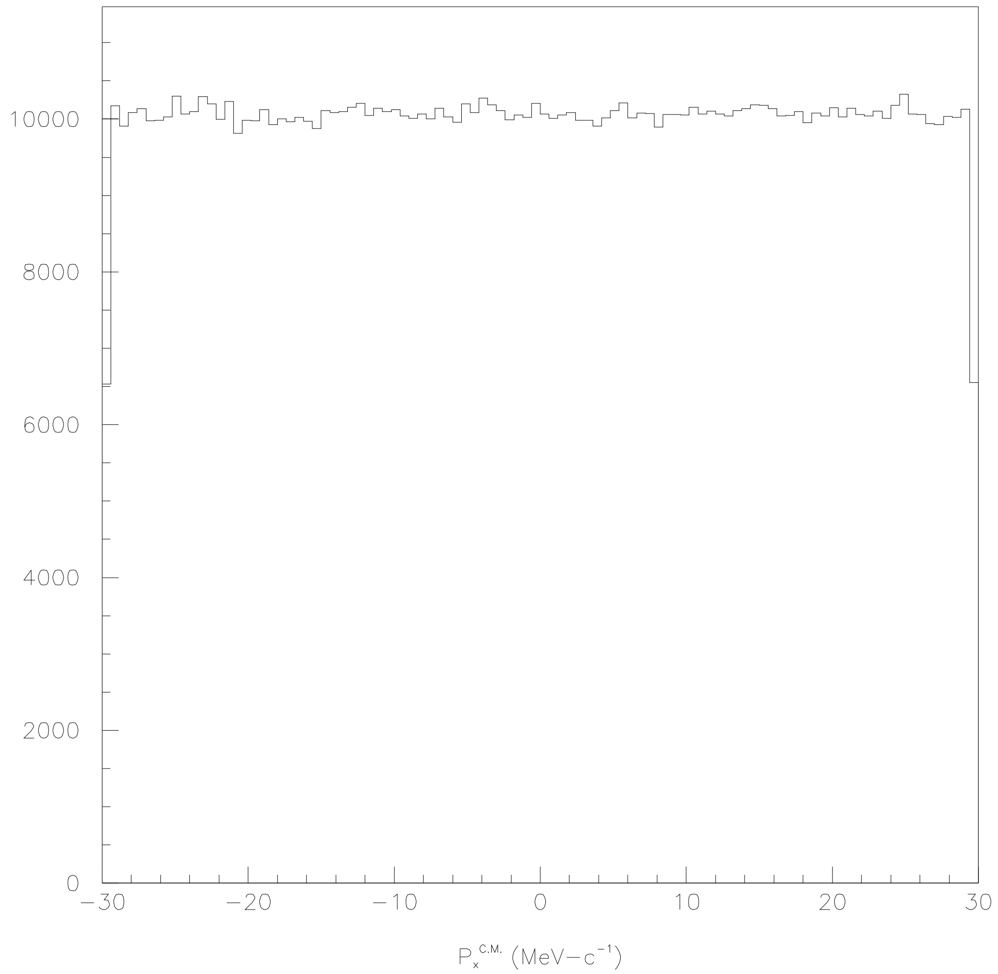

Figure 12: π C.M. P_x component of muon momentum.

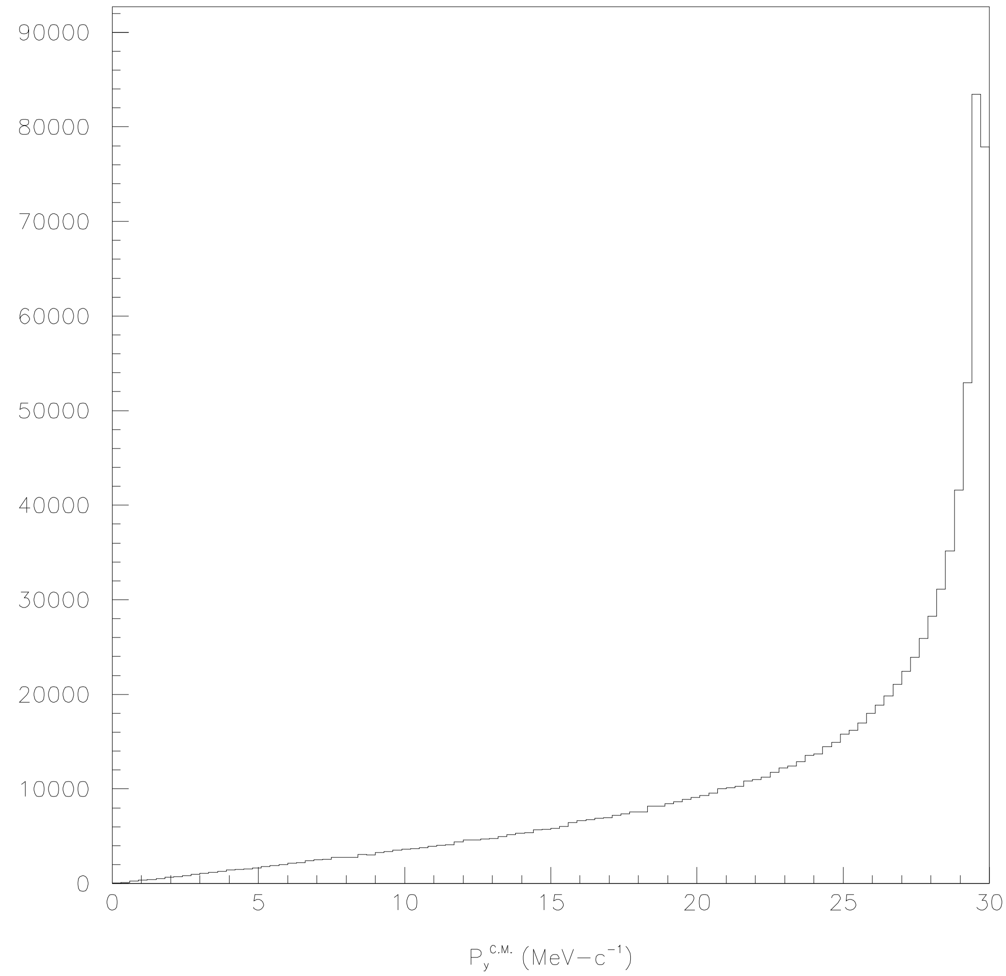

Figure 13: π C.M. P_y component of muon momentum.

The components of the LAB. muon momentum are obtained by applying a Lorentz boost:

$$\begin{aligned} P_x &= \gamma P_x^* + \beta E^* \\ P_y &= P_y^* \end{aligned} \tag{26}$$

As an example, let us consider the $K^+ \rightarrow \pi^+\pi^0$ decay. The momentum of the π^+ is 205.1 MeV/c, $\beta = 0.827$, $\gamma = 1.78$.

The components of the muon momentum are shown in Figures 14 and 15. The LAB. angle is given by:

$$\theta = \tan^{-1}(P_y/P_x) \tag{27}$$

and shown in Figure 16.

