Injection of a 30 MeV/c surface muon beam into the 2.3 Tesla solenoid for TRIUMF experiment E614

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1 Introduction and summary

TRIUMF experiment E614 aims to measure the Michel parameters of muon decay. A surface muon beam is stopped in the center of a 2.3 T solenoid. Surface muons have the spin in the direction of motion because they are produced by the decay of pions at rest in the surface of the production target. Since G=2 for muons, the spin follows the direction of motion during transport through magnetic elements.

An essential requirement of the experiment is that the beam is 100 percent polarized in the longitudinal Z direction. It is inevitable that the muons have a transverse momentum. Their direction makes an angle α with the axis of the solenoid and the component of the spin in the z direction is smaller than unity. If the α is 10 mr, than the depolarization is $1-cos(0.010) = 0.5*10^{-4}$. If α is 15 mr, the depolarization is $1.1 * 10^{-4}$,while it is $2.0 * 10^{-4}$ for α is 20 mr. The experiment requires a beam which has a small depolarization. $2.0 * 10^{-4}$ is too big.

A beam that is injected into a solenoid passes through a fringe field region, where the beam is being focussed by the radial component of the magnetic field. The angle in the beam becomes large, giving a large depolarzation. The injection of the muon beam from TRIUMF beam line M13 needs to be optimized to keep the increase in angle as small as possible. This note gives some very preliminary results from a study to understand the behaviour of the muon beam in the solenoid.

2 Method of calculation

Figure 1a shows the Z component of the magnetic field along the axis of the solenoid as obtained from calculations with the program OPERA2D. Z=0 corresponds to the center of the solenoid. This has been obtained from the full two dimensional map which extends to a radius of 30 cm. The field on the axis is sufficient to determine the field anywhere in the solenoid using the Maxwell equations. This is done in the raytracing program ZGOUBI, which traces particles stepwise through electric and magnetic fields in a calculation which is accurate to high order. Notice that the field rises steeply from Z=-160 cm on. Here is where the yoke of the solenoid ends. The radial component B_r of the field off the axis is to good approximation proportional to $(r/2)dB/dz$, where r is the distance from the axis. This was checked for the two dimensional field map for the region where the beam is. Figure 1b shows that B_r has a double maximum.

3 The behaviour of some individual rays in the solenoid

Figures 2a and 2c are for for four rays which start out parallel to the axis at 1, 3, 5 and 7 mm from the axis. Figure 2a shows how the radial position changes. The fringe field focusses the beam to the axis at a position which is independent of the inital radius.The radial focussing field B_r is proportional to the distance from the axis. Figure 2c gives the development of the angle α as the particle enters the solenoid. Figures 2b and 2d are for rays converging into the solenoid with initial angles of 5, 10, 15 and 20 mr. When the solenoid field is turned off they converge to a point 152 cm from the center. This is the optimum situation, as will be shown in the next section.

Figure 3 shows X-Y projections of how the indicated rays develop transversely.

Figure 4 shows the dependence of α in the center of the solenoid as function of the initial radius and of the initial angle. Even for a radius of 3 mm the final angle is already 18 mr. An initial angle of 10 mr gives a final angle of 17.5 mr.

4 Monte Carlo calculations for a collection of rays

The angle α in the center of the solenoid depends on the way the beam enters the solenoid. It is important to have the beam come to a waist somewhere in the region where the focussing by the fringe field is strongest. In order to study this we take a beam starting from a waist with a radius of 5 mm and a maximum angle with the axis of 20 mr. The projected emittance in both the vertical and the horizontal plane is then $\pi * 5 * 20 = 100\pi$ mm.mr. This is the phase space area of the beam projected on either the vertical or the horizontal plane. This beam is traced back and then injected into the solenoid.

4.1 Where to focus the beam?

In figure 5 S is a measure of where the beam would come to a waist if the solenoid is turned off. If $S = 0$, the beam comes to a waist 100 cm from the center of the solenoid. If $S = 100$ the beam comes to a waist at 200 cm from the center. The total number of traced particles is 617. Figures 5a and 5b show the number of particles which arrive in the center with angles less than 10 or 20 mr, while figure 5c shows the maximum angle in the beam. The optimum situation occurs is when the beam is aimed at a point 152 cm from the center. As figure 1b indicates, this is just behind the first bump in the radial field. It turns out that the optimum situation does not depend very much on the choice of the emittance.

