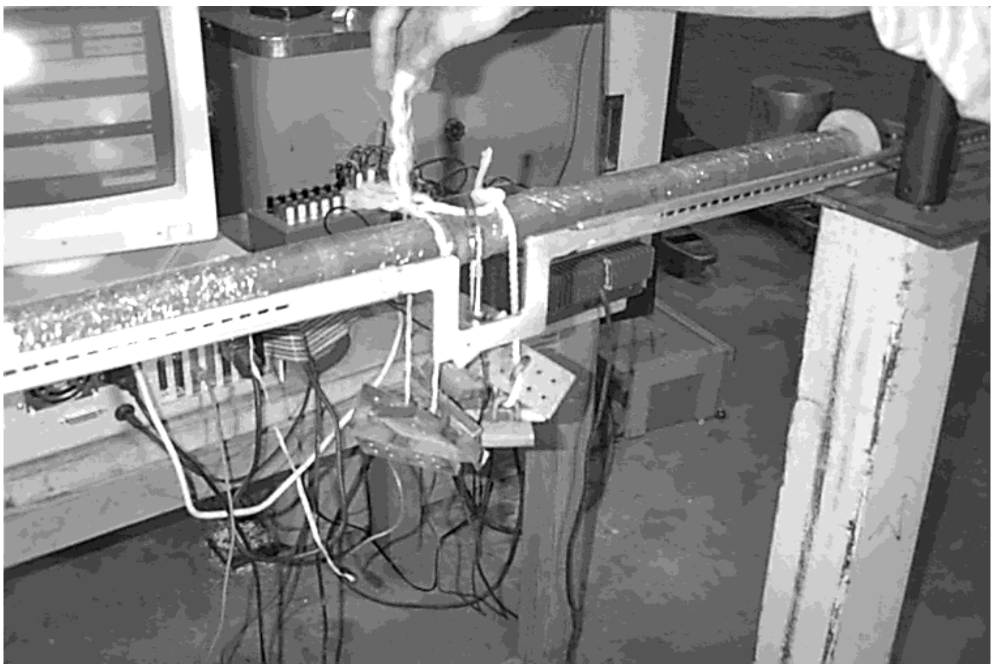
DESIGN PROPERTIES OF CITAL CO-115M
FOR USE IN
E614 DETECTOR SUPPORT STRUCTURE



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EXECUTIVE SUMMARY

Tests and analyses were carried out at the Civil Engineering Department of the University of British Columbia to predict the behaviour of the proposed E614 detector support structure design. Material properties of CITAL CO-115M were determined and analytical predictions of support structure behaviour were performed.

Experimental work determined the following material properties of CITAL CO-115M:
1) the elastic modulus was found to be 57.0 GPa; and
2) the maximum strength in compression was found to be about 900 MPa.

For a brittle material, such as CITAL CO-115M, the compression strength results are dependent on the load apparatus and specimen dimensions. A certain amount of variation in strength is to be expected depending on the particular application. For the expected range of operating loads, however, the maximum strength is well above the design stress maximums.

Friction between finished surfaces of adjacent segments of CITAL was not determined. Experiments aimed at determining load-slip behaviour of the friction surfaces resulted in fracture of the segments without ever exhibiting slip between adjacent segments.

Computer analysis for a 1.06 m long single beam predicts behaviour for two extreme end-support conditions: *simply-supported* and *fixed-fixed*. Equations based on the analysis are provided for determining the minimum requirements for combinations of axial stress and load per 4 mm CITAL segment. Also reported are prediction equations for maximum mid-span deflection for a variety of load cases.

The expected range of stresses and mid-span deflection for the 1.06 m long four-beam structure carrying 4 kg per load plane are:

	Simply-Supported	Fixed-Fixed
Minimum Required Axial Stress (MPa)	0.55 – 34.0	0.35 – 23.0
Mid-span Deflection (μm)	39 – 2500	0.8 - 500

The large range is due to the unknown amount of rigidity provided by the G10 plates which link the four beams together. If there is no rigidity, then each beam behaves independently and the higher values result. The best case, therefore, is the fixed-fixed condition with very rigid links between the beams. Although the minimum axial stress is quite low for the ‘best case,’ it is recommended that at least 20 MPa be provided.

