Michel spectrum fitting and sensitivity of the parameters to the energy scale.

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This is a current status of my attempt to look at a dependence of Michel parameters on the energy scale.

I don't' see a sound trend in variation of parameter values when the scale vary. Also, I'm surprised by big errors (of the order of 10^{-3} *!) returned by the fits. According to the MIG table they should be 10 times smaller.*

I would like to hear suggestions on improving performance of the fits.

A Michel spectrum function including first order radiative corrections (taken from the GEANT program) was used in the study. All histograming and fitting was done in the ROOT program.

15 2-D histograms were defined with 110 bins in [0., 1.1] dimensionless energy range and 200 bins in [-1., 1.] $\cos(\theta)$ range each. 10^9 entries were filled into each of the histograms at random with probability density function proportional to $f_{\text{Michel}}(x/(1+\text{escale}), \cos(\theta))$. Michel parameters were fixed to their SM values and P_μ was fixed[1] to -1, while deviation of the energy

[1] There is no P_μ in the GEANT functions. Rather, GEANT uses $F_{IS}(x)+\cos(\theta)*F_{AS}(x)$ to generate decay in the muon rest frame where the particle has a definite spin orientation. F_{AS} depends, among other parameters, on ξ (not $P_\mu\xi$). Direction of muon spin with respect to detector is taken care of separately on an event-per-event basis.

I introduced polarization P_μ in the formulae as $F_{IS}(x) + P_\mu \cos(\theta) * F_{AS}(x)$. Note that P_μ and ξ are two independent variables. They should be strongly correlated, but P_μ was fixed in this study anyway.

scale varied from -0.0014 to +0.0014 with step 0.0002 between histograms. ($1 \times 10^{-4} = 5.283\,\mathrm{KeV}$, so the range is approximately -75 KeV to +75 KeV.)

Then, the same function was used to fit the histograms. Since forward and backward halves of the detector will be calibrated independently, I defined two fit regions to study the sensitivity for both parts separately. The regions were:

1. $[0.4, 0.95]$ in energy, $[-0.98, -0.5]$ in $\cos(\theta)$
   Contained approximately 2.38×10^8 events.

2. $[0.4, 0.95]$ in energy, $[0.5, 0.98]$ in $\cos(\theta)$
   Contained approximately 1.44×10^8 events.

($\arccos(0.98) = 11.48^o$.) Note that if a region would contain 10^9 events statistical errors would be only 2–2.5 times smaller.

Both χ^2 and maximum likelihood fits were tried. To be exact, the option string to the fit function was "R0E" or "ROLE". My impression is that the maximum likelihood fits performed better. A fit status (apparently from MINOS?) for them alternated between "PROBLEMS" and "SUCCESSFUL", while for the χ^2 fits it was either "PROBLEMS" or "FAILURE". However, for all fits, including the failed ones, final χ^2/NDF was very close to 1. (As far as I understand, even a likelihood fit in ROOT returns it's equivalent χ^2.) I'm going to post examples of the fitting routine printout on the Web in addition to this document.

All four Michel parameters varied freely during the fits. Global normalization was the 5th free parameter in a set of fits. In another set of fits, f_{Michel} normalization was fixed so that it's integral over the fit region was always equal to an integral of the histogram contents over the same region.

Fit results were put into histograms you can see on Fig. 3–8. X-axis is the deviation of the energy scale. Leftmost and rightmost bin centers corresponds to -75 KeV and +75 KeV energy scale deviation. The central bin (x=0) contains results of the *undistorted* spectrum fit. Look, not only the errors are big, but the ρ value itself is off by more than 1×10^{-3}! (That is consistent with the fit errors...)

Random number generator is probably not to be blamed. When the histograms were filled using TRandom, which is default ROOT generator, χ^2 of the fits was 0.1 instead of 1. After I replaced the generator with a good one (TRandom3, claimed to have period of $2^{19937}-1$) χ^2 became "correct". (By the way, it's only 2 times slower than the first one!)

Mistakes in the spectrum function are also excluded, because the (undistorted) fit was made with the *same* function.