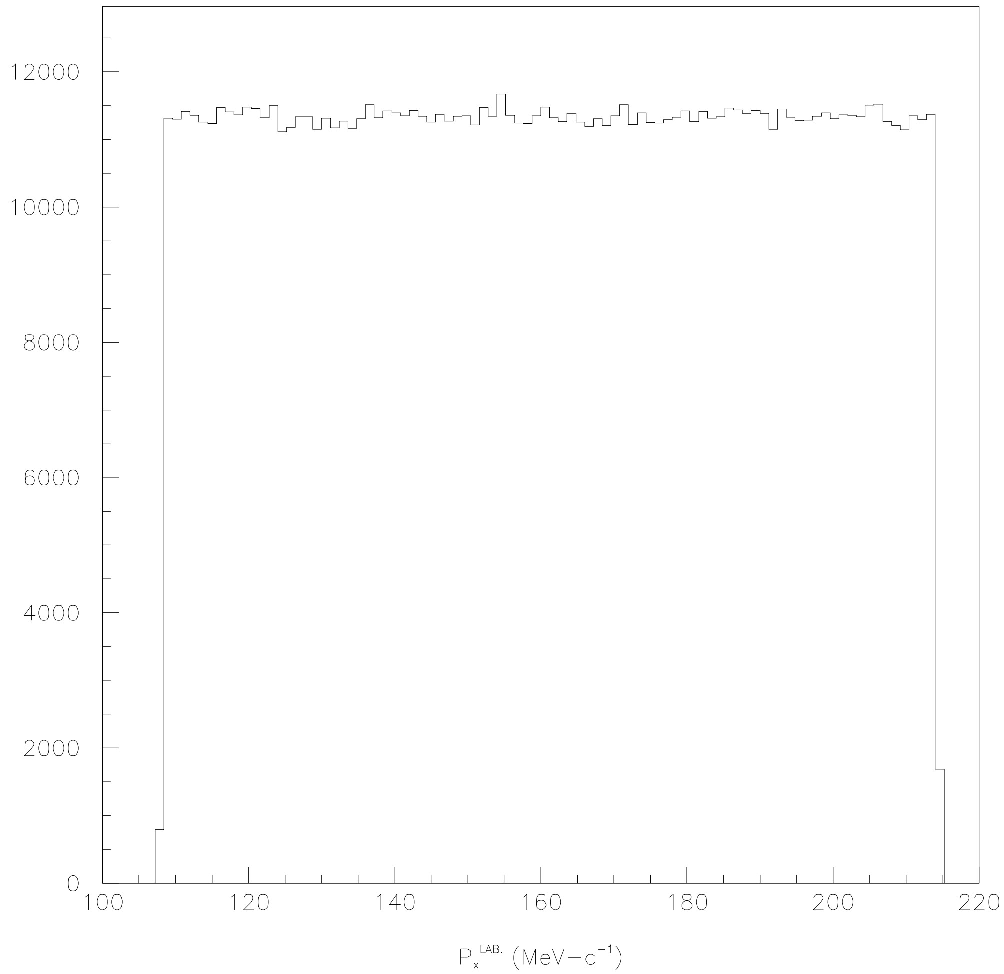

Figure 14: LAB. P_x component of muon momentum.

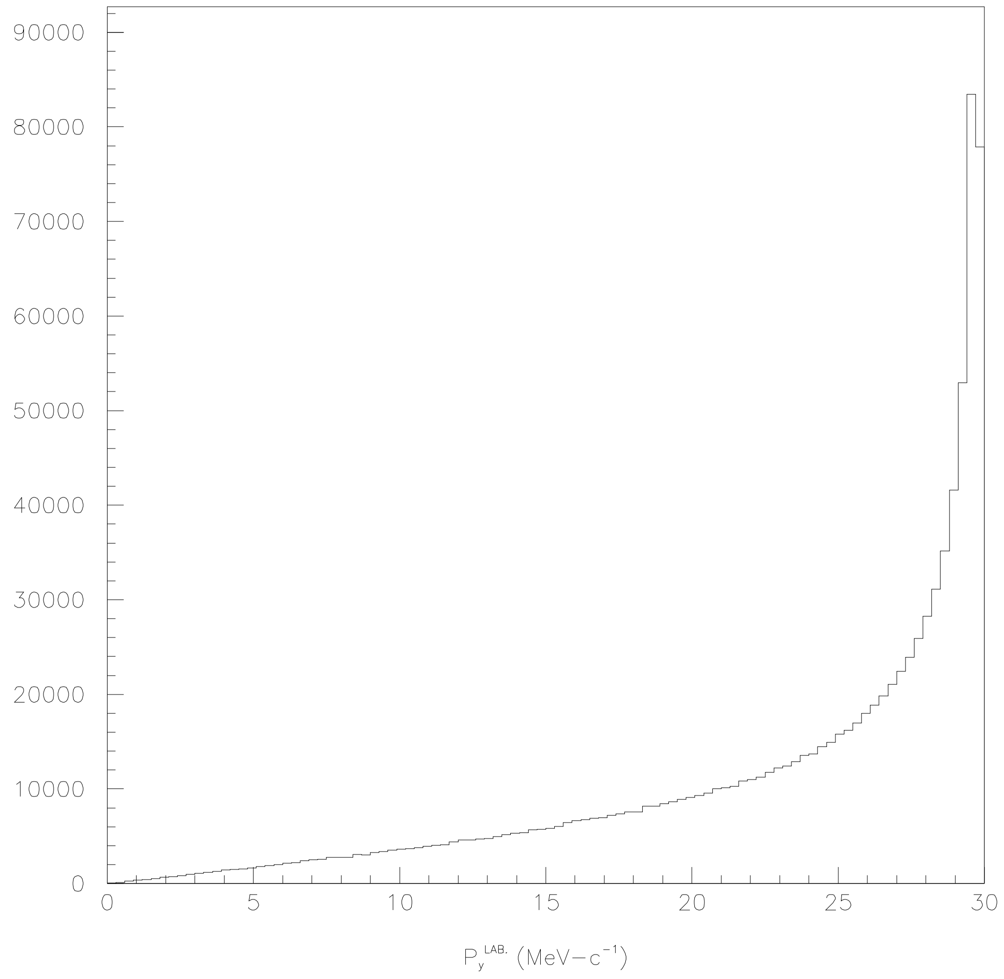

Figure 15: LAB. P_y component of muon momentum.

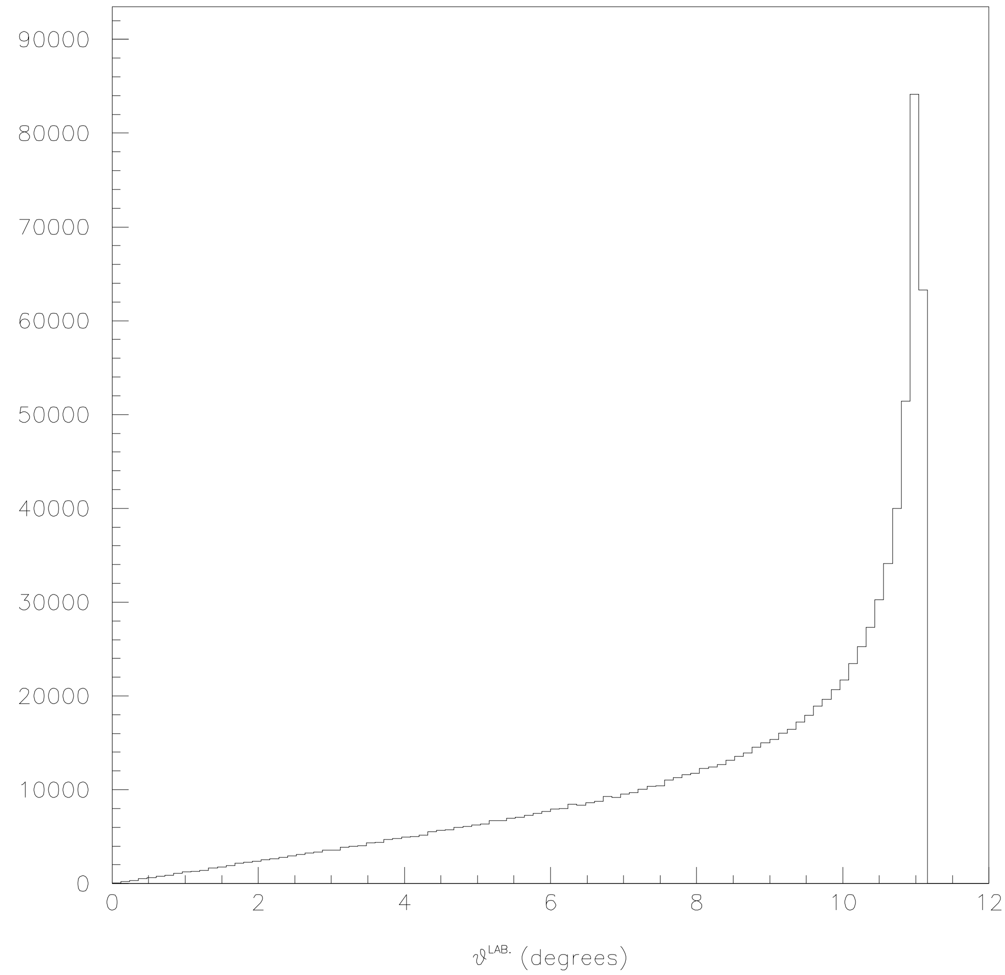

Figure 16: LAB. π-μ angle (in degrees).