4.2 Emittance dependence

How does the intensity within a chosen angle range in the center depend on the projected emittance?

This is shown in figure 6. To vary the emittance we change the radius from 3 to 10 mm in steps of 1 mm and, simultaneously, the angle from 12 to 40 mr in steps of 4 mr. The projected emittance varies from 36π mm.mr to 400π mm.mr. Figures 6a and 6b show the intensity within 10 and 20 mr in arbitrary units. For easy comparison, the intensity is normalized to 100 for an emittance of 100π mm.mr. The intensity is almost a linear function of the projected emittance. The total phase space and intensity of the incoming beam increases, in fact, as the square of the projected emittance. Figures 6c and 6d show the ratio of the total number of particles and the particles within 10 or 20 mr. For example for an emittance of 100π mm.mr the number of particles within 10 mr is about 8 percent.

4.3 Influence of the shape of the emittance

Does the number of particles within the chosen angle range depend on the shape of the projected emittance?

Results are given in figure 7. The emittance is 100π mm.mr which can be obtained with a radius of 3 mm and an angle of 33 mr, a radius of 5 mm and an angle of 20 mr, a radius of 10 mm and an angle of 10 mr, etcetera. In figure 7 the variation of the shape of the emittance is represented by the radius R. Figures 7a and 7b show the percentage of the total intensity within 10 or 20 mr, while 7c shows the maximum angle in the center of the solenoid. The conclusion is that 5 mm radius and 20 mr angle is an optimum choice.

4.4 Angle distribution for the optimum situation

Figure 8 shows for the choice of 5 mm and 20 mr the percentage of particles within the indicated range of radius or angle in the center. The bottom two figures give the accumulated percentages.

5 Consequences for the M13 beam

5.1 Intensity of M13 beam in 1 percent ΔP/P momentum bite

M13 takes of from proton beamline BL1A at an angle of 135 degrees. The production target is 1.0 cm long graphite. The proton spot on the target is of the order of 2 mm in diameter. This presents a source to the M13 channel which is horizontally $(10.0 + 2)sin(45) = 8.4$ mm. Vertically we take a size of 5 mm. This is bigger than 2 mm, because what counts is not the size of the proton beam but the effective size of the surface area of the target which contributes to the surface muon beam. M13 accepts vertical angles of up to 160 mr and horizontal angles up to 60 mr. The horizontal emittance is $60 * 8.4/2 * \pi = 250\pi$ mm.mr. The vertical emittance is $160 * 5.0/2 * \pi = 400\pi$ mm.mr. The momentum acceptance is 8.0 percentΔP/P. The measured total intensity of surface muons in this phase space is 1,600 K per second per 150 μA proton beam. The experiment needs a small momentum bite, not larger than 1.0 percent. This gives an intensity of 200 K per second.

5.2 Intensity of M13 in 100π mm.mr emittance and 1 percent ΔP/P momentum bite

It is not advisable to use the full emittance of M13, because it would give a large ratio of unwanted to wanted muons, the latter being muons within a 10 mr angle α in the center of the solenoid. For the sake of argument, assume that an emittance of 100π mm.mr in both planes is a good choice. In that case there will be 12 times as many muons with α larger than 10 mr, than muons with α smaller than 10 mr. There are $(100/250)*(100/400)*200K = 20K$ muons per second in the beam. Of these muons, there are 8 percent or 1.6 K within a 10 mr angle α. It is important to emphasize that this estimate depends crucially on the assumption that the vertical source size at the production target is 5 mm. If, in fact, the source size is larger, then the intensity within 100π mm.mr is less than 1.6 K per second. It is necessary to find out the source size.

5.3 Optimum distance between M13 and the E614 solenoid

Some calculations have been done to determine whether it is possible to get an optimum emittance shape for M13. It turns out that a distance of about 1.1 m between the last quadrupole of M13 and the coils of the solenoid ia optimal. In that case the size of the vertical spot would be 9 mm. The half size, which is used for calculating the emittance, is 4.5 mm, close to the optimum 5 mm. Horizontally, the size is 13 mm. The half size is 6.5 mm, not so far from the optimal 5 mm. If the distance is increased, the horizontal spot will increase, which is undesirable. It is not possible to decrease the horizontal spot size, because of the size of the production target. There is no freedom to vary the horizontally focussing quadrupoles in the beam line because they are determined by the requirement of making a dispersed focus at the momentum slit, and making the beam achromatic after the second bending magnet.

