The Measurement of ρ

The Michel spectrum can be written as the sum of two terms, $F(\rho, x, \eta)$ and $G(P_\mu, \cos\theta, x, \delta, \xi)$.

$$d^2\Gamma / [dx\, d(\cos\theta)] = F(\rho, x, \eta) + G(P_\mu, \cos\theta, x, \delta, \xi).$$

where

$$F = (x^2 - x_o^2)^{1/2}\, [6x(1 - x) + 4/3\, \rho(4x^2 - 3x - x_0^2) + 6\eta x_o(1 - x)]$$

and

$$G = P_\mu\, \xi \cos\theta\, (x^2 - x_o^2)[2(1 - x) + 4/3\delta(4x - 3 - x_o^2)]$$

By minimizing the value of G, one can limit the influence of its unknown parameters on the measurement of ρ. This can be done by minimizing P_μ and/or integrating the spectrum over $\cos\theta$. To this end, various experimental and computational studies are being proposed.

Experimentally-

- The rate of Michel positrons versus P_μ should be determined for a range of momentum about the nominal surface muon momentum.
  If it is determined that $P_\mu \approx 0$ for a given momentum selection and the rates are sufficient then data will be collected using that momentum selection. If it is found that P_μ is opposite in sign to the surface muon P_μ and the rates are sufficient, then data will be collected using that momentum in combination with data collected from surface muons.
- A search should be made to locate and quantify any asymmetry in the detector (or analysis package).
  A difference between front and back detector efficiencies would produce a distribution dependent on θ such that integrating over $\cos\theta$ would not eliminate G and thus would not eliminate dependencies on δ and ξ.
- The detector must be calibrated relative to the positron energy.
- Reconstruction efficiency as a function of θ and x should be determined.

Computationally-

- The sensitivity of the ρ measurement method to the following should be determined:
  - $\Delta\rho \neq 0$
    - (Where $\rho = 3/4 + \Delta\rho$) This sensitivity should be maximized. It should also be compared to those listed below.
  - $\eta \neq 0$
  - $x_o \neq 0$
  - $P_\mu\, \xi \neq 0$
  - asymmetry of the detector
    - It is necessary to understand how a change in any of the above values effects the determination of $\Delta\rho$. This information will be used to ascertain the precision with which we are able to measure ρ or conversely the precision with which we must know these values. An effort will be made to minimize these sensitivities once they are understood.
  - range in θ and in x
    - The sensitivity to θ and x will be taken in combination with information about the reconstruction efficiency as a function of θ and x to determine the range of data analyzed. Rates as a function of θ and x will also be considered.
  - energy calibration

- The number of events necessary for the required precision should be determined.
  The length of time required to observe the necessary events, as a function of the momentum selection, must be feasible.

The computational studies are in progress. The measurement of ρ is being treated as a measurement of the correction to the standard model value of 3/4. ($\rho = 3/4 + \Delta\rho$) As such the angle independent distribution (F, from above) is written as the standard model distribution, Λ, plus a correction term, $\Delta\rho B$.

$$d\Gamma/dx = \Lambda(x, \eta) + \Delta\rho B(x)$$

Where

$$\Lambda(x, \eta) = 2(x^2 - x_o^2)^{1/2} [6x(1 - x) + (4x^2 - 3x - x_o^2) + 6\eta x_o(1 - x)]$$

and

$$B(x) = 2(x^2 - x_o^2)^{1/2} [4/3(4x^2 - 3x - x_o^2)]$$

The observed spectrum will be fit to the above distribution. The current studies on sensitivities and precision are being done assuming this method.

As an example, consider the study of sensitivity to detector efficiency asymmetry.

Let all Michel parameters take the Standard Model values. Let $P_\mu = 1$ and note that $x_o \approx 0$. Then

$$F = x^2(-2x + 3) \qquad\qquad G = \cos\theta\, x^2(2x - 1)$$

and

$$\Lambda(x) = 2x^2(-2x + 3) \qquad\qquad B(x) = 8/3\, x^2(4x - 3)$$

If the asymmetry in detector efficiency were to make a 1% difference between upstream and downstream then $F \Rightarrow F_m$ and $G \Rightarrow G_m$ where

$$F_m = \begin{cases} F & \text{for } 0 < \theta < \pi/2 \\ .99F & \text{for } \pi/2 < \theta < \pi \end{cases}$$

and

$$G_m = \begin{cases} G & \text{for } 0 < \theta < \pi/2 \\ .99G & \text{for } \pi/2 < \theta < \pi \end{cases}$$

Integrating over $\cos\theta$ yields

$$(d\Gamma/dx)_m = 1.99x^2(-2x + 3) + .01x^2(2x - 1)$$

Fitting this spectrum to $d\Gamma/dx = N[\Lambda(x) + \Delta\rho B(x)] = N[2x^2(-2x + 3) + \Delta\rho\, 8/3\, x^2(4x - 3)]$ yields

$N = .997$ (normalization)

$\Delta\rho = -2.6 \times 10^{-3}$

In a similar manner, I am exploring the effect including a limited range of θ acceptance. For instance, using the same setup, but limiting collection to $10° < \theta < 70°$ (and $110° < \theta < 170°$) yields the following:

$N = 0.642$

$\Delta\rho = 4.07 \times 10^{-3}$

Likewise, using the original setup, but limiting collection to $0.3 \leq x \leq 0.9$ yields the following:

$N = 0.997$

$\Delta\rho = 2.51 \times 10^{-3}$

Limiting both angle and energy acceptance, $10° < \theta < 70°$ (and $110° < \theta < 170°$) and $0.3 \leq x \leq 0.9$ yields:

$N = 0.642$

$\Delta\rho = 3.90 \times 10^{-3}$

From these studies, one can see the importance of understanding detector efficiency asymmetry. Ignoring such an asymmetry could result in a significant inaccurate adjustment to ρ.

Currently these studies are being done crudely to examine broad characteristics. I will be recalculating these with more realistic samples and techniques. This is posted in hopes of garnishing input concerning errors or omissions regarding the methodology.