4 The polarization vector of the muon

In $V - A$ theory the matrix element is:

$$M =< 0|J_V^\alpha + J_A^\alpha|\pi > [\overline{u}_\mu\gamma_\alpha(1-\gamma_5)u_\nu] = F(q^2)m_\mu[\overline{u}_\mu(1-\gamma_5)u_\nu] \tag{28}$$

$$F(q^2) = F(m_\pi^2) = F_\pi \tag{29}$$

We have to calculate the trace:

$$M^2 = F_\pi^2 m_\mu^2 \; Tr[(1-\gamma_5)u_\nu\overline{u}_\nu(1+\gamma_5)u_\mu\overline{u}_\mu] \tag{30}$$

We use the projection operators:

$$u_\nu\overline{u}_\nu = \not{p}_\nu \qquad u_\mu\overline{u}_\mu = (-\eta m_\mu + \not{p}_\mu)(1+\gamma_5 \not{s}_\mu) \tag{31}$$

with the convention:

$$\eta = +1 \quad \text{for } \mu^+ \text{ decay} \tag{32}$$

$$\eta = -1 \quad \text{for } \mu^- \text{ decay} \tag{33}$$

$$M^2 = 8F_\pi^2 m_\mu^2[(p_\nu \cdot p_\mu) - \eta m_\mu(p_\nu \cdot s_\mu)] \tag{34}$$

One has:

$$(\tilde{p}_\nu \cdot \tilde{p}_\mu) = E_\nu(E_\mu - p_\mu\cos\theta_{\mu\nu}) = \frac{m_\pi^2 - m_\mu^2}{2} = m_\mu p_0 \tag{35}$$

$$m_\mu(\tilde{p}_\nu \cdot \tilde{s}_\mu) = E_\nu(\vec{\xi}\cdot\vec{p}_\mu) - \vec{p}_\nu . \left[m_\mu\vec{\xi} + (\vec{\xi}\cdot\vec{p}_\mu)\frac{\vec{p}_\mu}{E_\mu + m_\mu}\right] \tag{36}$$

The polarization vector is given by:

$$\vec{P} = \eta\frac{1}{m_\mu p_0}\left\{-\left[E_\nu - \frac{\vec{p}_\mu\cdot\vec{p}_\nu}{E_\mu + m_\mu}\right]\vec{p}_\mu + m_\mu\vec{p}_\nu\right\} \tag{37}$$

It is easy to check that $(\vec{P})^2 = 1$.

In the rest system of the pion:

$$\vec{p}_\nu = -\vec{p}_\mu \quad \text{and} \quad p_\mu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \tag{38}$$

This gives:

$$\vec{P} = -\eta\vec{n}_\mu \tag{39}$$

where $\vec{n}_\mu$ is a unit vector along the muon momentum.

The longitudinal polarisation is 100%. Since it has the negative sign for a μ^+, or a positive sign for a μ^-, the decay is helicity-suppressed.

In the general case the longitudinal polarisation is:

$$P_L = \eta\vec{P}\cdot\frac{\vec{p}_\mu}{p_\mu} = \frac{1}{m_\mu p_0}E_\nu(E_\mu\cos\theta_{\mu\nu} - p_\mu) \tag{40}$$

where P_L can be positive or negative.

$$\vec{P}_L = \eta \frac{E_\nu}{m_m u p_0} \left[\frac{E_\mu}{p_\mu} \cos\theta_{\mu\nu} - 1 \right] \vec{p}_\mu \qquad P_L^2 = \frac{E_\nu^2}{m_\mu^2 p_0^2} (E_\mu \cos\theta_{\mu\nu} - p_\mu)^2 \tag{41}$$

The transverse polarization is:

$$\vec{P}_T = \vec{P} - \vec{P}_L = \eta \frac{E_\nu}{p_0} \left[-\frac{\vec{p}_\mu}{p_\mu} \cos\theta_{\mu\nu} + \frac{\vec{p}_\nu}{p_\nu} \right] \qquad P_T^2 = \frac{E_\nu^2}{p_0^2} \sin^2\theta_{\mu\nu} \tag{42}$$

The longitudinal polarization is equal to zero if:

$$\cos\theta_{\mu\nu} = \frac{p_\mu}{E_\mu} = \frac{v_\mu}{c} \tag{43}$$

Then the transverse polarization is:

$$P_T^2 = \frac{E_\nu^2}{p_0^2} \frac{m_\mu^2}{E_\mu^2} \tag{44}$$

Using the relation:

$$(E_\mu + E_\nu)^2 - (\vec{p}_\mu + \vec{p}_\nu)^2 = m_\pi^2 \tag{45}$$

one gets:

$$\frac{E_\nu}{E_\mu} = \frac{p_0}{m_\mu} \qquad E_\mu = E_\pi \frac{m_\mu}{E_0} \qquad E_\nu = E_\pi \frac{p_0}{E_0} \tag{46}$$

This leads to:

$$P_T^2 = 1 \tag{47}$$

In this kinematical configuration the muon is transversely polarized. It is interesting to calculate the angle $\theta_{\pi\mu}$ between the pion and the muon:

$$\cos\theta_{\pi\mu} = \frac{2E_\pi E_\mu - m_\pi^2 - m_\mu^2}{2 p_\pi p_\mu} = \sqrt{1 - \frac{p_0^2}{p_\pi^2}} \tag{48}$$

If the condition:

$$p_\pi \sin\theta_0 = p_0 \tag{49}$$

is satisfied, then:

$$\cos\theta_{\pi\mu} = \cos\theta_0 \tag{50}$$

This correspond to the maximum angle for $\theta_{\pi\mu}$.

Finally, we give simple expressions for the polarization vector:

$$\vec{P}_L = \eta \frac{E_\mu \cos\theta_{\mu\nu} - p_\mu}{E_\mu - p_\mu \cos\theta_{\mu\nu}}\ \vec{n}_\mu \qquad \vec{P}_T = \eta \frac{m_\mu}{E_\mu - p_\mu \cos\theta_{\mu\nu}}\ (\vec{n}_\nu - \vec{n}_\mu \cos\theta_{\mu\nu}) \tag{51}$$

where $\vec{n}_\mu$ and $\vec{n}_\nu$ are unit vectors along the directions of the muon and neutrino, respectively.

For $m_\mu = 0$ we get:

$$\vec{P}_L = -\eta \vec{n}_\mu \qquad \vec{P}_T = 0 \tag{52}$$

and the decay is helicity-forbidden.

It is also possible to express the polarization in terms of the angle between pion and muon. Starting from:

$$m_\mu p_0 \vec{P} = -\eta \left[E_\nu - \frac{\vec{p}_\mu \cdot \vec{p}_\nu}{E_\mu + m_\mu} \right] \vec{p}_\mu + m_\mu \vec{p}_\nu \tag{53}$$

with:

$$E_\nu = E_\pi - E_\mu \tag{54}$$

$$\vec{p}_\nu = \vec{p}_\pi - \vec{p}_\mu \tag{55}$$

one finds:

$$\vec{P}_L = \eta \frac{1}{m_\mu p_0} \left[E_\mu p_\pi \cos\theta_{\pi\mu} - E_\pi p_\mu \right] \vec{n}_\mu = \eta \frac{1}{m_\mu p_0} \left[E_\mu p_\pi (\vec{n}_\pi \cdot \vec{n}_\mu) - E_\pi p_\mu \right] \vec{n}_\mu \tag{56}$$

The longitudinal polarization is equal to zero if:

$$\cos\theta_{\pi\mu} = \frac{p_\mu}{E_\mu} \frac{E_\pi}{p_\pi} = \frac{v_\mu}{v_\pi} \tag{57}$$

The transverse polarization is:

$$\vec{P}_T = \eta \frac{p_\pi}{p_0} \left[\vec{n}_\pi - (\vec{n}_\pi \cdot \vec{n}_\mu) \vec{n}_\mu \right] \tag{58}$$

We also have:

$$P_T^2 = \frac{p_\pi^2}{p_0^2} \sin^2\theta_{\pi\mu} \tag{59}$$

or:

$$P_T = \frac{\sin\theta_{\pi\mu}}{\sin\theta_0} \tag{60}$$

The transverse polarization is linear in $\sin\theta_{\pi\mu}$ and reaches its maximum value $P_T = 1$ when $\theta_{\pi\mu}$ is equal to the limiting angle θ_0.

It should be noted that the angular distribution of the muon is peaked at the limiting angle, therefore most muons will be transversely polarized. If the beam has cylindrical symmetry, this transverse polarization will average to zero.

4.1 Transversely polarized muons used in experiments

Bardon et al.[1] have used transversely polarized muons in a Mott scattering experiment. They produced a parallel beam of 28 ± 2.5 MeV negative π mesons and used a selection on the angle to produce transversely polarized muons with a polarization $P_y = 0.9$. The kinetic energy of the muons was 9 MeV $< T_\mu <$ 33 MeV.