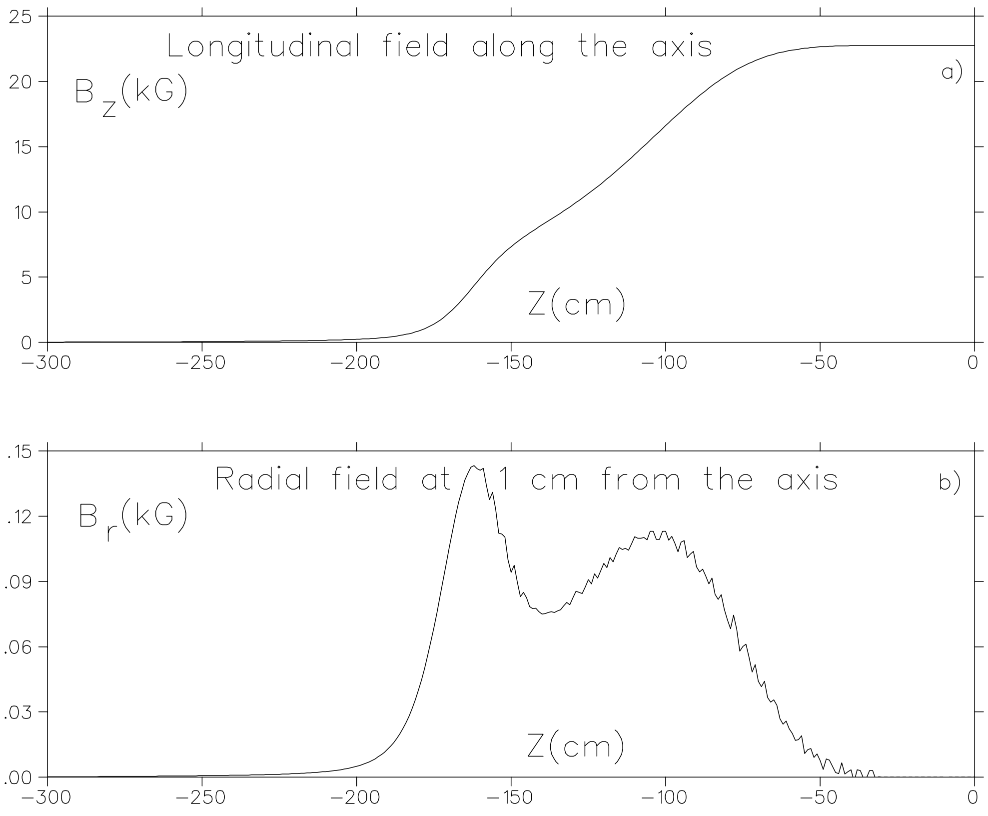

Figure 1: Longitudinal and radial field of the solenoid.