To study the E-scale dependence, one of things to try is to fill the histograms more carefully. The FillRandom() ROOT method uses function value in the bin center to calculate probability of filling an entry into the bin. This is not good enough for bins at the edge of Michel spectrum. So, I rewrote the method to use integral of the function over the bin instead of it's value in the center. Still, I'm not sure that the integral catches the spectrum edge for small E-scale deviations. (Probably it's not.)

But this can't explain poor results of the undistorted spectrum fit.

It's also possible to combine the two halves of the spectrum and fit (upstream+downstream) for ρ and η, and (upstream-downstream) for ξ and δ. But for these fits E-scale dependence of Michel parameters should be considered as a function of two variables instead of one. (Upstream and downstream E-scale deviations will be, in general, different.) So, I didn't want to start with it. (Were the MIG statistical errors on parameters obtained in this approach?)

Other thoughts/suggestions?

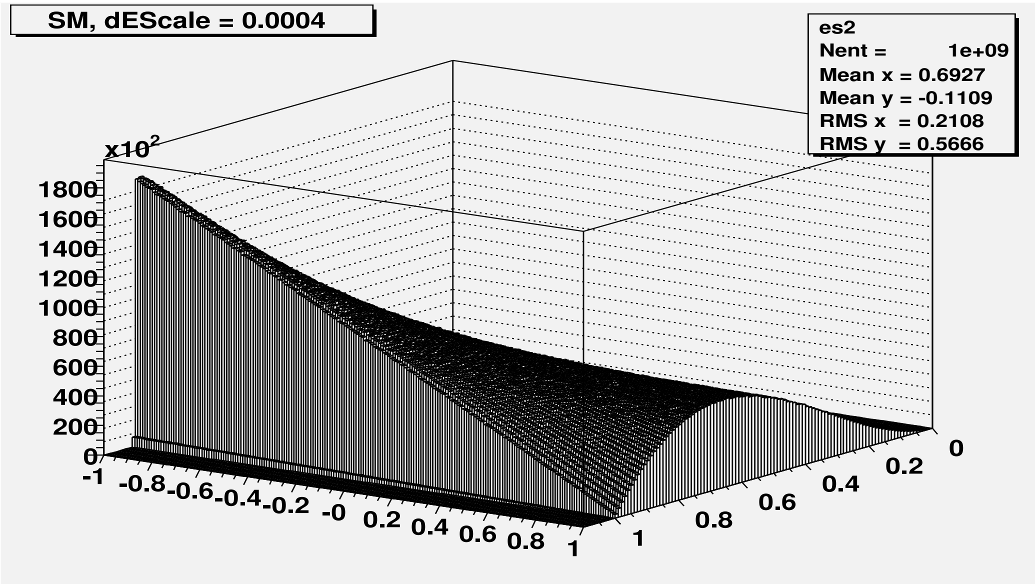

Figure 1: An example of Michel distribution.

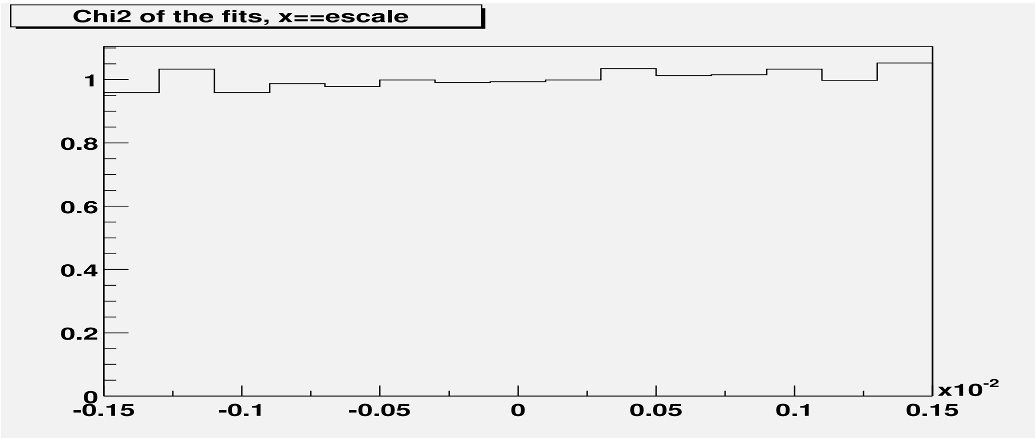

Figure 2: χ^2 of a set of fits. (Looks the same for all sets.)