Later on, the same technique was used by Beltrami et al.[2, 3] in a μSR experiment to measure $P_\mu \xi = 1.0027 \pm 0.0084$. The design of the apparatus and associated Monte Carlo calculations are described in a paper by Bünzli[4] (this work, done at PSI, was supervised by Peter Gumplinger).

4.2 A visual representation of the polarization

For a pion decaying from rest the angular distribution has spherical symmetry. The polarization vector of the muon is aligned on the momentum (longitudinal polarization). Figure 17 shows the components p_x et p_y of the muon momentum. The circles represent the polarization vector. Figure 18 shows the same but the pion momentum is 50 MeV (above the critical momentum). For the muons emitted at the limiting angle the polarization vector is orthogonal to the momentum (transverse polarization).

When the pion is Lorentz-boosted the sphere takes an elongated shape resembling an ellipsoid. In Figure 19 the pion momentum is 20 MeV/c (below the critical momentum). The long lines represent the muon momenta (the momentum of the pion is along the X axis). The short lines represent the polarization vector of the muon (in the rest system). Only muons emitted in the same direction as the pions (forward or backward) are longitudinally polarized (negative polarisation for the positive muons).

In Figure 20 the pion momentum is 50 MeV/c (above the critical momentum). The long lines represent the muon momenta (the momentum of the pion is along the X axis). The two groups of muons emitted forward in the direction of the pion have opposite polarizations. In the case of positive muons the polarization is negative for the high-energy muons and positive for the low-energy ones. For the muons emitted at the limiting angle the polarization vector is orthogonal to the momentum (transverse polarization). See Figure 21.

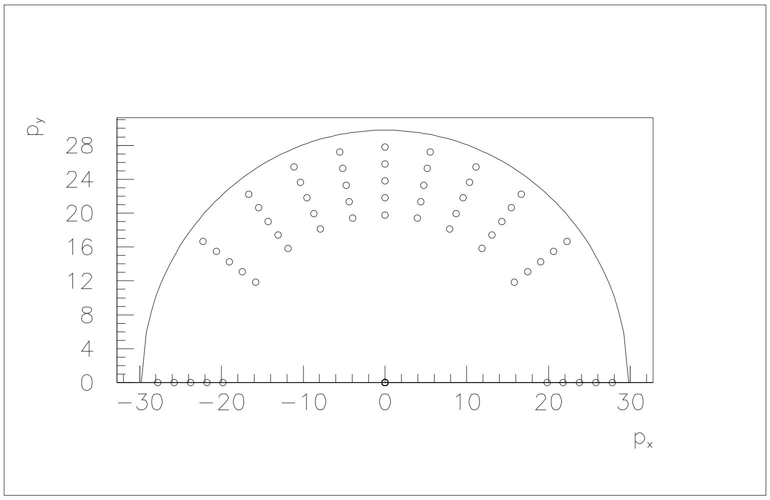

Figure 17: Polarization vector in $\pi_{\mu 2}$ decay. The pion is at rest.

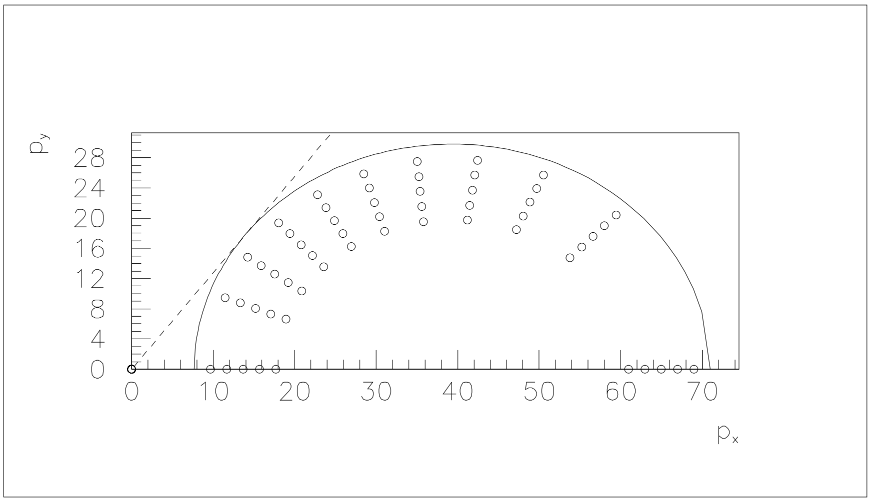

Figure 18: Polarization vector in $\pi_{\mu 2}$ decay. The momentum of the pion is 50 MeV/c.

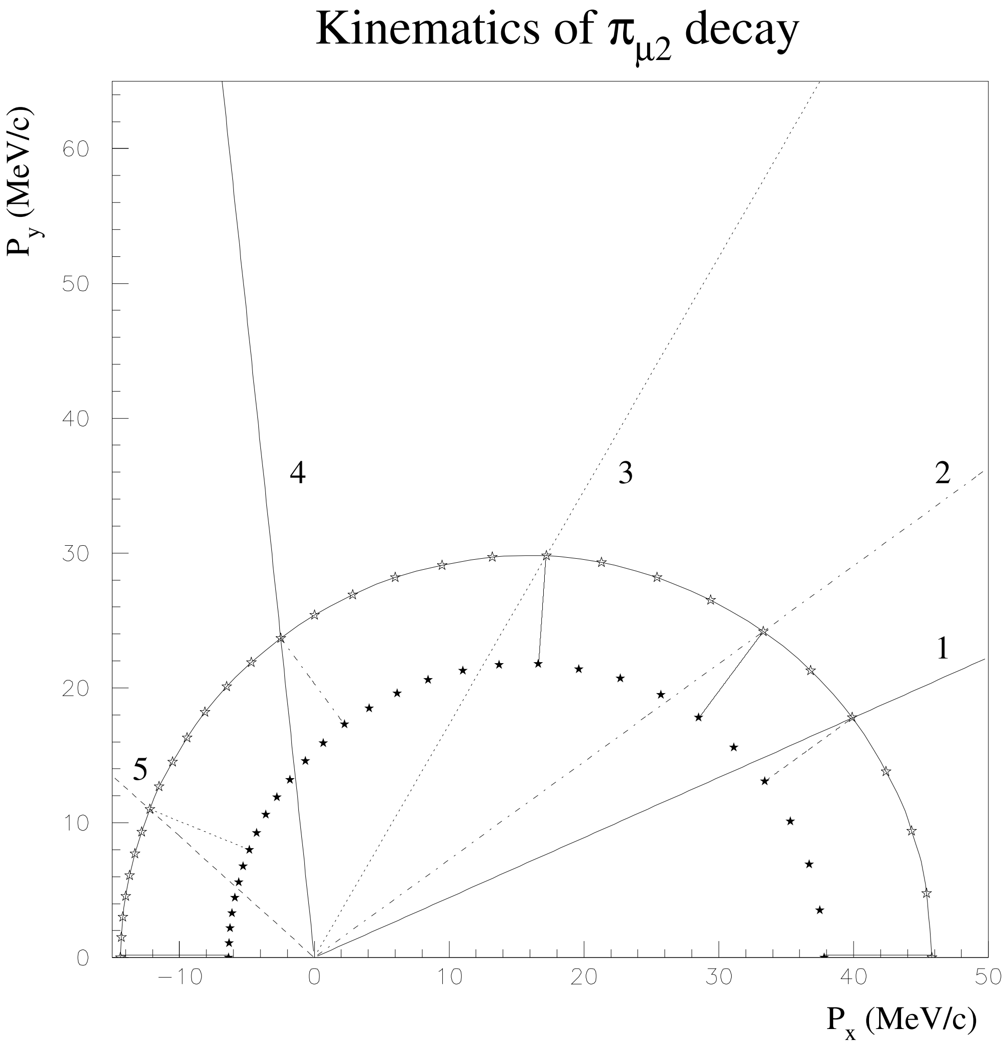

Figure 19: The momentum of the pion is 20 MeV/c, below the critical momentum.