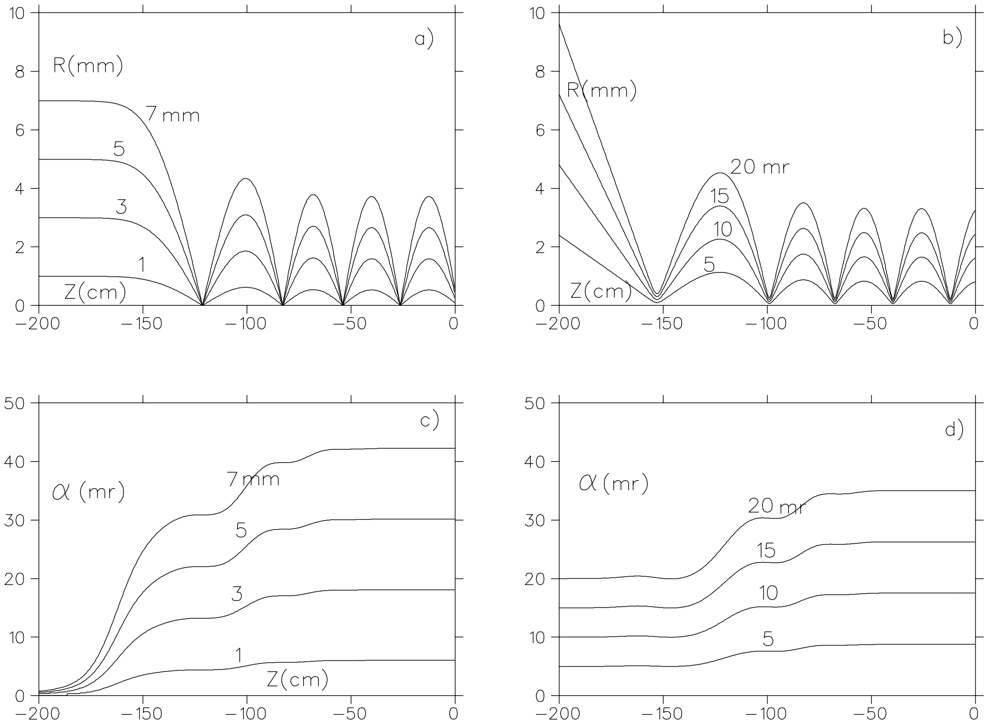

Figure 2: The radius and the angle α along the solenoid for selected rays. Figures 2a and 2c are for four rays parallel to the axis, 1, 3, 5 and 7 mm from the axis. Figures 2b and 2d are for four rays aimed at a point 152 cm from the center of the solenoid, starting with angles of 5, 10, 15 and 20 mr.

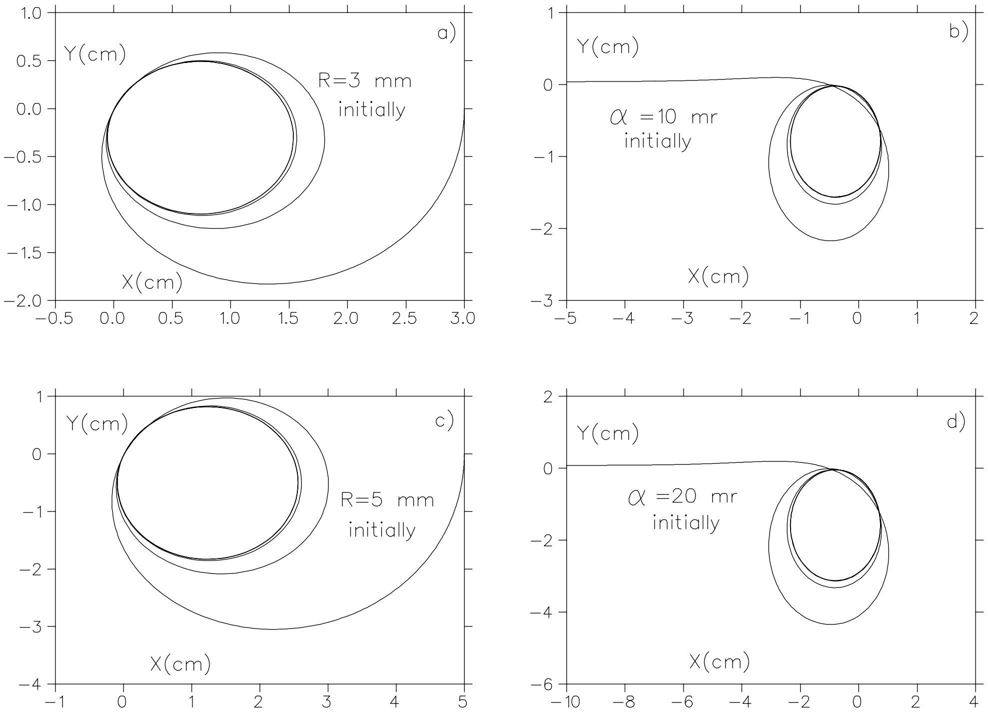

Figure 3: The transverse X-Y motion of a subset of the rays of figure 2.