Design equations are provided and further recommendations are made.

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Appendix C: Load-slip Tests Load and Time, Load-displacement Curves
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Appendix F: Calculation of Moment of Inertia, I for Various Cross-sections
Appendix G: Strain Gauge Schematics

1. Introduction

A test programme was carried out to determine the elastic modulus and strength properties of CITAL CO-115M ceramic/glass segments. These values are required for the final design of the detector support-structure.

The CITAL CO-115M pieces are hollow cylindrical segments and are available in three (3) lengths: 4, 20 and 40 mm as shown in Figure 1. The segments will be aligned, end-to-end, and compressed to form one long beam, as shown in the sketch in Figure 2.

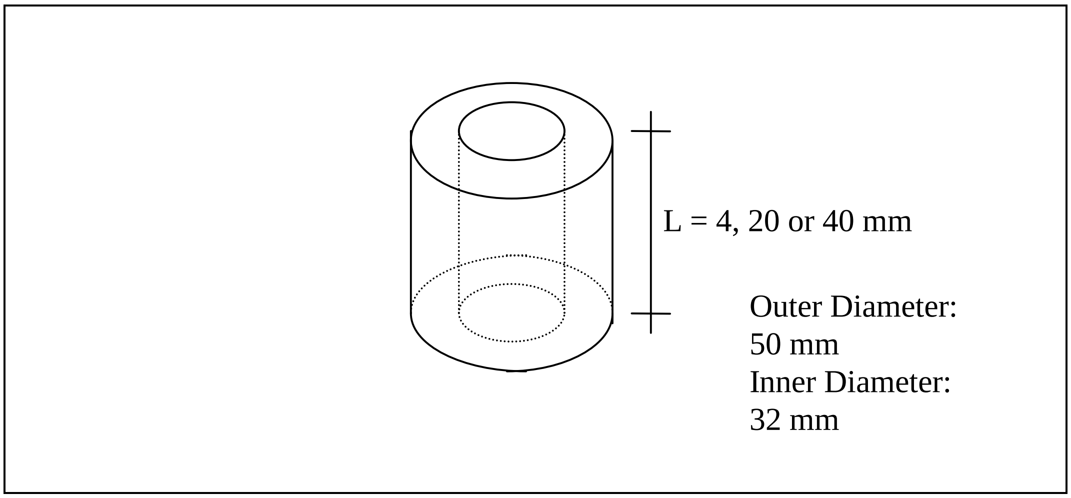

Figure 1. CITAL CO-115M Segment.

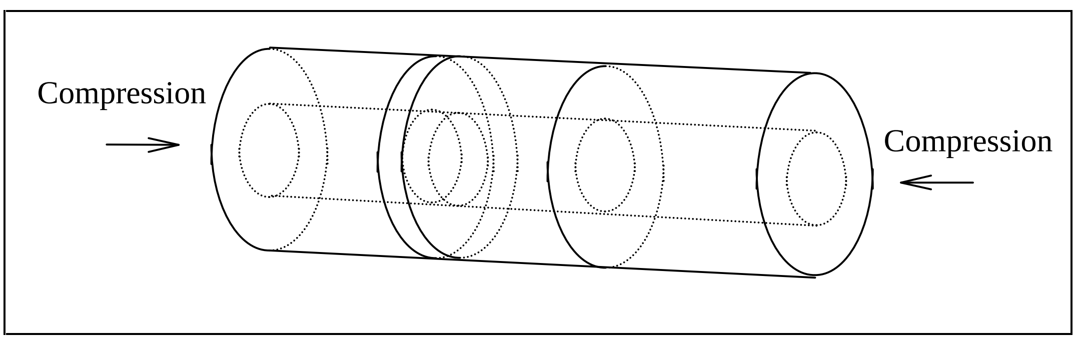

Figure 2. Sample Single Beam.

