We need to estimate the gas contamination after applying the psPACT cut.

First of all, what DOESN'T work:

1. Using the simulation, since the PC response is not properly reproduced.
    See my previous posting on psPACT plots for different z-ranges in the simulation, which
    shows that the simulation does not reproduce the tail of gas stops:
    https://twist.phys.ualberta.ca/forum/view.php?bn=twist_software&key=1208815645

2. Trying to fit the sum of two exponential functions in the mostly-metal and mostly-gas zones.
    I tried an iterative fit (fit mostly-gas zone with lambda_metal fixed, then fit mostly-metal zone
    with lambda_gas fixed etc.), but the data does not have the necessary sensitivity.
    [Aside: We tried this in muSR where there were muons stopping in the metal target and the
    trigger scintillator]

Dick guided me through an alternative approach that uses **only the data**:

Consider the distribution of PC5 vs PC6 widths for s68-a3, chosen because the stopping distribution
was peaked a little upstream ("1/3-stops"), maximising PC6 gas stops:
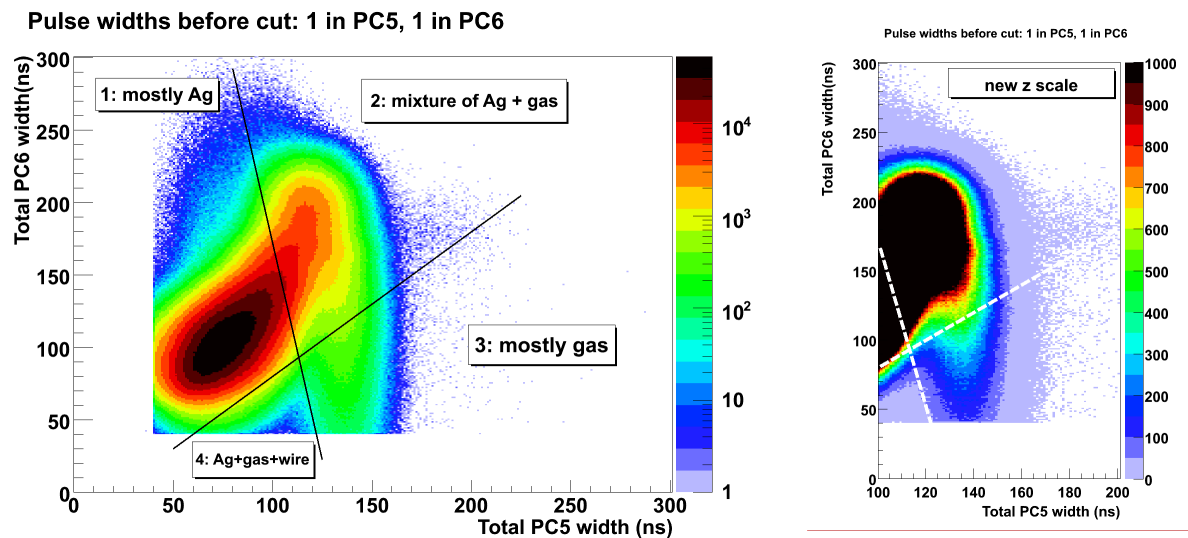
The nearly-vertical cut line runs parallel with the gas band. The gas band has a tail that leaks over this
cut line – this is more obvious in the right hand figure, where the scale is changed. We need to estimate
how much of the gas band leaks across the zone 1 / 2 boundary to estimate the systematic uncertainty
for the psPACT. Specifically, this systematic is "how much gas contamination is left after the psPACT
cut".

Three pieces of information are needed:

1. Depolarisation in the gas (lambda, and any fast depol).
2. Fraction of gas leaking across the zone 1 / 2 boundary.
3. Fraction of gas stops for PC6.