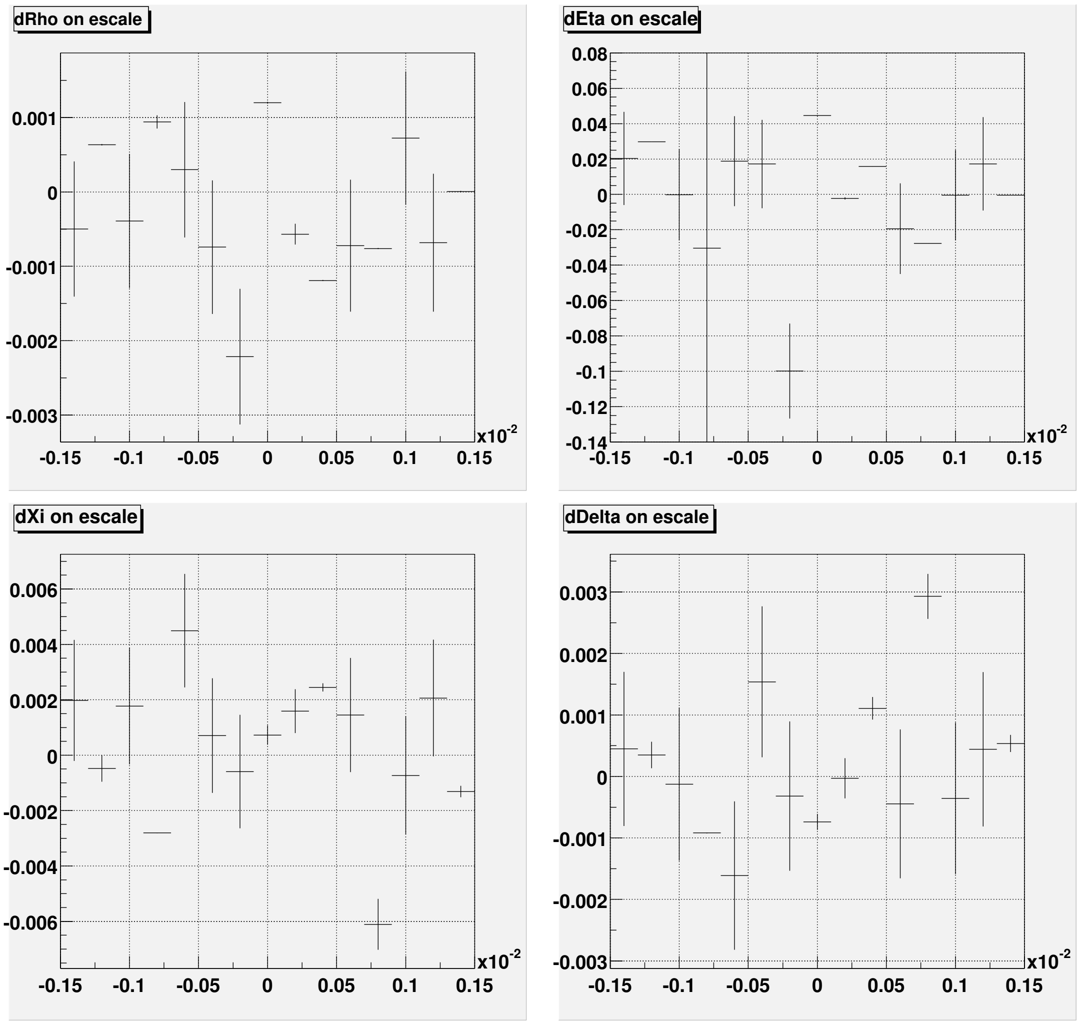

Figure 3: Dependence of Michel parameters on energy scale deviation. χ^2 fit results. Adjustable normalization. Fit region: $0.4 < x < 0.95$ and $-0.98 < \cos(\theta) < -0.5$.