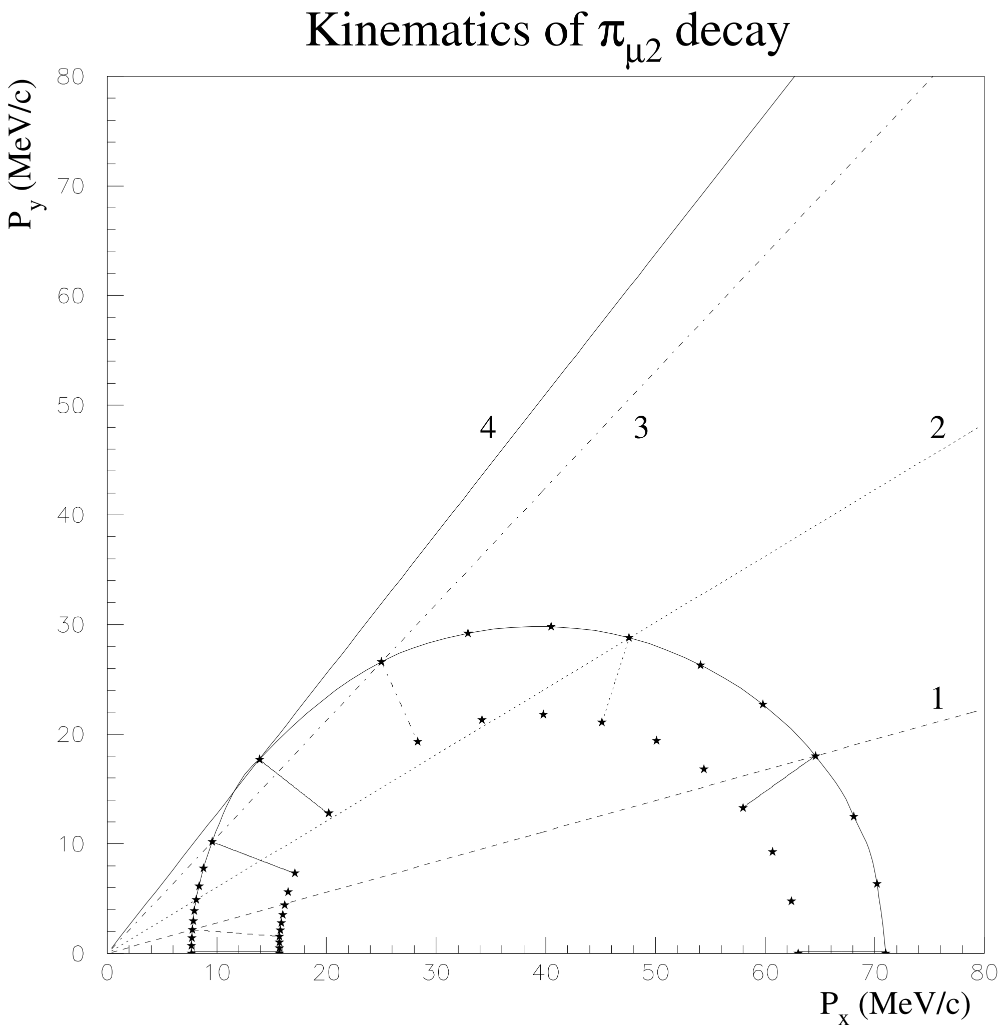

Figure 20: The momentum of the pion is 50 MeV/c, above the critical momentum.

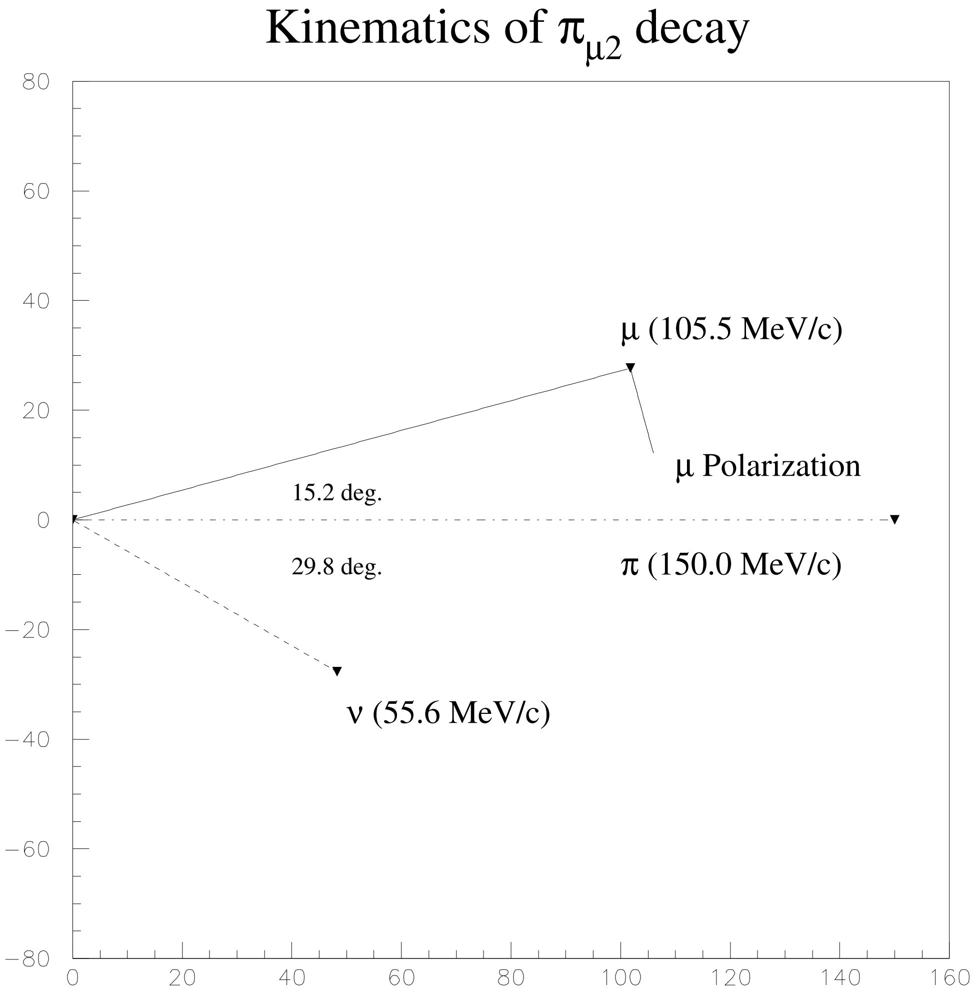

Figure 21: The kinematics for a pion momentum = 150 MeV/c and the muon emitted at the maximum angle. The muon is transversely polarized. Note the direction of the neutrino momentum.

4.3 Effect of pion thermal motion

It has been suggested that one should consider the influence of the thermal motion of the pion on the polarization of the muon. The question is: is this effect important for E614? A rough calculation is presented here.

Assuming a temperature of 500 K in the target, the thermal kinetic energy of the pion is:

$$E = \frac{3}{2}kT = 8.617 \times 10^{-5}\ \text{ev K}^{-1} \times 500\ \text{K} = 0.0646\ \text{eV} \tag{61}$$

This correspond to a pion momentum of:

$$p = \sqrt{2mE} = 3.70\ \text{KeV} \tag{62}$$

A calculation with $p = 4$ KeV gives the following results:

- Figure 22: muon momentum vs pion-muon angle. The momentum spread is $\sim 2 \times 10^{-4}$.
- Figure 23: muon kinetic energy vs pion-muon angle. The energy spread is $\sim 4 \times 10^{-4}$.
- Figure 24: neutrino energy vs pion-muon angle. The energy spread is $\sim 5 \times 10^{-5}$.
- Figure 25: muon-neutrino angle vs pion-muon angle. What is plotted is the difference ($\times 10^8$) between the cosine of the angle (-1 for a pion decaying from rest) and -1. The angular spread is $\sim 10^{-8}$.
- Figure 26: muon longitudinal polarisation vs pion-muon angle. What is plotted is the difference ($\times 10^8$) between the longitudinal polarisation (-1 for a pion decaying from rest) and -1. The deviation from -1 is $\sim 5 \times 10^{-9}$.
- Figure 27: muon transverse polarization ($\times 10^4$). This transverse polarization reaches a maximum $\sim 10^{-4}$.
- Figure 28: muon polarization component on the neutrino momentum. What is plotted is the difference ($\times 10^8$) between this component (-1 for a pion decaying from rest) and -1. The deviation from -1 is very small, $\sim 5 \times 10^{-10}$. At such a low momentum the polarization of the muon tends to "follow" the polarization of the neutrino.

The effects are of the order of 10^{-8} on the muon-neutrino angle and on the longitudinal polarization of the muon. On the other hand, the transverse polarization could be of the order of 10^{-4}.

