More on Kalman Filtering

Outline

- Helix fitting in 3-D.
- Alternating UV measurements.
- Performance of the KF with a small number of measurements (22).
- Are there any failure modes?
  - Helices generated over a range of the radius of curvature (R).
  - Helices generated over a range of the dip angle (λ).
- Sensitivity to starting parameters.
- Convergence of the KF iterations.
- Sensitivity to starting values for the covariance matrix of predicted residuals (Cp).

Tests of the Kalman Filter

Helix Fitting in 3D

- Particle's momentum at the target

$$P_x = \frac{cB}{2|K|}\cos\varphi$$

$$P_y = \frac{cB}{2|K|}\sin\varphi$$

$$P_z = \frac{cB}{2|K|}\tan\lambda$$

B: magnetic field

K: inverse of radius of curvature R

ϕ: azimuthal angle

λ: dip angle

(parameters at the target)

- Helix equations

$$u = u_0 + \frac{1}{K}(\cos\varphi - \cos\varphi_0)$$

$$v = v_0 + \frac{1}{K}(\sin\varphi - \sin\varphi_0)$$

$$z = z_0 + \frac{1}{K}\tan\lambda(\varphi - \varphi_0)$$

- State vector

$$x_k = \begin{bmatrix} U \\ V \\ K \\ \tan\lambda \\ \varphi \end{bmatrix}$$

- Measurement Vector

$$m_k = \begin{bmatrix} U \\ V \end{bmatrix}$$

- Measurement transformation matrix

$$H^{uz} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H^{vz} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Extrapolation and Transport

- Extrapolate from detector k-1 to detector k
  - Calculate the ϕ angle at detector k

$$\varphi_k = \varphi_{k-1} + K\frac{z_k - z_{k-1}}{\tan\lambda}$$

  - Calculate the X/Y coordinates

$$u_k = u_{k-1} + \frac{1}{K}(\cos\varphi_k - \cos\varphi_{k-1})$$
$$v_k = v_{k-1} + \frac{1}{K}(\sin\varphi_k - \sin\varphi_{k-1})$$

  - Assume that

$$K_k = K_{k-1}$$
$$\tan\lambda_k = \tan\lambda_{k-1}$$

- The Transport matrix

$$F_{k-1} = \begin{bmatrix} \frac{\partial u_k}{\partial u_{k-1}} & \frac{\partial u_k}{\partial v_{k-1}} & \frac{\partial u_k}{\partial K_{k-1}} & \frac{\partial u_k}{\partial \tan\lambda_{k-1}} & \frac{\partial u_k}{\partial \varphi_{k-1}} \\ \frac{\partial v_k}{\partial u_{k-1}} & \frac{\partial v_k}{\partial v_{k-1}} & \frac{\partial v_k}{\partial K_{k-1}} & \frac{\partial v_k}{\partial \tan\lambda_{k-1}} & \frac{\partial v_k}{\partial \varphi_{k-1}} \\ \frac{\partial K_k}{\partial u_{k-1}} & \frac{\partial K_k}{\partial v_{k-1}} & \frac{\partial K_k}{\partial K_{k-1}} & \frac{\partial K_k}{\partial \tan\lambda_{k-1}} & \frac{\partial K_k}{\partial \varphi_{k-1}} \\ \frac{\partial \tan\lambda_k}{\partial u_{k-1}} & \frac{\partial \tan\lambda_k}{\partial v_{k-1}} & \frac{\partial \tan\lambda_k}{\partial K_{k-1}} & \frac{\partial \tan\lambda_k}{\partial \tan\lambda_{k-1}} & \frac{\partial \tan\lambda_k}{\partial \varphi_{k-1}} \\ \frac{\partial \varphi_k}{\partial u_{k-1}} & \frac{\partial \varphi_k}{\partial v_{k-1}} & \frac{\partial \varphi_k}{\partial K_{k-1}} & \frac{\partial \varphi_k}{\partial \tan\lambda_{k-1}} & \frac{\partial \varphi_k}{\partial \varphi_{k-1}} \end{bmatrix}_{k-1}$$

Sensitivity to Starting Parameters & Number of Measurements

(1) Sensitivity to R

- Generated helix parameters:

  $\phi_0 = 45^0$

  $u_0 = +5$ cm

  $v_0 = -5$ cm

  $\lambda = 60^0$

- Fit starting parameters:

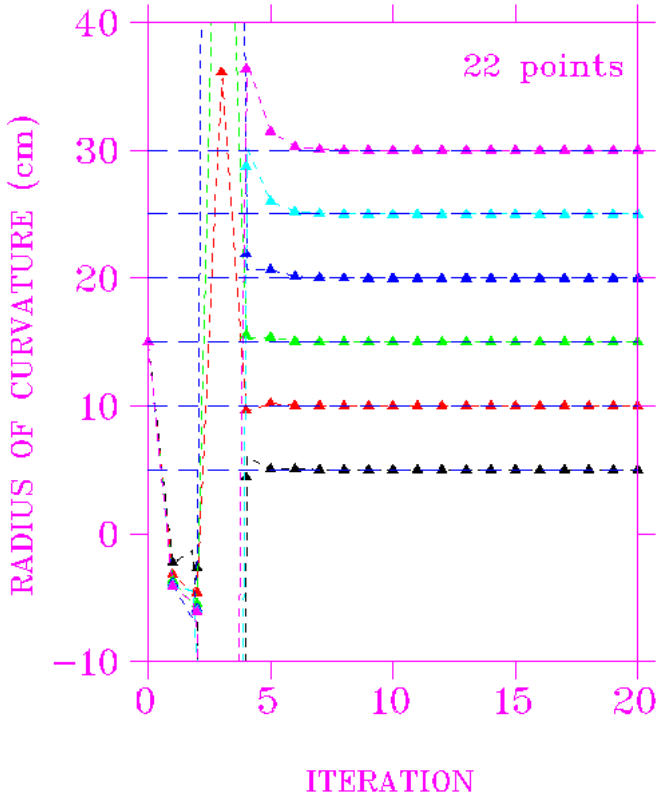
  $\phi_0 = 0^0$

  $u_0 = 0$ cm

  $v_0 = 0$ cm

  $\lambda = 40^0$

  $R = 5$ cm



Sensitivity to Starting Parameters & Number of Measurements

(2) Sensitivity to λ

- Generated helix parameters:

  $\phi_0 = 45^0$

  $u_0 = +5$ cm

  $v_0 = -5$ cm

  R = 5 cm

- Fit starting parameters:

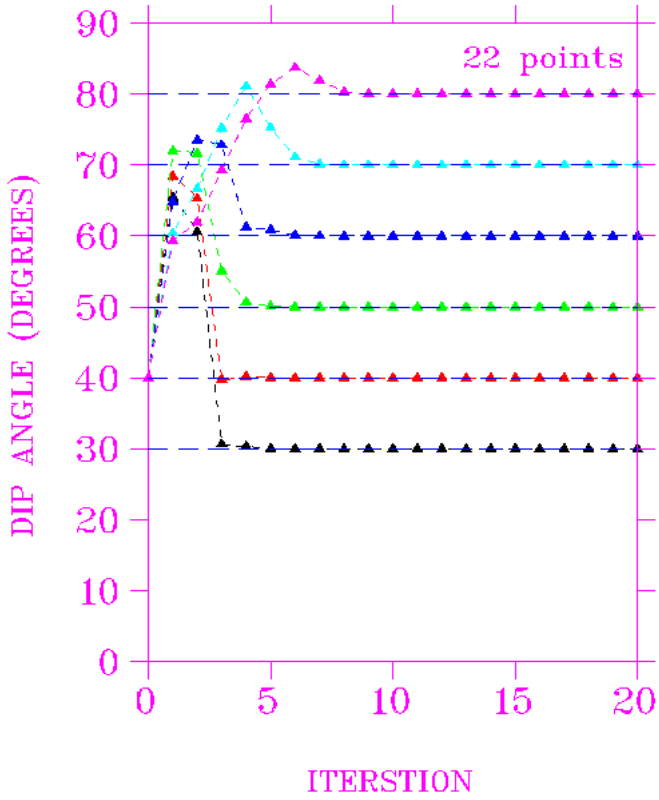
  $\phi_0 = 0^0$

  $u_0 = 0$ cm

  $v_0 = 0$ cm

  R= 15 cm

  $\lambda = 40^0$



Sensitivity to Starting Parameters
&
Convergence Tests

Generated helix parameters:

$\phi_0 = 45^0$

$u_0 = +5$ cm

$v_0 = -5$ cm

R = 15 cm

$\lambda = 50^0$

Fit starting parameters:

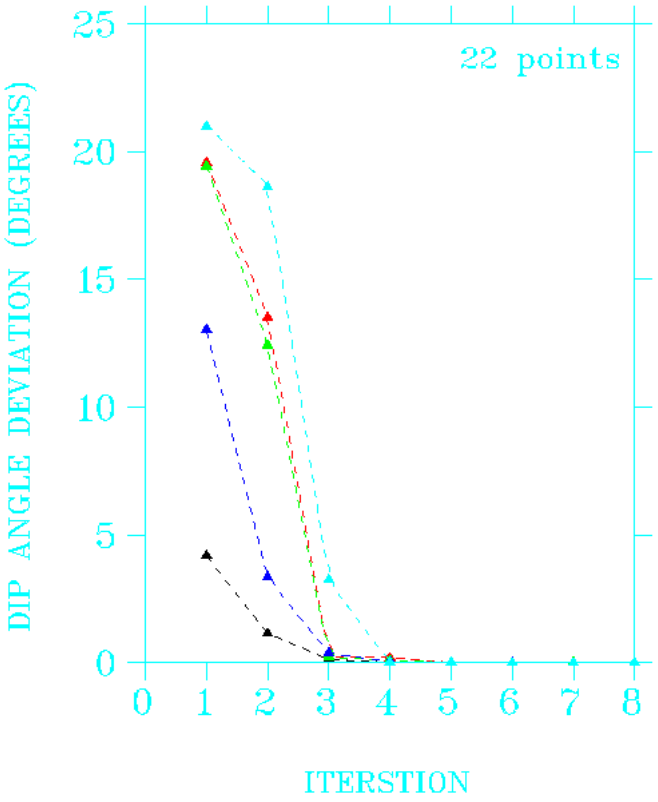
- - - - 10% off

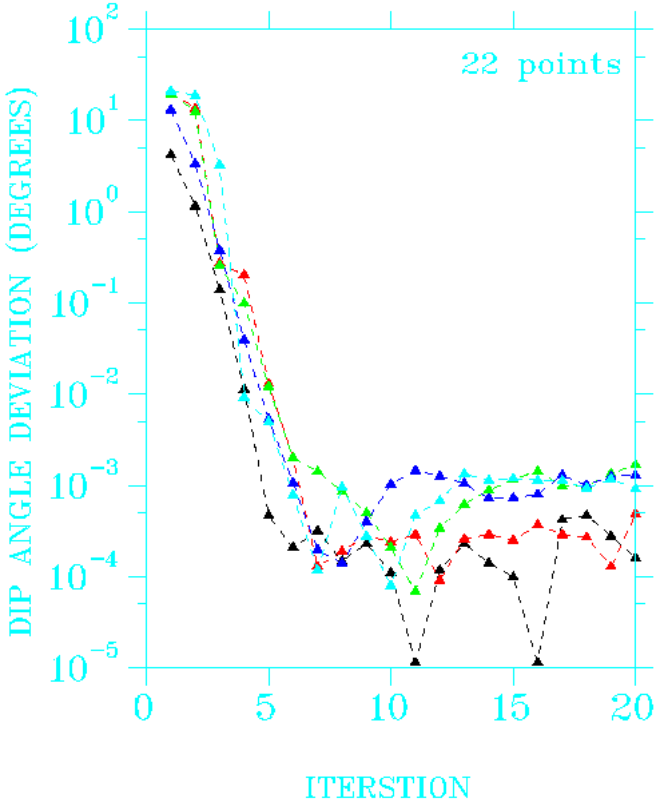
- - - - 30% off

- - - - 50% off

- - - - 70% off

- - - - 90% off





Convergence & Sensitivity to Covariance Matrix

Generated helix parameters:

$\phi_0 = 45^0$

$u_0 = +5$ cm

$v_0 = -5$ cm

$\lambda = 60^0$

Fit starting parameters:

$\phi_0 = 0^0$

$u_0 = 0$ cm

$v_0 = 0$ cm

R = 15 cm

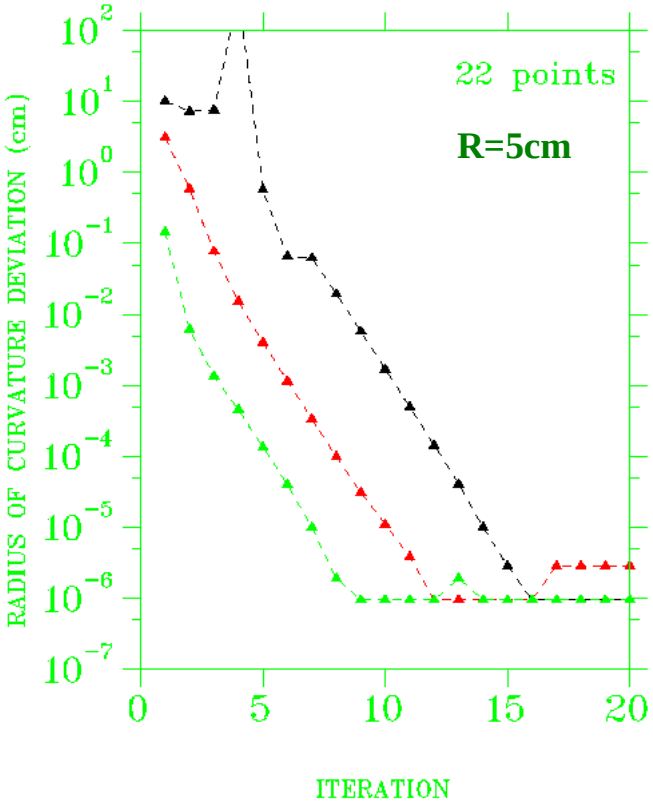
$\lambda = 40^0$

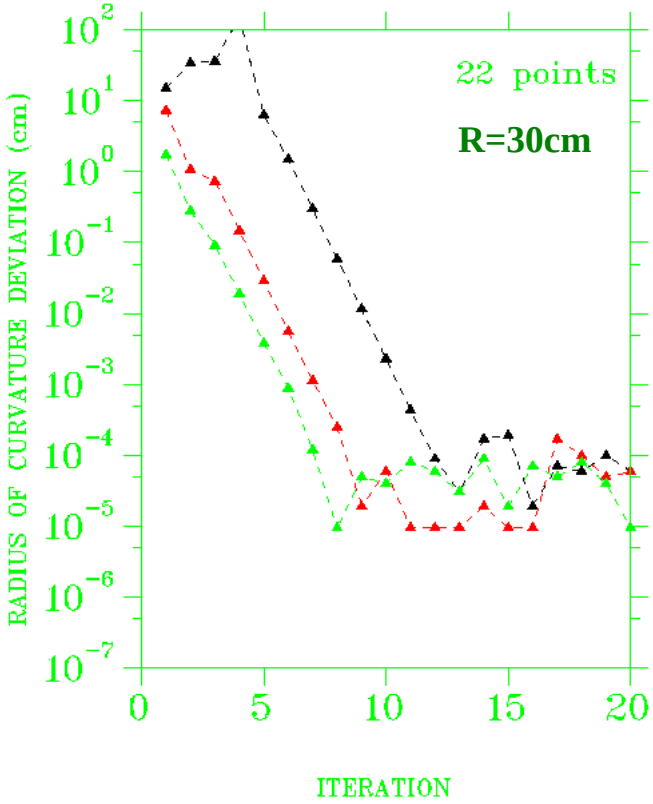
Fit starting parameters:

- - - - $C_p = 0.1$

- - - - $C_p = 0.001$

- - - - $C_p = 0.00001$





Conclusions

• Helix fitting in 3-D.	Works fine! Verifies correct formulation, derivatives and code.
• Alternating UV measurements.	
• Performance of the KF with a small number of measurements (22).	Works fine when iterated.
• Are there any failure modes?	None intrinsic to KF (Cp sensitive).
• Sensitivity to starting parameters.	Not too sensitive (might change with correct choice of Cp).
• Convergence of the KF iterations.	Converges over a range of about 6-8 iterations (Cp dependent).
• Sensitivity to starting values for the covariance matrix of predicted residuals (Cp).	Fairly sensitive.