Four of these long beams will be used to form the structural frame of the E614 Detector at the TRIUMF facility in Vancouver. They will be arranged in the pattern shown in Figure 3. The distance, R, shown in the figure is 0.260 m. For the single beam and the group of four beams, an analysis can be performed assuming *fixed-fixed* support at the ends or *simple-support* at the ends, as will be discussed in sections 1.2 and 4.

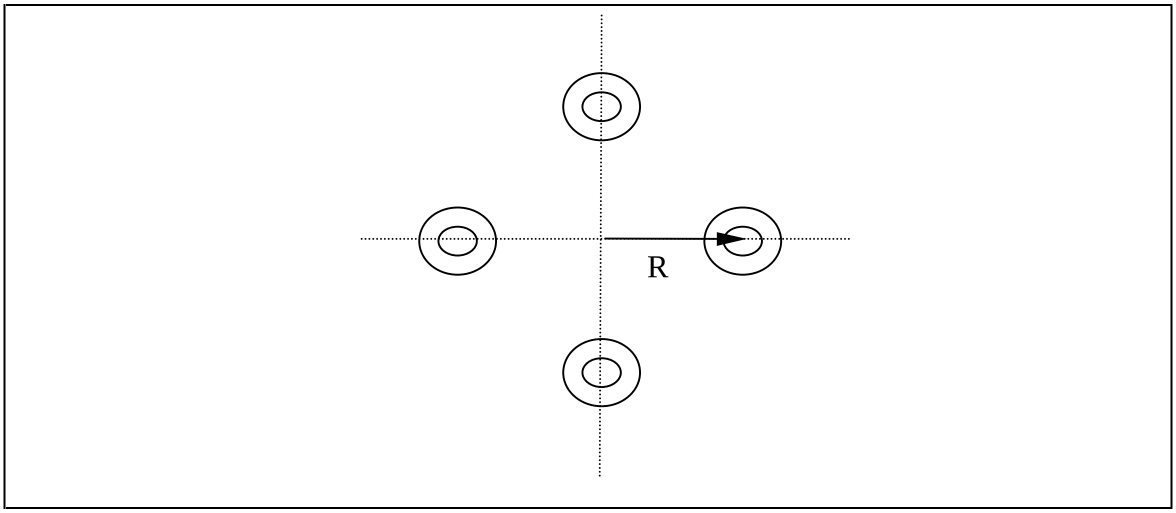

Figure 3. Cross-section of Detector Support-Structure

1.1. Loading

Loads will only be placed on the 4 mm rings. It is estimated that the load per ring will be between 1 kg to 5 kg. There will be seventy-five (75) 4 mm rings distributed along the length of the beam, for a total load of between 75 and 375 kg. Thus, the worst case of shear load between segments occurs near the ends of the beam where approximately half of the entire load is transmitted to the supports. If the load per ring is 5 kg, then the shear load at the end rings is in the range of 200 kg.

The total load per ring is, however, distributed between the four supporting beams. If it is assumed that the load is equally distributed between the beams, then the load per ring per beam only ranges from 0.25 to 1.25 kg, giving a maximum shear load at the end rings of approximately 50 kg.

1.2. Basic Beam Theory

Beams, such as those shown in Figure 4, where support is provided at each end to resist lateral forces, will deflect an amount which is dependent on the type of constraint at the end supports. The *simply-supported* case, shown in the left hand figure, is defined as allowing rotations to occur at each end. This is, as its name implies, a simple case. It is the easiest to build since the beam need only rest on top of supports as the supports resist movement in the vertical direction.

Figure 4. Beam Support Conditions.

A more rigid configuration is the *fixed-fixed* support condition. In this case, each end is 'fixed' against rotations, as well as displacements in the horizontal and vertical directions. Because of the increased rigidity, a fixed-fixed beam will deflect less than one which is simply-supported. The amount of deflection depends on:

1) the magnitude and location of the applied forces;
2) the type of material from which the beam is fabricated; and
3) the geometry of the beam (length and cross-sectional properties).