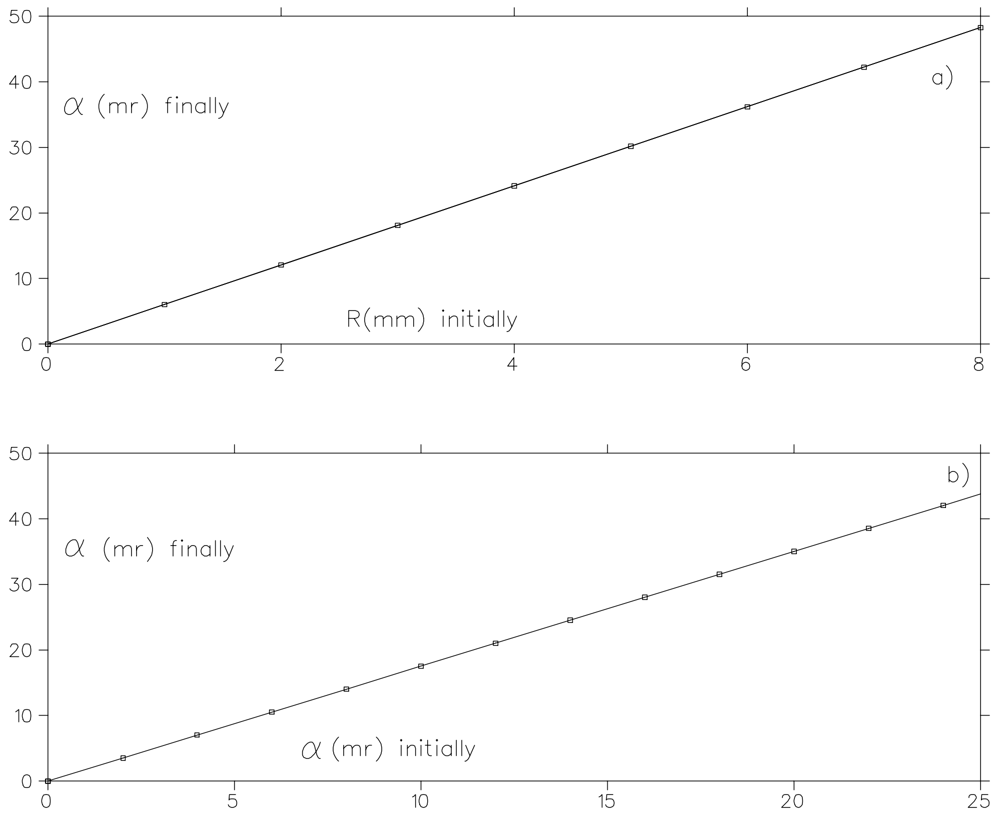

Figure 4: The dependence of α in the center of the solenoid on the initial angle in figure 4b, and on the initial distance to the axis in figure 4a

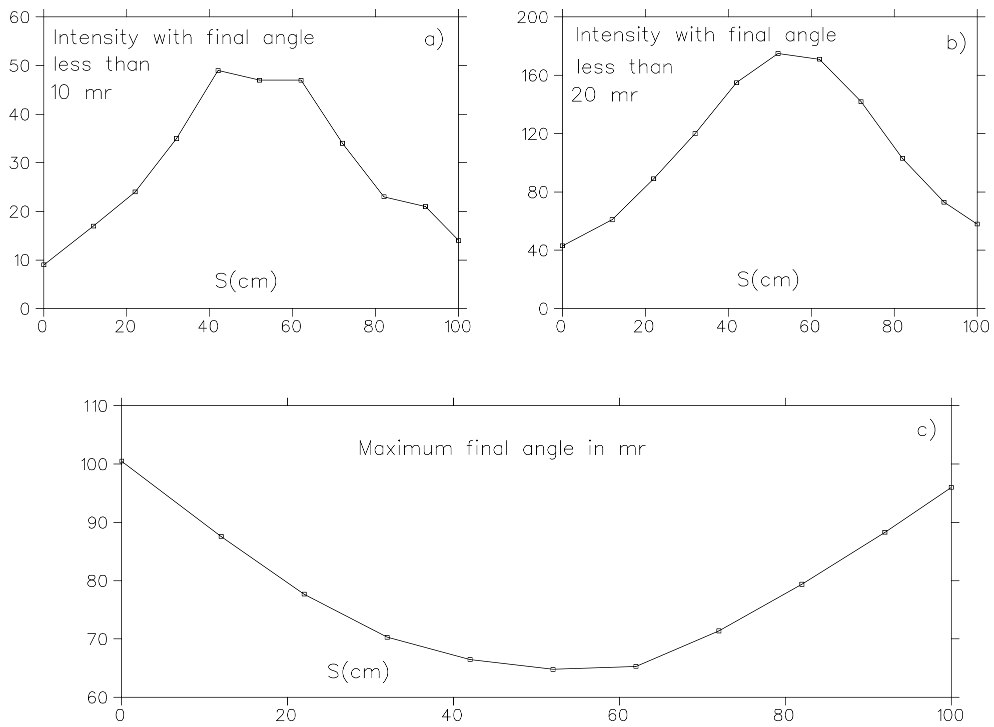

Figure 5: Dependence of the intensity within α less than 10 or 20 mr in figures 5a and 5b. S indicates where the beam is focussed when the solenoid is turned off. $S = 0$ corresponds to a point 100 cm upstream of the center of the solenoid, $S = 100$ corresponds to a point 200 cm upstream of the center. This calculation is for a beam with a 100π mm.mr projected emittance, a radius of 5 mm, and an angle of 20 mr. Figure 5c gives the maximum angle in the center