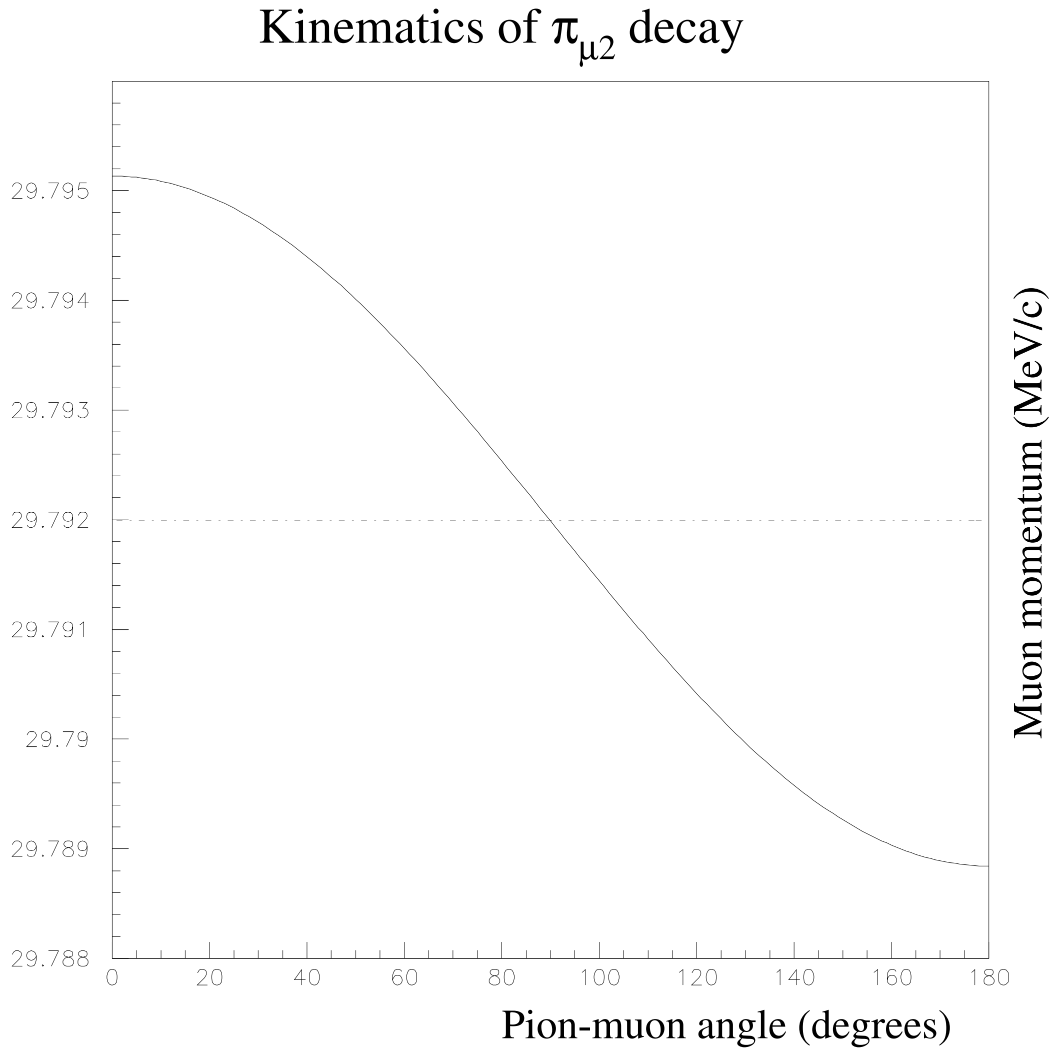

Figure 22: Muon momentum vs pion-muon angle.

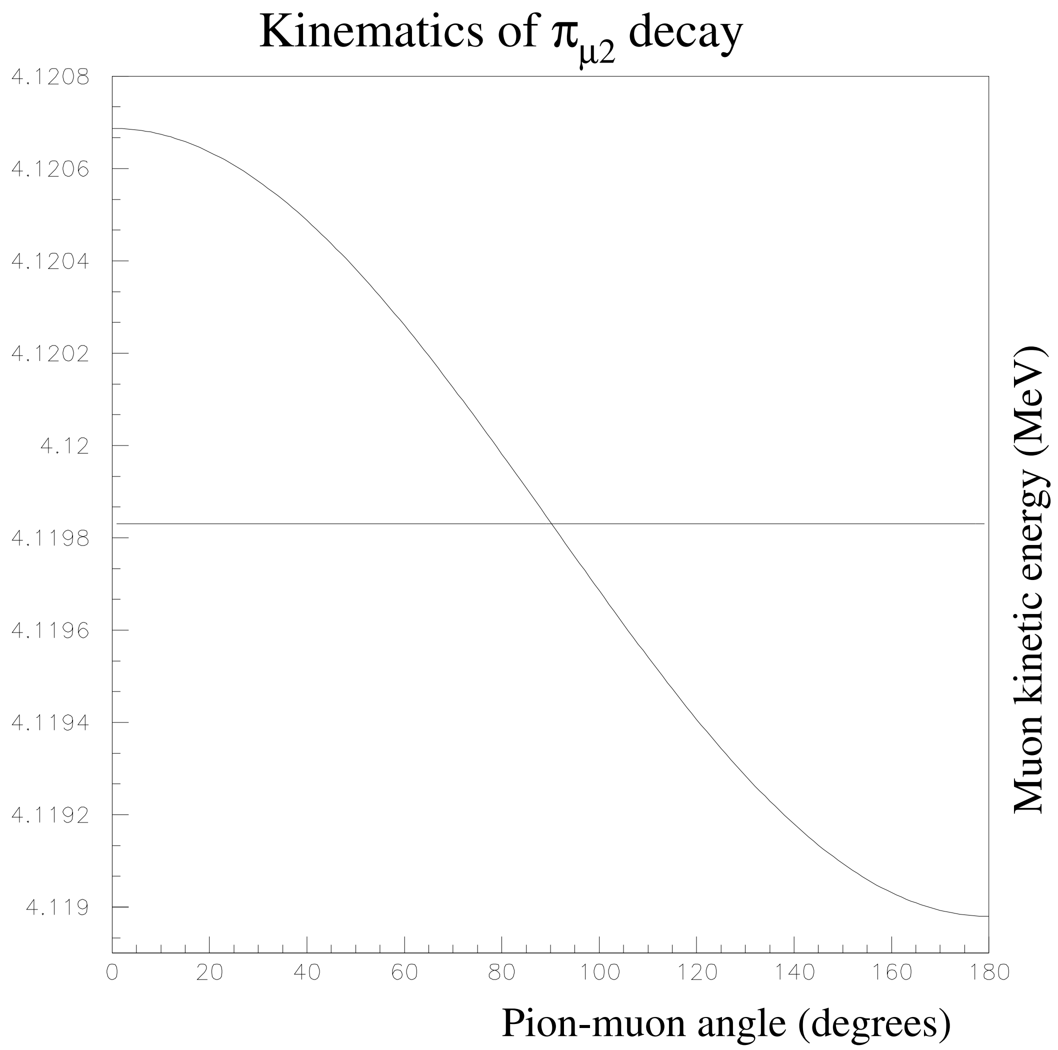

Figure 23: Muon kinetic energy vs pion-muon angle.