1. Depolarisation in the gas

A special treesum for **just zone 3** allowed the depolarisation in the gas to be determined.
For set 68 (Ag) the results were:

Zone	Lambda (per ms)	Pmu0
1	0.9 +- 0.2	0.9972 +- 0.0007
3	28 +- 3	1.00 +- 0.01

There are other posted results for set 83 (Al) that indicate a lack of fast depolarisation when selecting pure gas stops in PC6, but find ~4% change in pmu0 when selecting PC5/7/8 = gas+foil. See:
https://twist.phys.ualberta.ca/forum/view.php?site=twist&bn=twist_physics&key=1250713050

The difference in <Pmu> between the gas and metal can be calculated using the "lever-arm" method:

$$\frac{\int_{t_1}^{t_2} N(t) \cdot P_\mu(0) \exp\left(-\lambda_2 t\right) dt}{\int_{t_1}^{t_2} N(t) dt} - \frac{\int_{t_1}^{t_2} N(t) \cdot P_\mu(0) \exp\left(-\lambda_1 t\right) dt}{\int_{t_1}^{t_2} N(t) dt},$$
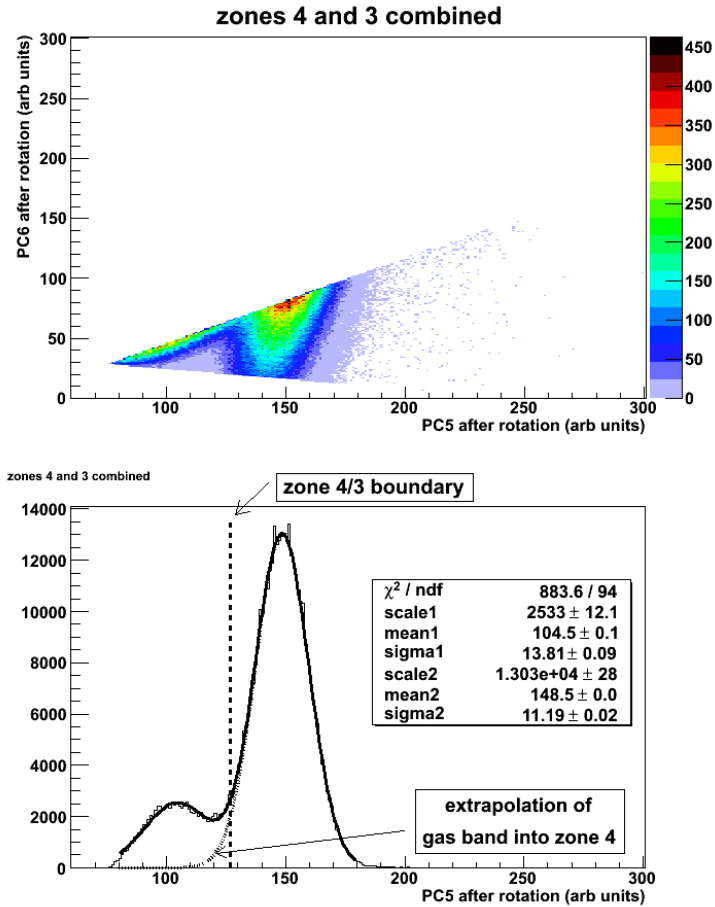
where $N(t) = N(0) \exp\left(-t/\tau_\mu\right)$ and τ_μ is the muon lifetime, and λ_1 and λ_2 are the relaxation rates

This gives **7.7% as the depolarisation (<Pmu>) in gas.**

2. Fraction of gas leaking across the zone 1 / 2 boundary.

The metal and gas distributions are on top of each other in zone 2, so we can't easily isolate the gas distribution in this zone and see how much leaks into zone 1. However, the gas band continues roughly parallel to the nearly-vertical cut line with about the same width, so the contamination of zone 3 into zone 4 will be about the same as the zone 2 / 1 contamination.