For the simply-supported beam, with one load (force) at centre span, the maximum mid-span deflection (i.e. the point directly under the load), is:

[1] $$\Delta_{max} = \frac{PL^3}{48EI}$$

where:

Δ_{max} is the maximum vertical displacement (mid-span) in metres,
P is the vertical force in Newtons,
L is the total length of the beam in metres,
E is the modulus of elasticity of the beam material in Pascals, and
I is the moment of inertia (second moment of area) in metre^4, see section 1.2.2.

Equation [1] is modified by a constant for the fixed-fixed case and becomes:

[2] $$\Delta_{max} = \frac{1}{192}\frac{PL^3}{EI}$$

If a load is continuously distributed over the length of the beam, such that load W, expressed in Newtons per metres is applied, then equation [1] becomes equation [3] for a simply-supported beam:

[3] $$\Delta_{max} = \frac{5}{384}\frac{WL^4}{EI}$$

and [2] becomes [4] for a fixed-fixed beam:

[4] $$\Delta_{max} = \frac{1}{384}\frac{WL^4}{EI}$$

Again, the mid-span deflection is dependent on the position and number of lateral loads, P or W. For loading situation where many equal loads are spread over the length of a beam, the maximum deflection tends towards equations [3] and [4]. Note: Only when loads are distributed symmetrically about the mid-span point (L/2) does the maximum deflection occur at L/2. For example, if all forces are on one side of mid-span, the maximum deflection will occur at a point other than the mid-span.

1.2.1. Principle of Superposition

An interesting property is the linear increase in deflection due to the addition of loads along a beam. If one places a single force at a point 1/3 of the beam length from one end of the beam, as shown in Figure 5, a deflection at mid-span will result. If a second load of equal magnitude is placed at 2/3 of the distance from the same end, then the mid-span deflection will be double. This is the *principle of superposition*. The resulting effect of several loads acting on a member simultaneously is the sum of the contributions from each of the loads applied individually.

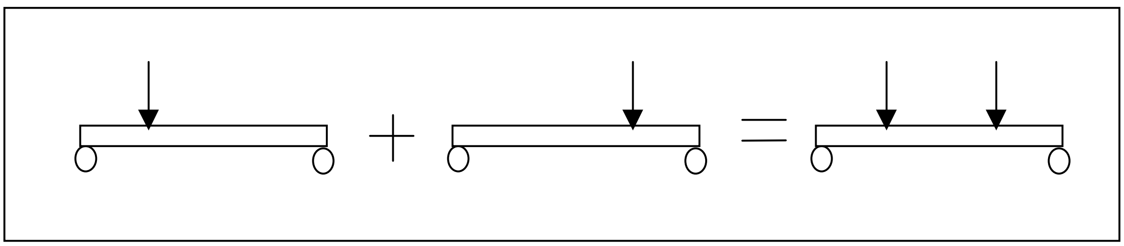

Figure 5. Principle of Superposition: Total Deflection Summation.

This principle can be used to create graphs, known as *influence line diagrams*, which indicate the deflection at mid-span due to a load at any given point along the beam. Figure 6 shows a sample influence diagram for some beam. If the deflection at mid-span is required for the case when a unit load is applied to point A, then the value of deflection at point A can be read from the figure. The same holds for a unit load at point B. If the two loads are in place simultaneously, the total mid-span deflection is the summation of the two individual deflections. The diagram is based on unit loads (1 Newton). If the load is a value which differs from 1, the deflection values from the diagram can be multiplied by the real load level since deflections vary linearly with load. This can be described mathematically, as will be performed in section 3.6.4.

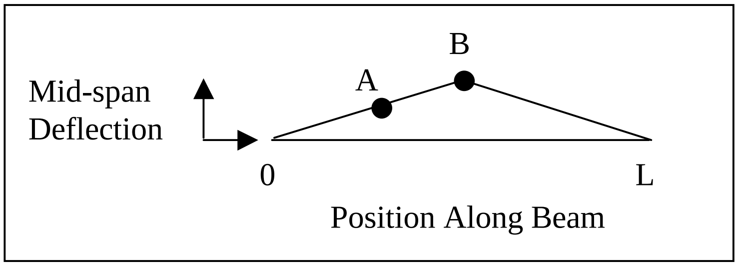

Figure 6. Example Influence Line Diagram.