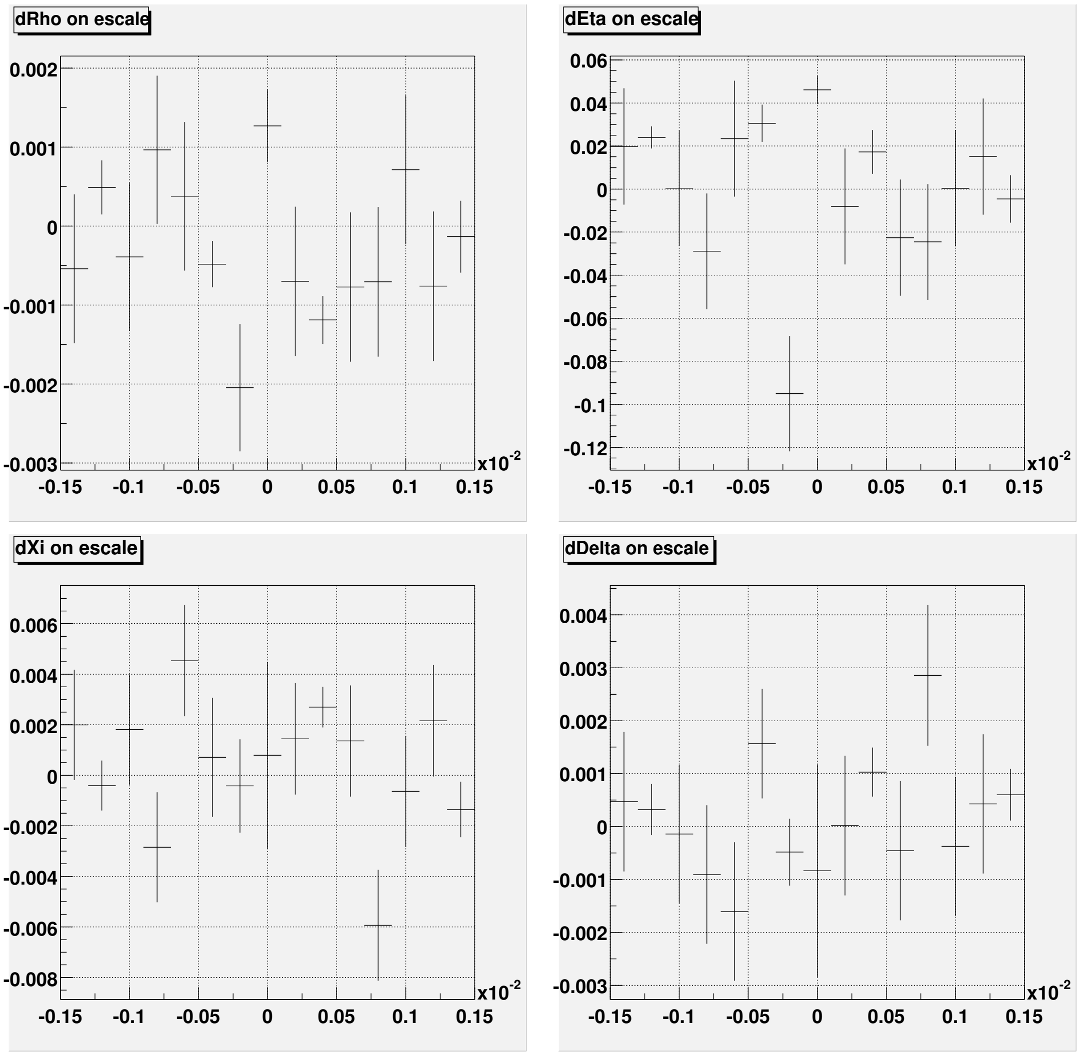

Figure 4: Dependence of Michel parameters on energy scale deviation. Maximum log likelihood fit results. Adjustable normalization. Fit region: $0.4 < x < 0.95$ and $-0.98 < \cos(\theta) < -0.5$.