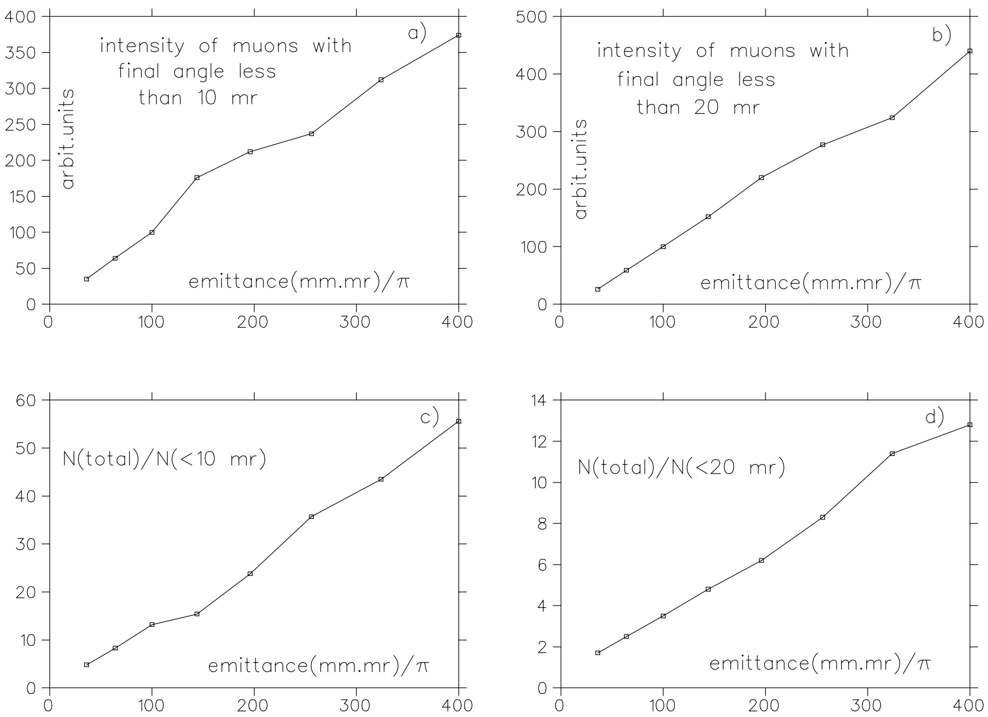

Figure 6: The intensity in the center of the solenoid with α less than 10 or 20 mr in figures 6a and 6b, as function of the projected emittance. The intensities are normalized to 100 for an emittance of 100π mm.mr. In fact, there are roughly four times as many muons in the case of 20 mr, than in the case of 10 mr. Figures 6c and 6d show the ratio of the total number of muons to the muons in the indicated angle range.