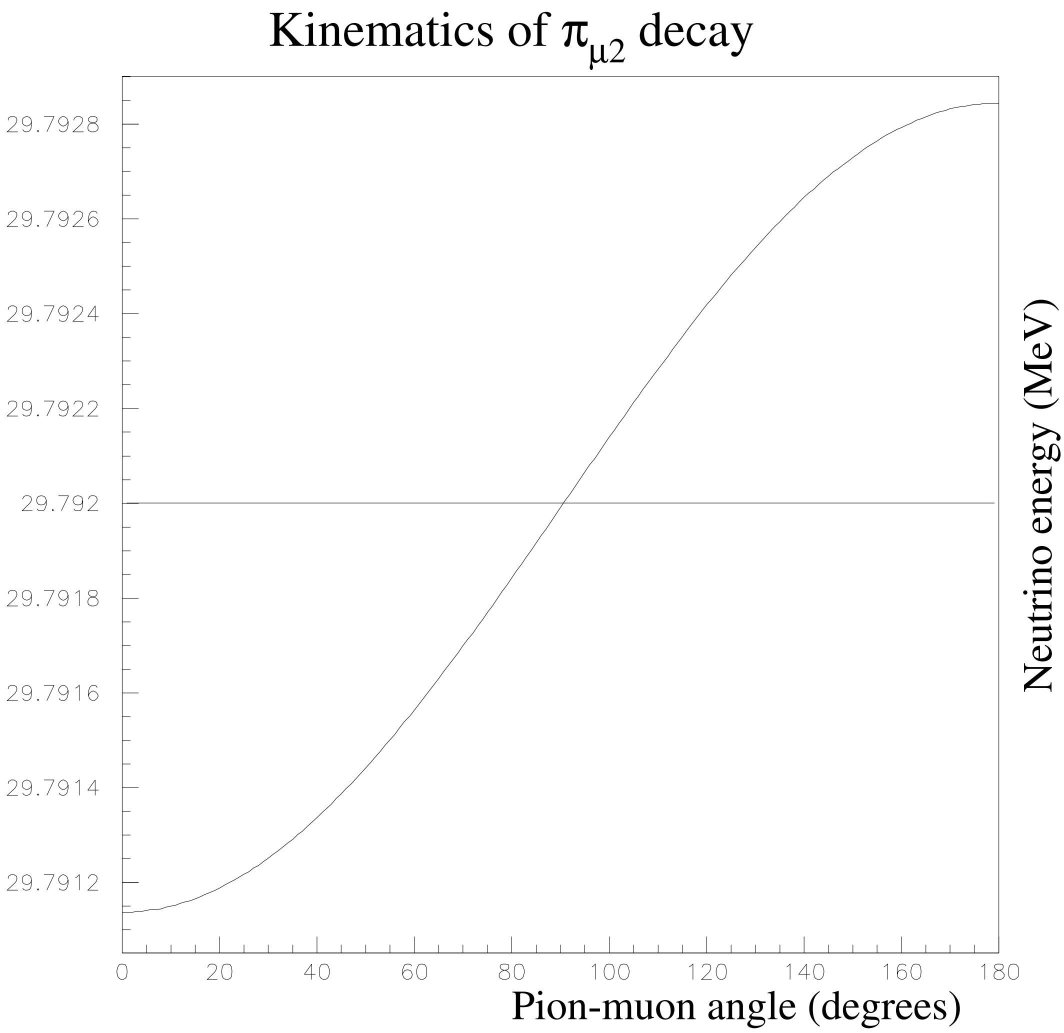

Figure 24: Neutrino energy vs pion-muon angle.

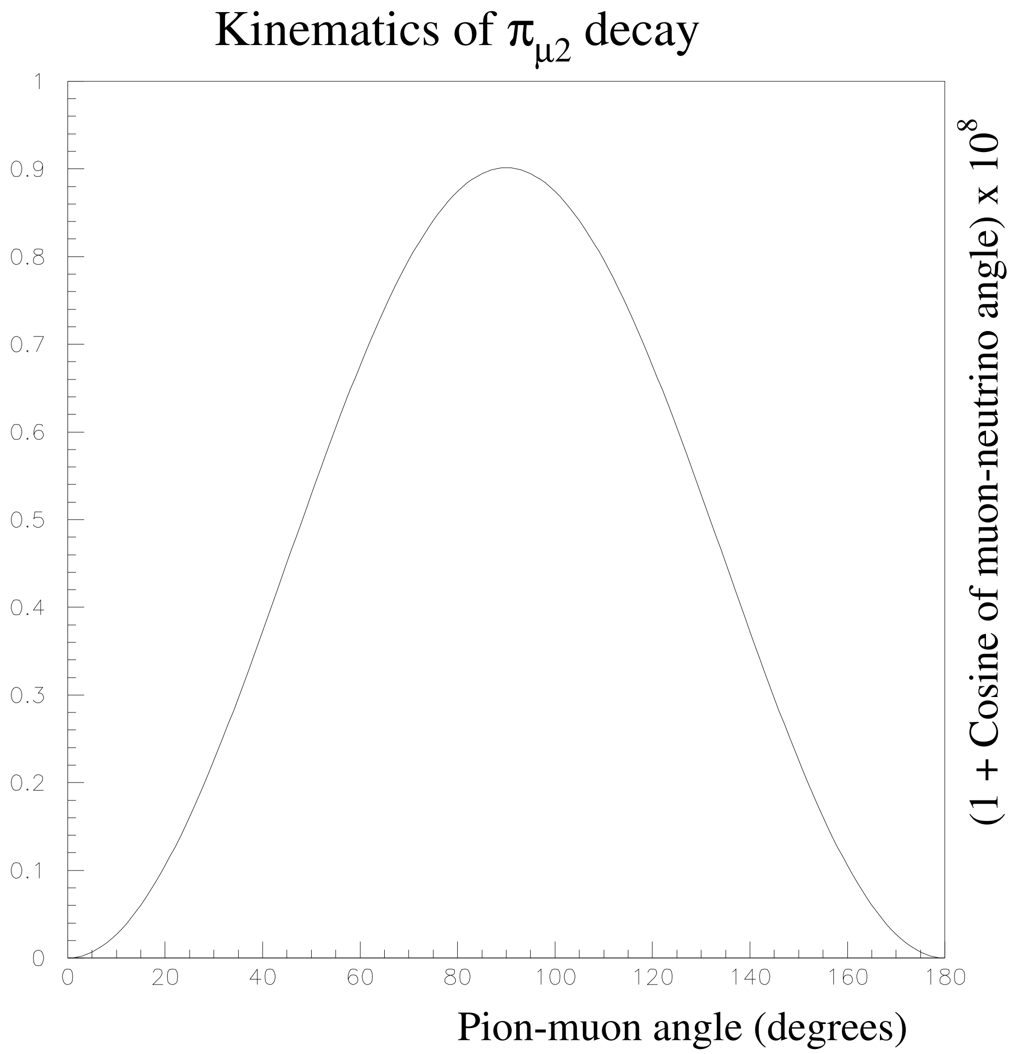

Figure 25: Muon-neutrino angle vs pion-muon angle.

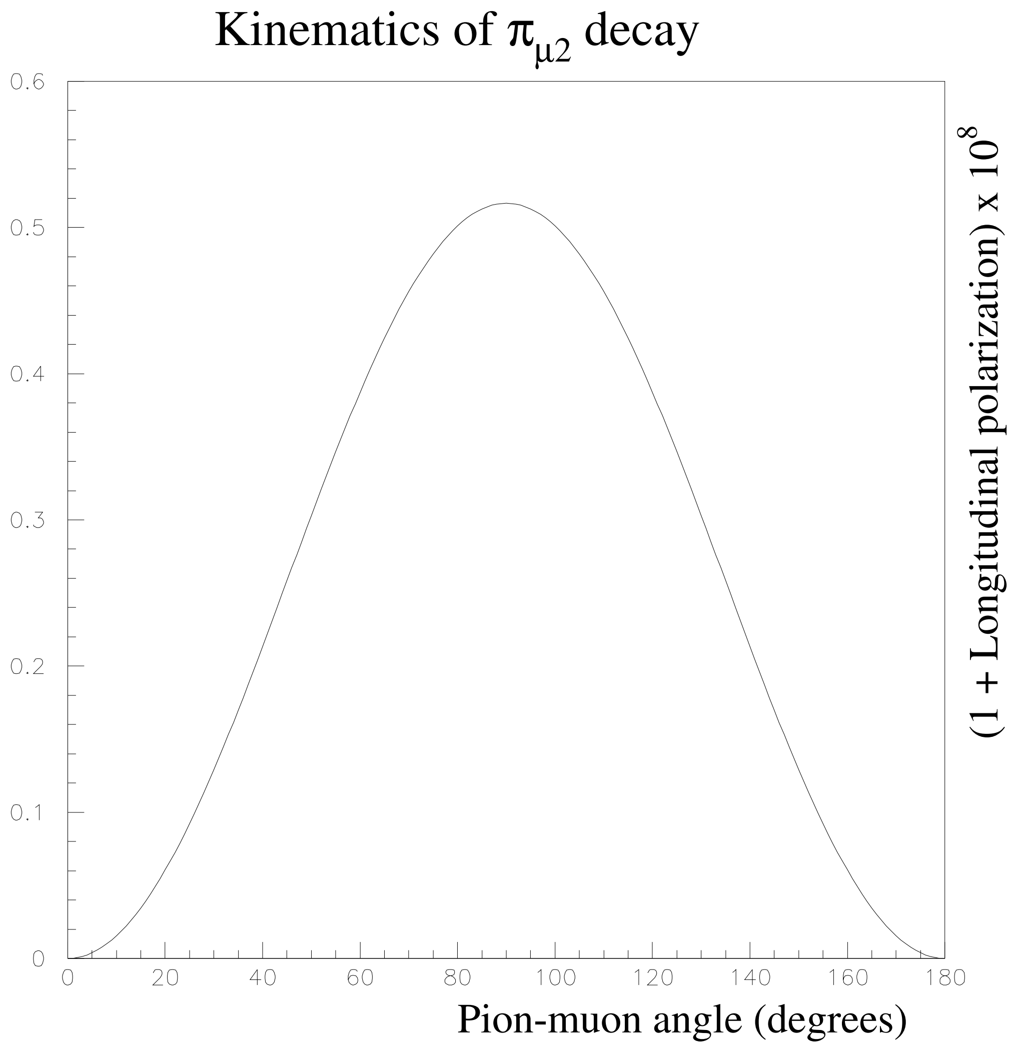

Figure 26: Muon longitudinal polarization vs pion-muon angle.

