Valentine Bargmann

Oral history interview with V. Bargmann, 1984 Apr.12.

Go to the web Site:

http://infoshare1.princeton.edu/libraries/firestone/rbsc/finding_aids/mathoral/pmc02.htm

Bargmann tells of his immigration to the United States in 1937. He became a member of the Institute for Advanced Study and worked as an assistant to Einstein. Bargmann talks about the relationship between mathematicians and physicists at Princeton and elsewhere, and about his wartime work. In 1948 he accepted a tenured position in the physics department at Princeton. Bargmann and Tucker describe conditions in Princeton in the 1930s and talk about von Neumann, Einstein, Milton White, and others.

See the web site http://newton.nap.edu/html/biomems/vbargmann.html

Louis Michel

Louis Michel was born in 1923 in Roanne. He was a student of the École Polytechnique in Paris in 1943-44, and spent 1944-45 in the army. He then entered the French "Service des Poudres" and persuaded his superiors to let him do research in physics, first experimental and very soon theoretical. As a visitor in Manchester, he published his first work on the decay of the μ meson. His thesis of 1953 was a masterly analysis of Fermi's theory of weak interactions. In the same year he discovered isotopic parity, later to become G-parity in the work of Lee and Yang. His "μ-parameter" has been continuously in use from those early times to this day. In 1955 he came back to France after stays in Copenhagen and Princeton. He became a "Maître de Conférences" in Lille and at the École Polytechnique, then in Orsay in 1958. In those years he was one of a small group of people responsible for the revival of theoretical physics and its teaching in France. At the same time he founded the Centre de Physique Théorique of the École Polytechnique.

His work in elementary particle theory ranges from the phenomenological to the most mathematical. He was the world's expert on problems involving the polarization of particles. A follower and friend of Wigner, he was an ardent proponent of the use of group theory, at a time when this was far from popular among physicists. At the same time he spent much time talking and listening to experimentalists. In 1962 he became a permanent Professor at the just founded IHÉS. His work on possible extensions of the symmetries of particle physics led him to mathematical investigations in invariant theory, then, starting in the seventies, to crystallography. He was a member of the Académie des Sciences.

Louis Michel had a continuous stream of students and collaborators from all parts of the world, who all retained the deepest gratitude and affection for him. He had an indefatigable passion for science. His activity and enthusiasm did not abate to his last day.

He died on December 30, 1999.

http://www.math.tu-berlin.de/iamp/old_bulletins/1998ff/Obituaries.html

Louis Michel gave a talk at TRIUMF on December 15, 1999 on the subject "Relativistic Theory of Particle Polarization: Applications", Louis Michel, IHES, Bures-sur-Yvette.

Valentine Telegdi

CERN BULLETIN

Monday 21st January 2002

Professor Valentine Telegdi celebrated his 80th birthday on Friday, 11th January. A brilliant American physicist of Hungarian origin, Professor Telegdi was a professor at the University of Chicago, the Swiss Federal Institute of Technology in Zurich (ETHZ) and the California Institute of Technology and took part in many CERN experiments, of which NA10 and L3 were the most recent. He served as Chairman of CERN's Scientific Policy Committee from 1981 to 1983. A member of numerous scientific academies, he shared the prestigious Wolf Prize with Maurice Goldhaber in 1991 in recognition of their separate seminal contributions to nuclear and particle physics, particularly those concerning weak interactions involving leptons.

TRIUMF Experiment E614
Technical Note No. 49
Muon polarisation in electromagnetic fields

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January 3, 2001

1 Introduction

This note was originally written for the KEK-E246 experiment (Search for a transverse polarization of the muon in the $K^+ \to \pi^0\mu^+\nu$ decay), following extensive discussions with Peter Gumplinger who wrote the KEK-E246 and TRIUMF-E614 Monte Carlo codes, including the rotation of the muon polarization in a magnetic field by using the Bargmann-Michel-Telegdi (BMT) equation.

This note will also be of interest for the TRIUMF-E614 experiment. Its purpose is mainly pedagogical.

It starts with the description of the polarization in the rest system of the particle. It is based on the density matrix formalism and the Stokes vector (a three-vector).

Then the relativistic treatment of the polarization is presented. If the particle, initially at rest, is submitted to a Lorentz boost, the polarization vector becomes a four-vector. It reduces to a three-vector only in the rest system of the particle.

The Bargmann-Michel-Telegdi (BMT) equation, which describes the motion of the polarization vector (a four-vector) in an electromagnetic field, is introduced. An expression is obtained for the motion of the Stokes vector (defined in the rest system of the particle, a three-vector).

As an application, the special case of a uniform magnetic field is considered and an analytical solution is given. A computer program based on this analytical solution has been written. This program has been used to test a subroutine of the Monte Carlo simulation, which applies to the more general case of a non-uniform (slowly varying) magnetic field.