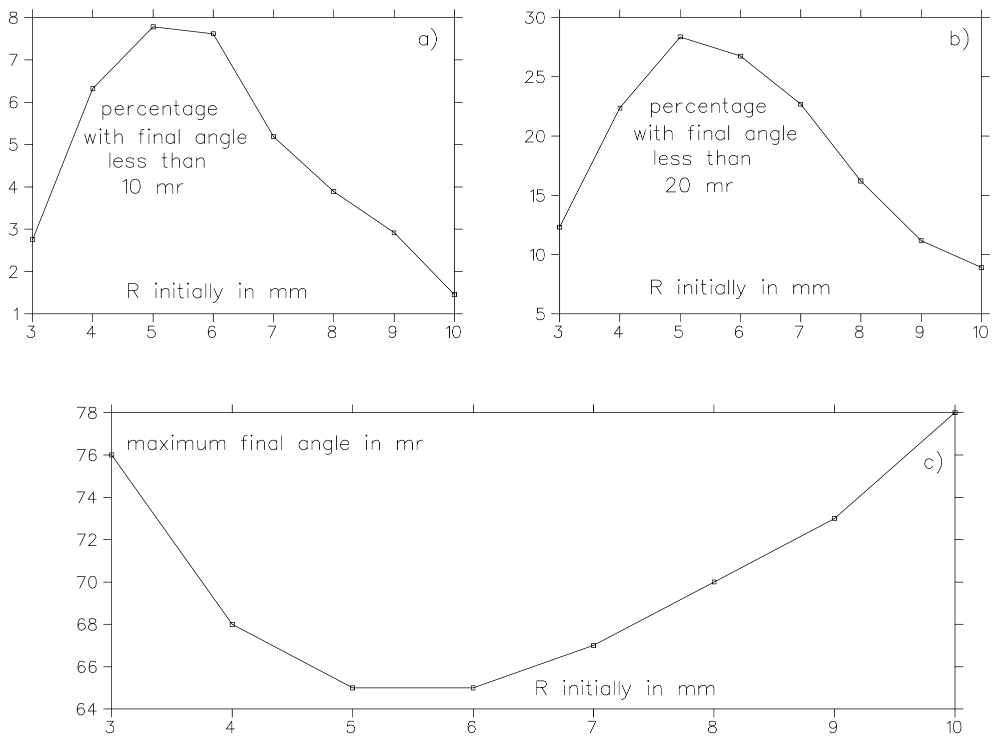

Figure 7: The intensity for different shapes of a 100π mm.mr emittance in figures 7a and 7b. The angle varies inversely to the radius to keep the emittance constant. If R is 5 mm, the angle is 20 mr. This is the optimum choice. The maximum angle in the center of the solenoid is shown in figure 7c.

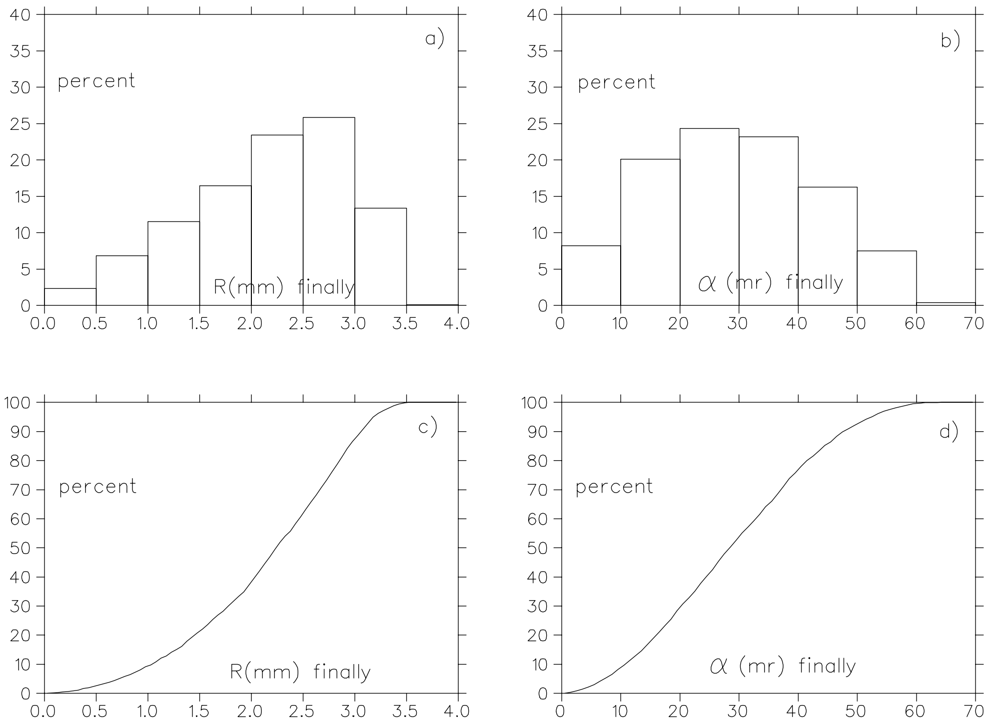

Figure 8: Figures 8a and 8b give for a 100π mm.mr emittance, radius of 5 mm and angle of 20 mr, the percentage of beam as function of R or α in the center of the solenoid. Figures 8c and 8d give the accumulated percentages.