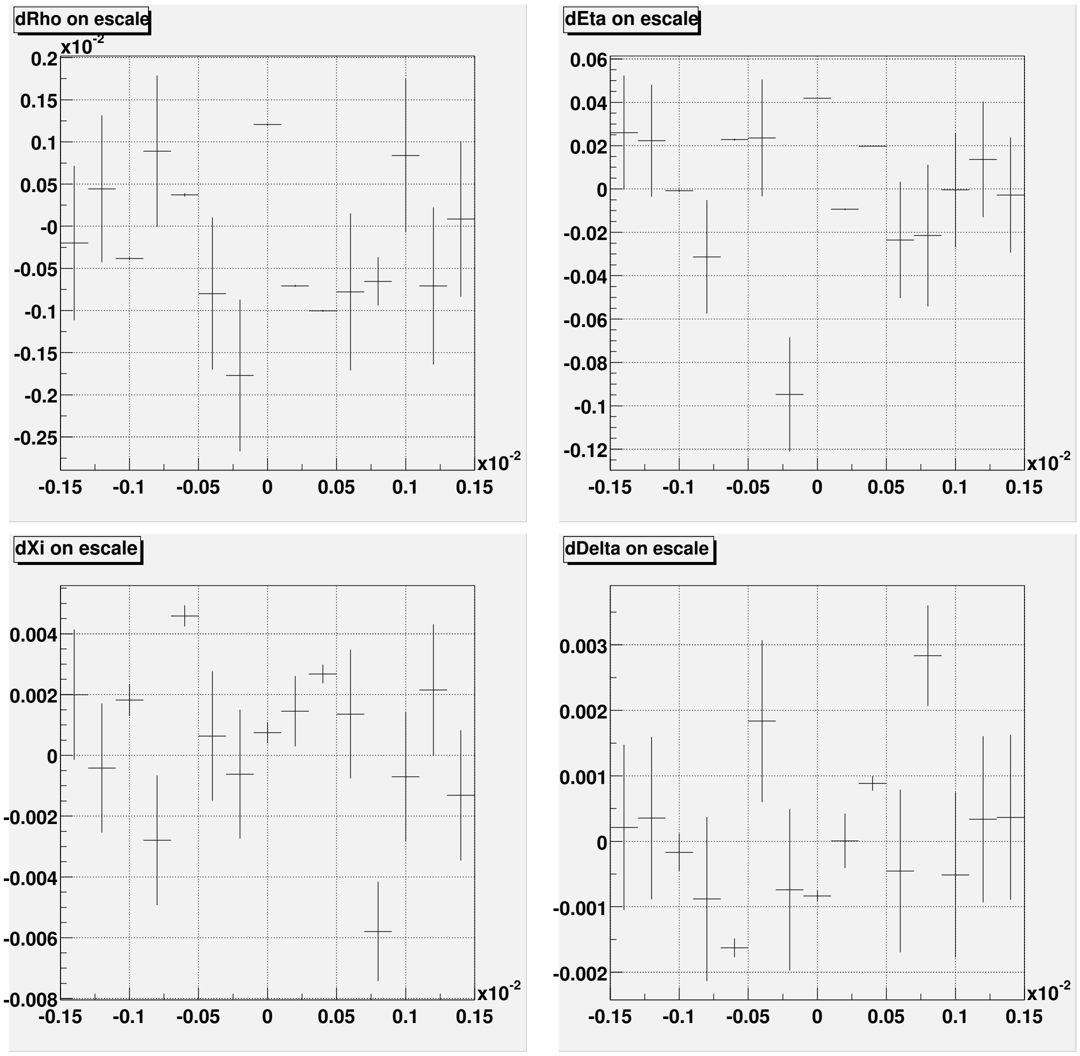

Figure 5: Dependence of Michel parameters on energy scale deviation. χ^2 fit results. Fixed normalization. Fit region: $0.4 < x < 0.95$ and $-0.98 < \cos(\theta) < -0.5$.