1.2.2. Moment of Inertia, I

The value of I depends on the geometry of the cross-section. The inner radius, r_i, of one CITAL segment is 0.015 m and the outer radius, r_o, is 0.025 m. The major radius, R, is 0.26 m. These are shown in various configurations in Appendix F. Note that there is not much change in I if the four-beam structure is rotated 45°. The deflections are inversely proportional to I, as indicated in equations [1] through [4].

The four figures in Appendix F each show a different beam geometry. Only F.1 and F.4 will be considered here. Figure F.1 shows the single beam. This is the simplest case. Figure F.4 shows the four-beam configuration. The value of I for Figure F.4 assumes that there is a rigid connection between all of the members so that the radius, R, remains constant throughout loading. If this is not the case, then each of the four beams acts individually, thus having the same moment of inertia as Figure F.1.

The benefit of using the four beam structure is two-fold. First, it reduces the load which must be carried by each beam, and second, it increases the moment of inertia by a factor of more than 600. This increase helps to reduce deflections. Since deflections are inversely proportional to I, deflections will be reduced by a factor of 600. Again, however, this is only possible if the four beams are rigidly linked to one another.

For the detector, the four-beam structure will approach the rigid connection condition, allowing the use of the higher I value, however, the real value will be somewhere between the two cases, F.1 and F.4.

2. Objectives

2.1. Material Properties

The following material properties of CITAL CO-115M are required:

1) Elastic modulus;
2) Ultimate strength;
3) Load-rate sensitivity; and
4) Shear load resistance.

2.2. Analysis

The following parameters must be calculated for the single-beam and four-beam set-ups:

5) Maximum compressive stress;
6) Maximum deflection; and
7) Maximum shear load.

Items 5) and 7) will be compared to items 2) and 4), respectively, to determine if the proposed structure exceeds the material capacities. Item 6) is a serviceability requirement which must be within the client's acceptable limits.

3. Material Properties - Experiments

The following tests were carried out in the Structural Engineering Laboratory at the University of British Columbia.

1) Elastic modulus of CITAL in compression;
2) Time-dependency of CITAL in compression
3) Load-slip behaviour of CITAL;
4) Ultimate strength of CITAL;
5) Load-displacement of G10; and
6) Beam deflection of a single CITAL beam.

The set-up and results of each test will now be discussed.

3.1. Elastic Modulus

3.1.1. Apparatus

One 40 mm long segment was fitted with strain gauges. The gauges were mounted on the outer surface and were oriented to measure strain along the length of the segment. The gauges formed a half-bridge and with a scale factor of 1475.797 $\mu\varepsilon$/V (micro-strain per Volt), as shown in Appendix G.

A Baldwin loading frame was used. The maximum capacity of its load cell is 1.8 MN; the full scale was adjusted to suit the scale of measurement required for these tests.

The data acquisition system was PC-based. The acquisition rate was 2 Hz. Two channels were recorded: load and strain.

3.1.2. Specimen

A stack of five (5) segments was tested. The segment with the strain gauges was placed in the middle of the stack. The specimen was pre-assembled and surrounded in plastic wrapping to prevent dust from entering between the segments. The specimen was placed between two pieces of G10 material to prevent scratching of the ends. It is shown in Figure 7.

Figure 7. Compression specimen.

3.1.3. Results

A total of seven (7) tests were performed. Data was recorded during loading and unloading to determine if any permanent deformation occurred. The maximum load reached was 37.1 kN (32.0 MPa) and did not produce any failures. The results are summarised in Table 1.

Table 1. Compression test results.

