A note on χ^2 for straight lines

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I have looked at χ^2 distributions resulting from fitting straight lines to real data (run 1823) under different conditions. I make reference to three figures posted along with this text. Here is a description of the algorithm:

1. I start by considering for analysis only those planes which have exactly 1 hit. I insist that there be at least six such planes in both U and V.

2. I fit a straight line to wire centers, assuming an equal weight ($\sigma = 1$mm) for all points. I calculate χ^2 and residuals and reject any event for which any individual residual is greater than 0.5 mm, or the reduced χ^2 is greater than 3.0.

3. I use the time information and the $y = 0$ part of dc_str.00010 to calculate new hit positions. I use the straight line fits from step 2 to assign left-right. I fit a straight line to these new coordinates, still assuming an equal weight ($\sigma = 1$mm) for all points.

4. I calculate residuals for all points. I try flipping the left-right assignment of the point with the largest residual, and refitting. If the new χ^2 is lower than the old one, I again try flipping left-right for the point with the largest residual, and continue until the χ^2 increases after a flip.

The first subplots on the left of fig1 show the χ^2 distributions after step 2. The upper subplots are for U planes, while the lower ones are for V planes. The central subplots are after step 3, and the rightmost ones are for after step 4. (Note that the scales are all logarithmic.) The means of the distributions decrease from 0.74 to 0.19 to 0.05 for the U planes, and from 0.79 to 0.16 to 0.05 for the V planes. From this, I think we can conclude that the numbers in dc_str.00010 bear some relation to reality, and that we have to be careful about assigning left-right from fits to wire centers. Note that to generate these particular figures, I analyzed 100000 events, of which 78549 were good (icfail=0). This means that $67805/78549 = 86\%$ of events survive the cuts applied in step 1.

In the upper plots of fig 2 I show the 'experimental' x-t relations. That is, on the vertical axis I plot the time associated with each hit of each event. On the horizontal axis I plot the distance between the line fit in step 4 and the center of the hit wire. The 'U' shapes visible are the same as the input values from dc_str.00010, which at least shows that the method is self-consistent.

In the lower plots of fig 2 I plot the time on the horizontal axis. On the vertical axis I plot the distance between the line fit in step 4 and the calculated position based on time (i.e. the residual). I think this plot shows that our resolution is terrible for short times (i.e. less than 50 ns), and improves, or at least stays the same, for longer times.

Based on the information from the lower plot of fig 2, I reanalyzed the data from run 1823. But now, instead of assuming equal weights for all points, I assigned a σ of 0.02 cm to all points with times less than 50 ns, and a σ of 0.01 cm to those points with times greater than 50 ns. The resulting χ^2 distributions are shown in fig 3. The mean of the distributions in this plot is less than 1.0, and 18% of events have χ^2 greater than 2.0, which must mean than the simple assignment of σ's I choose is not a good represention of the truth.