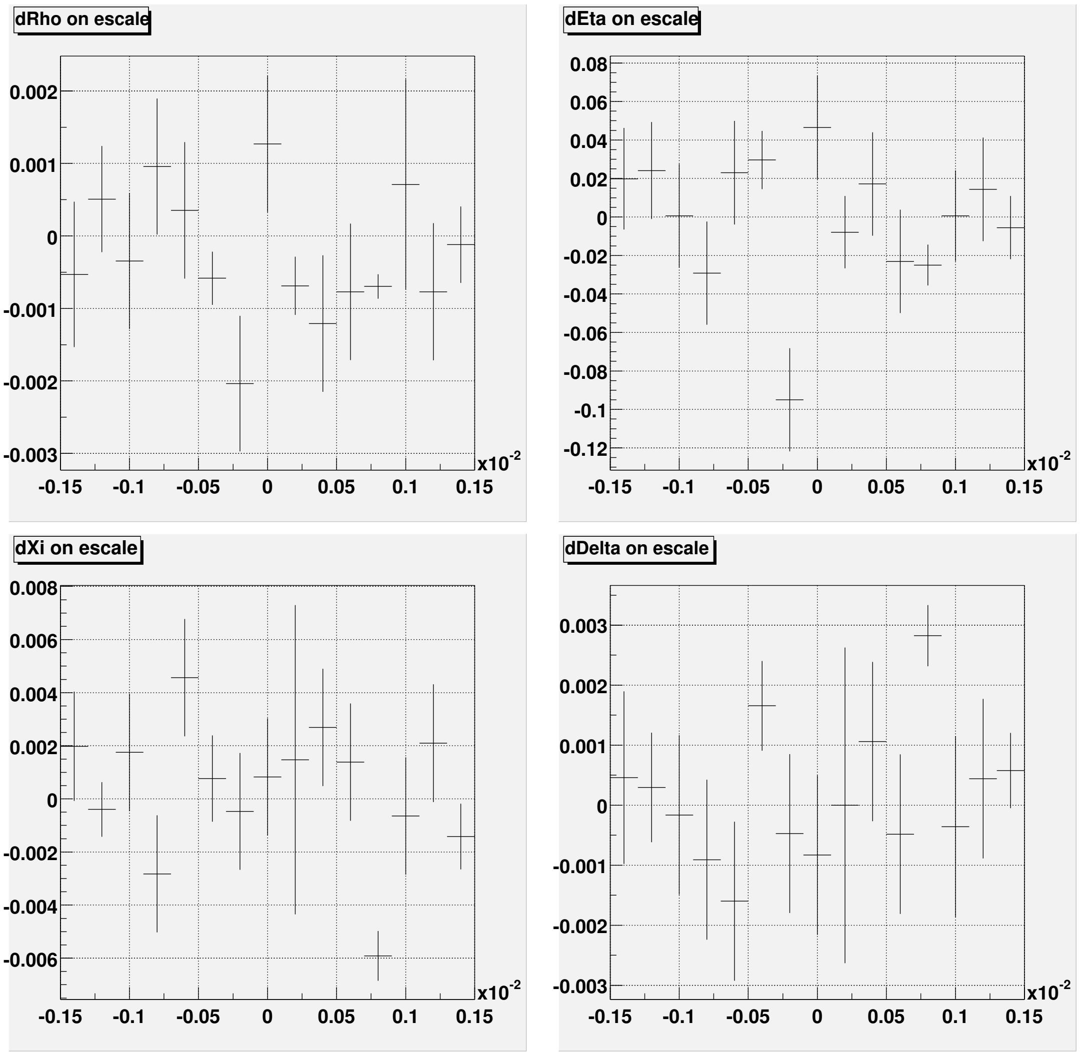

Figure 6: Dependence of Michel parameters on energy scale deviation. Maximum log likelihood fit results. Fixed normalization. Fit region: $0.4 < x < 0.95$ and $-0.98 < \cos(\theta) < -0.5$.