Summary Information
Compression Specimens

Sample Name	Elastic Modulus (GPa) Loading	Elastic Modulus (GPa) Unloading	Loading Rate (kPa/s) Loading	Loading Rate (kPa/s) Unloading
TR1	56.5	55.0	66	59
TR2	56.2	55.4	99	85
TR3	58.3	62.9	290	463
TR4	59.7	64.3	380	303
TR5	58.6	67.1	602	458
TR6	55.5	55.1	189	1062
TR7	54.2	56.1	207	1769
Average	57.0	59.4		
S.D.	1.9	5.2		
C.O.V. (%)	3.4	8.7		

Load displacement curves are available in Appendix A. It was found that no significant difference in loading and unloading behaviour occurred and that there was no apparent permanent deformation of the specimen at this level of loading. Thus, behaviour was elastic and the value of elastic modulus, E, is 57.0 GPa.

Using this information, one can predict the amount of shortening in μm which occurs under axial stress. The shortening over a 1.06 m length (the full length of the CITAL beams in the detector) is:

[5] $$\Delta = \sigma \frac{L}{E}$$

where:

Δ is the amount of shortening in metres,
σ is the axial stress in the beam in Pascals,
L is the length of the beam in metres (1.06 m), and
E is the modulus of elasticity in Pascals (57×10^9 Pa)

For example, if the axial stress is 20 MPa, the shortening of the beam is equal to: $(20 \times 10^6)(1.06)/ 57 \times 10^9) = 372$ μm.

Equation [5] can be re-written in terms of mass, m, in kilograms (applied **axially**, not laterally) if one uses the cross-sectional area of a segment, A=1.963 x 10^{-3} m^2:

[6] $$\Delta = \frac{mgL}{AE} = \frac{m(9.81\frac{metre}{second^2})1.06}{1.963x10^{-3} \cdot 57x10^{9}} \cdot 10^{6} = 0.0929 \cdot m$$

where: Δ is displacement in μm, and
m is load in kilograms.

3.2. Load Rate Dependency

The results of the compression tests were also used to determine if the rate of loading had any effect on the value of elastic modulus. This test does not conclude if long term displacements under constant load (creep) occur in CITAL.

3.2.1. Results

The results are shown in Table 1 and in graphs in Appendix B. The data from the table (excluding outliers) are plotted in Figure 8 and showed only slight sensitivity to rate of loading in the range of loads and rates observed.

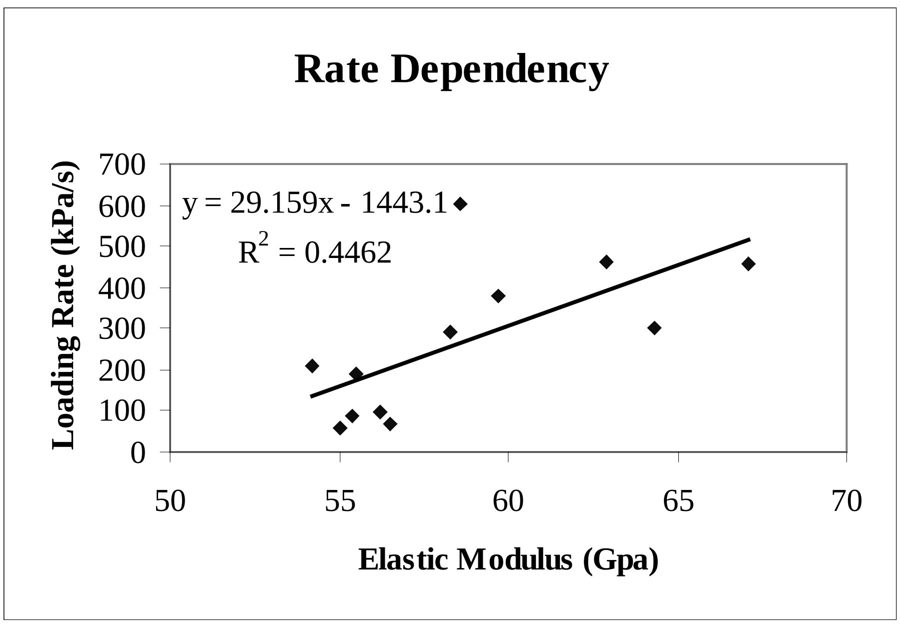

Figure 8. Load-rate dependency.