A special treesum was produced that selected zones **3 and 4**, with the 45-degree cut line lowered to conservatively remove metal stops from zone 3. The projection (parallel to the nearly-vertical cut line) is shown below, along with a fit to two Gaussians:

zones 4 and 3 combined

zones 4 and 3 combined

zone 4/3 boundary

χ^2 / ndf	883.6 / 94
scale1	2533 ± 12.1
mean1	104.5 ± 0.1
sigma1	13.81 ± 0.09
scale2	1.303e+04 ± 28
mean2	148.5 ± 0.0
sigma2	11.19 ± 0.02

extrapolation of
gas band into zone 4

Integrating the Gaussian for the gas stops:

range (arb rotated PC5 units)	ncounts (1E3)
-500 < PC5' <127	9.9
127 < PC5' < 500	355.6

Therefore the % of gas distribution leaking over the cut line is 9.9/(9.9+355.6) = **2.7 %** for set 68.
The analysis was repeated for a few other sets:

| Set	Fraction
s68	2.7%
s74	1.3%
s83	6.8%
s87	8.4%

3.   Fraction of gas stops for PC6

Previously Blair used the simulation to determine that 2.5% of muons stop in the PC6 gas, and the rest in the metal target.

For the modern data, Anthony found the PmuXi change for each set between psPACT on/off. See
https://twist.phys.ualberta.ca/forum/view.php?site=twist&bn=twist_physics&key=1249411865
Since the depolarisation in gas is <Pmu>=7.7%, the fraction of muons stopping in PC6 can be
estimated using change in pmuxi divided by 7.7%.

The results for each set are

set	pmuxi change (psPACT on/off)	% muons stopping in PC6 gas
68	54 +- 7	7.0
70	37 +- 6	4.8
71	35 +- 7	4.5
72	31 +- 7	4.0
74	32 +- 7	4.2
75	33 +- 6	4.3
76	40 +- 7	5.2
83	53 +- 7	6.9
84	50 +- 7	6.5
86	55 +- 6	7.1
87	50 +- 7	6.5
91	56 +- 12	7.3
92	52 +- 10	6.8
93	50 +- 9	6.5
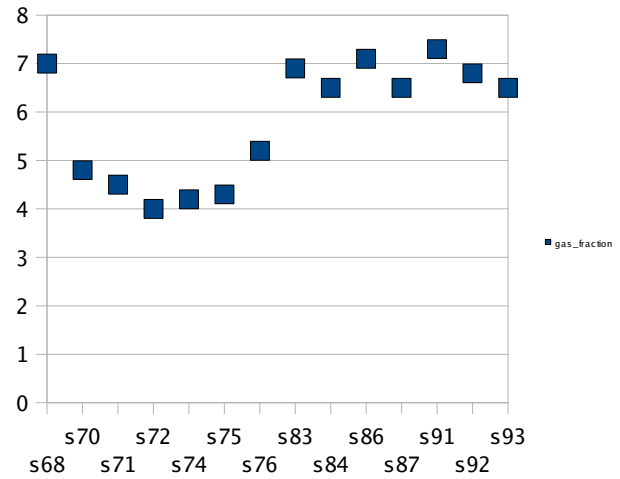
Estimate of systematic

Now we have all the information.

Let N_1^g be the number of muons stopping in the gas for zone 1. The numbers N_2^g, N_3^g and N_4^g are
defined similarly.

We know that for set 68:

$$N_1^g + N_2^g + N_3^g + N_4^g = [7.0\%] \, N^{total} \quad (1) \quad (7.0\% \text{ of PC6 muons stop in gas for s68})$$

$$N_1^g = [2.7\%] \, (\, N_1^g + N_2^g) \quad (2) \quad (\text{from the double Gaussian fit for s68})$$

(1) and (2) give $N_1^g / N^{total} = [2.7\%] \, [7.0\% - (N_3^g + N_4^g)/N^{total}]$

The depolarisation in gas is 7.7%, leading to a systematic uncertainty for set 68 of

$$[7.7\%]\ [2.7\%][7.0\% - (N_3^g + N_4^g)/N^{total}]$$

If we make the conservative assumption that all the gas stops are in zones 1 and 2, then the term $(N_3^g + N_4^g)/N^{total}$ vanishes, and an upper limit on the systematic for set 68 is 1.5e-4.

For the other sets:

set	% muons stopping in PC6 gas	% gas stops leaking from zone 2 to 1	systematic (1e-4)
68	7.0	2.7	1.5
74	4.2	1.3	0.4
83	6.9	6.8	3.6
87	6.5	8.4	4.2

This systematic ~~could~~ will be reduced significantly by resumming with a tigher psPACT cut.