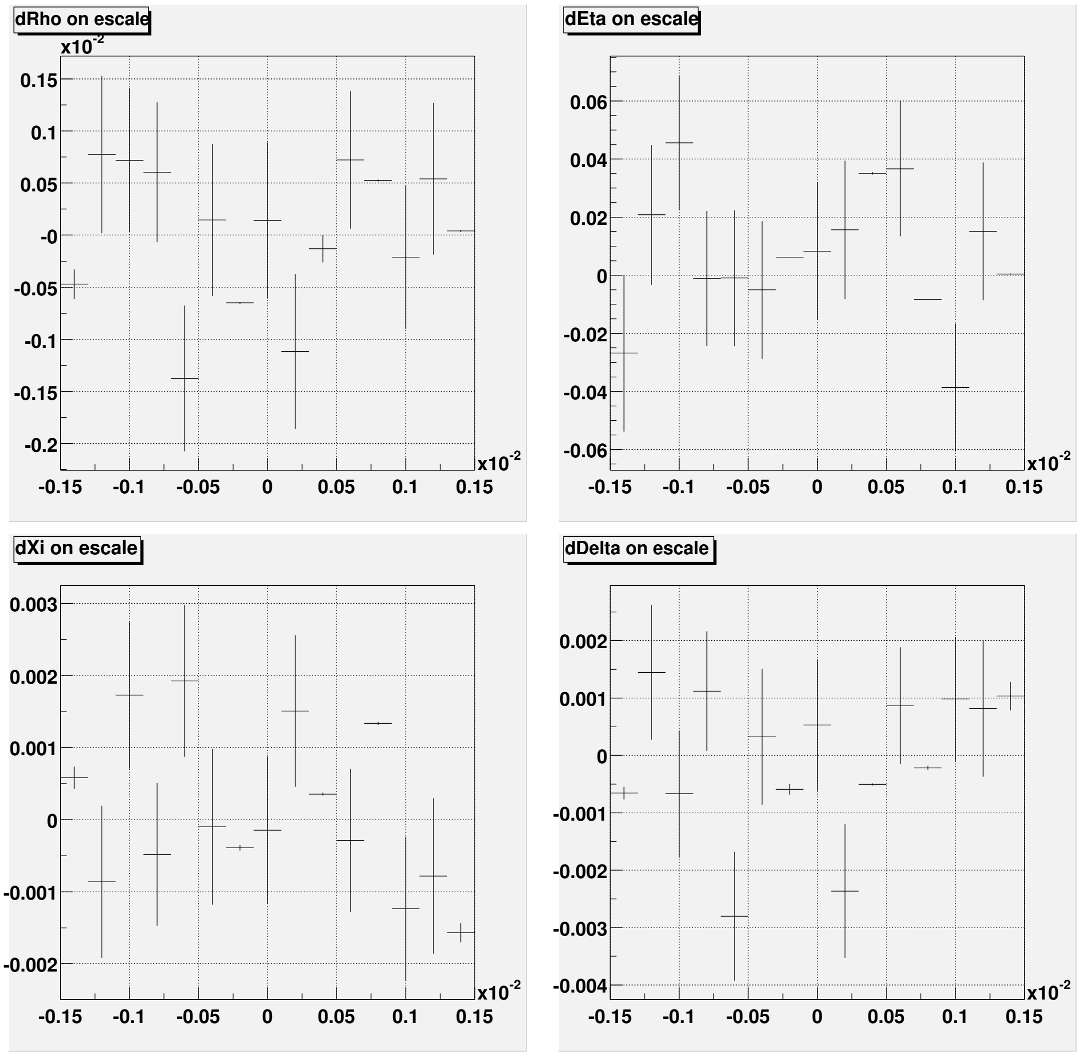

Figure 7: Dependence of Michel parameters on energy scale deviation. χ^2 fit results. Fixed normalization. Fit region: $0.4 < x < 0.95$ and $0.5 < \cos(\theta) < 0.98$.