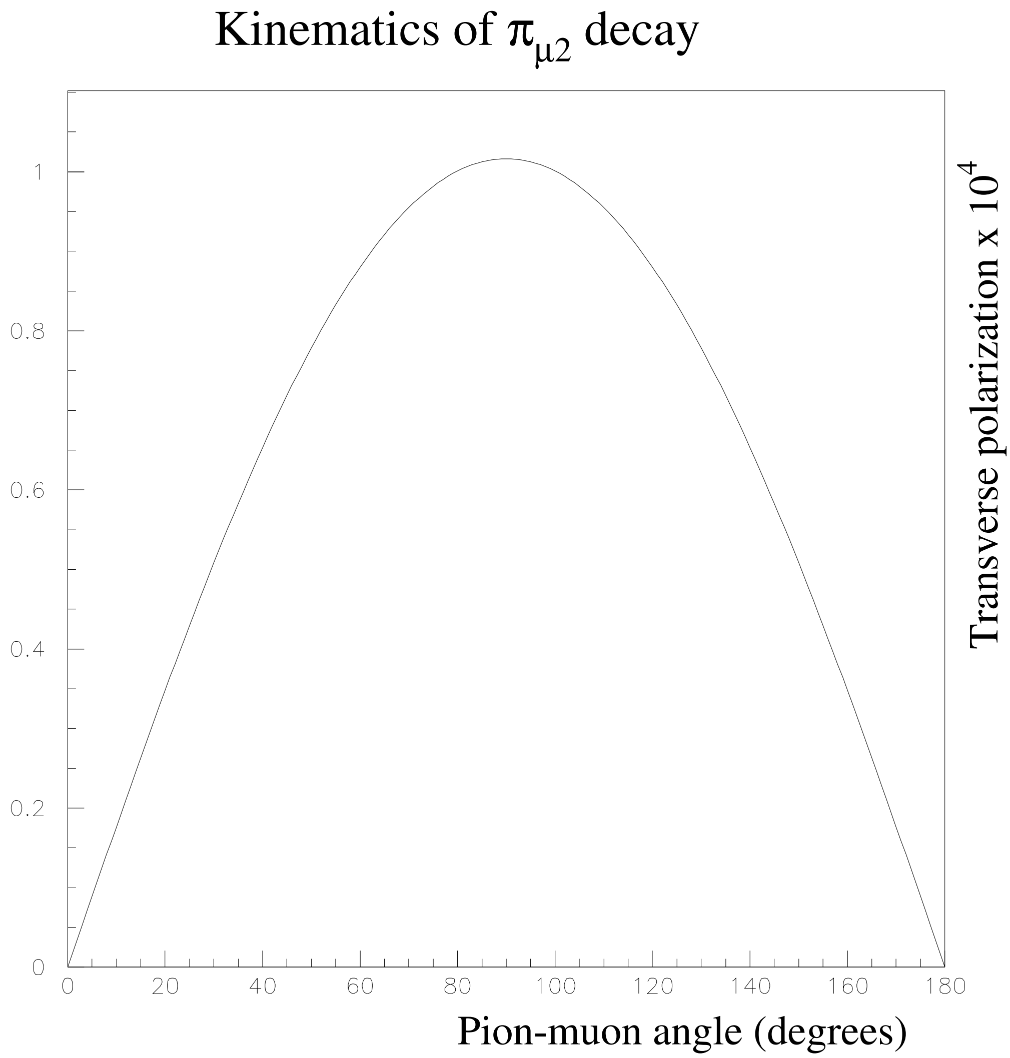

Figure 27: Muon transverse polarization vs pion-muon angle.

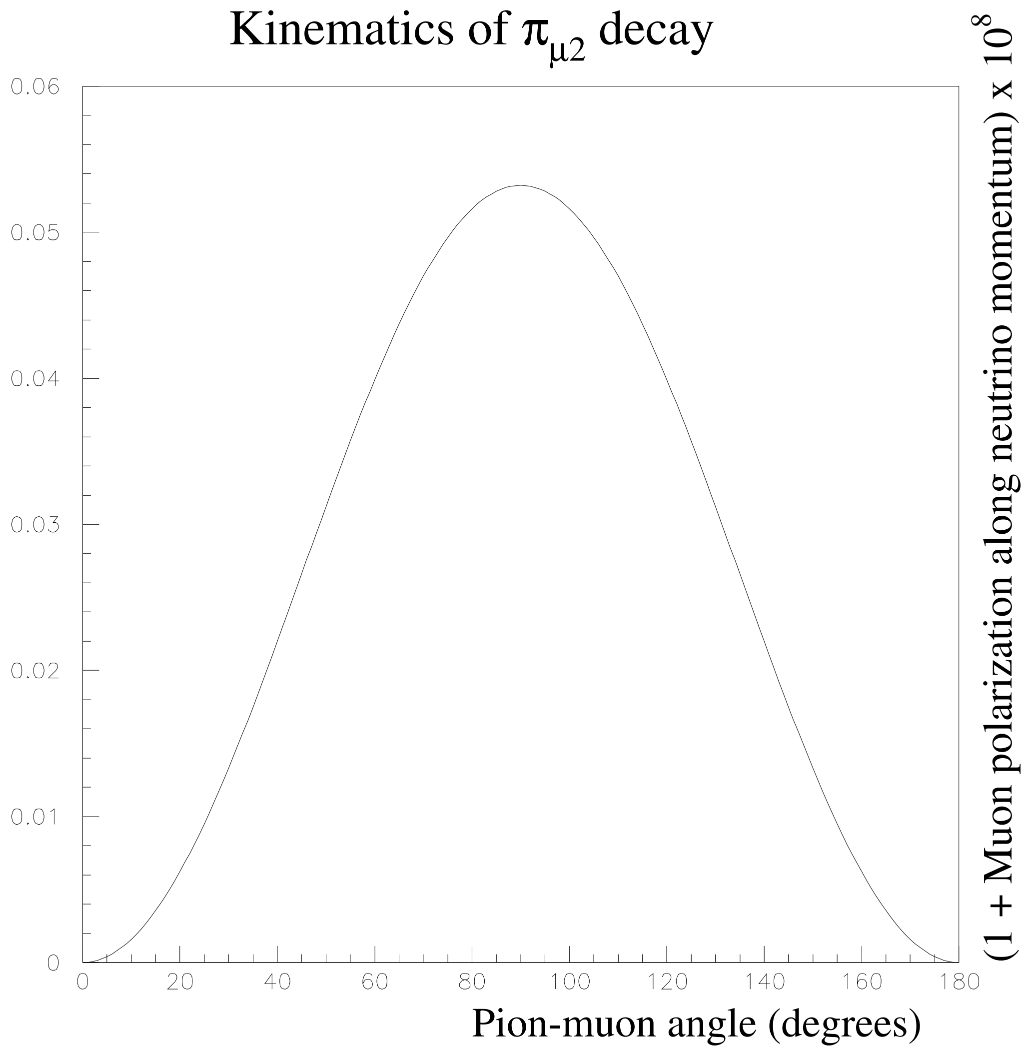

Figure 28: Muon polarization component on the neutrino momentum vs pion-muon angle.

5 Acknowledgments

The author wishes to thank Peter Gumplinger for interesting discussions and for bringing to his attention the paper by Bünzli[4].

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