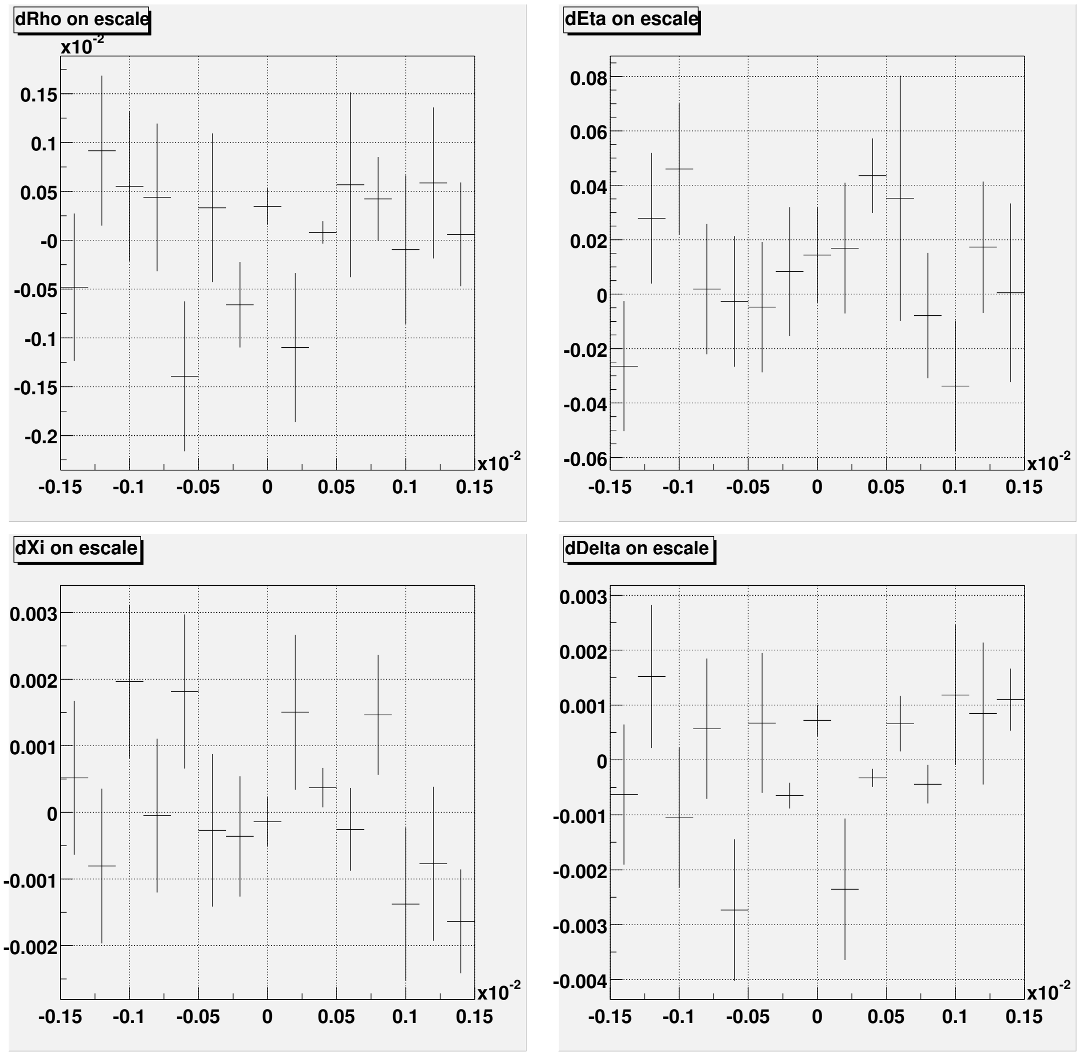

Figure 8: Dependence of Michel parameters on energy scale deviation. Maximum log likelihood fit results. Fixed normalization. Fit region: $0.4 < x < 0.95$ and $0.5 < \cos(\theta) < 0.98$.