In the publication:

"PRECESSION OF THE POLARIZATION OF PARTICLES MOVING IN A HOMOGENEOUS ELECTROMAGNETIC FIELD",

V. Bargmann, L. Michel and V.L. Telegdi, Phys. Rev. Letters, Vol. 2., Number 10, Page 435, May 15, 1959

The BMT equation is presented as:

$$\frac{ds}{d\tau} = \frac{e}{m}\Big[\Big(\frac{g}{2}\Big)\ F \cdot s + \Big(\frac{g}{2} - 1\Big)\Big(s \cdot F \cdot u\Big)\ u\Big]$$

More explicitly:

$$\frac{ds^{\mu}}{d\tau} = \frac{e}{m}\Big[\Big(\frac{g}{2}\Big)\ F^{\mu\nu}\ s_{\nu} + \Big(\frac{g}{2} - 1\Big)\Big(s_{\alpha}\ F^{\alpha\beta}\ u_{\beta}\Big)\ u^{\mu}\Big]$$

where u_{μ} is the <u>four-vector</u> velocity of the particle:

$$u^{\mu} = \begin{pmatrix} u_0 \\ \vec{u} \end{pmatrix} = \begin{pmatrix} \gamma \\ \gamma\vec{v} \end{pmatrix} \qquad u_{\mu} = \begin{pmatrix} u_0 \\ -\vec{u} \end{pmatrix} = \begin{pmatrix} \gamma \\ -\gamma\vec{v} \end{pmatrix}$$

s_{μ} is the <u>four-vector</u> polarization of the particle:

$$s^{\mu} = \begin{pmatrix} s_0 \\ \vec{s} \end{pmatrix} = \begin{pmatrix} \vec{s}\cdot\vec{v} \\ \vec{s} \end{pmatrix} \qquad s_{\mu} = \begin{pmatrix} s_0 \\ -\vec{s} \end{pmatrix} = \begin{pmatrix} \vec{s}\cdot\vec{v} \\ -\vec{s} \end{pmatrix}$$

$F^{\mu\nu}$ is the <u>four × four</u> antisymmetric electromagnetic <u>tensor</u>:

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

Ref.: *Group Theoretical Discussion Of Relativistic Wave Equations*, V. Bargmann, Eugene P. Wigner (Princeton U.), Proc. Nat. Acad. Sci. <u>34</u>, 211, 1948.

Properties of the BMT equation:

- **The BMT equation presented above possesses relativistic invariance. But it has nothing to do with quantum mechanics. The "spin" is deduced from the structure of the Poincaré group (full Lorentz group, including translations of space-time). In fact, the constant $\hbar$ does not appear in the equation. Therefore:**

  ## How about quantum effects? How important are they?

- **The BMT equation is valid for homogeneous electromagnetic fields. Perfectly homogeneous fields do not exist. Therefore:**

  ## What are the limits of applicability of the BMT equation?

Are you afraid of the four-dimensional formalism?

Dont you like the polarization vector of the muon being a four-vector? It is possible to use a three-vector for the polarization of the muon, but there is a price to pay.

Let us introduce the Stokes vector $\vec{\xi}$, the polarization vector in the rest system of the particle (a three-vector and unit vector).

G.G. Stokes, *"On the composition and resolution of streams of polarized light from different sources"*, Trans. Cambridge. Philos. Soc. 9, 399, 1952.

H. A. Tolhoek, *"Electron Polarization, Theory and Experiment"*, Rev. Mod. Phys. 28, 277, 1956.

Here are the relations between the three-vector $\vec{\xi}$ and the four-vector s_μ

$$s_0 = \frac{\vec{\xi}\cdot\vec{p}}{m} \qquad \vec{s} = \vec{\xi} + \frac{(\vec{\xi}\cdot\vec{p})\,\vec{p}}{m(E+m)} \qquad s^\mu s_\mu = (s_0)^2 - (\vec{s})^2 = -1 \qquad s^\mu p_\mu = 0$$

The motion of the Stokes vector (polarization vector) is given by:

$$\frac{d\vec{\xi}}{dt} = \vec{\Omega}\times\vec{\xi}$$

There are, in the litterature, several expressions for $\vec{\Omega}$. One of them:

$$\vec{\Omega}(t) = -\frac{e}{2m_0c}\, g\left[\vec{B} - \frac{\gamma-1}{\gamma}\left(\frac{\vec{v}}{v}\right)\left(\vec{B}\cdot\frac{\vec{v}}{v}\right) + \left(\vec{E}\times\frac{\vec{v}}{c}\right)\right] - (\gamma-1)\left[\left(\frac{\vec{v}}{v}\right)\times\frac{d}{dt}\left(\frac{\vec{v}}{v}\right)\right]$$

The kinematics are done in the LABORATORY FRAME (position of the muon, its velocity, the components of the electric and magnetic fields), but the polarization vector (a three-vector) is defined in the MUON FRAME. This is the price to pay if you want the polarization to be a three-vector.

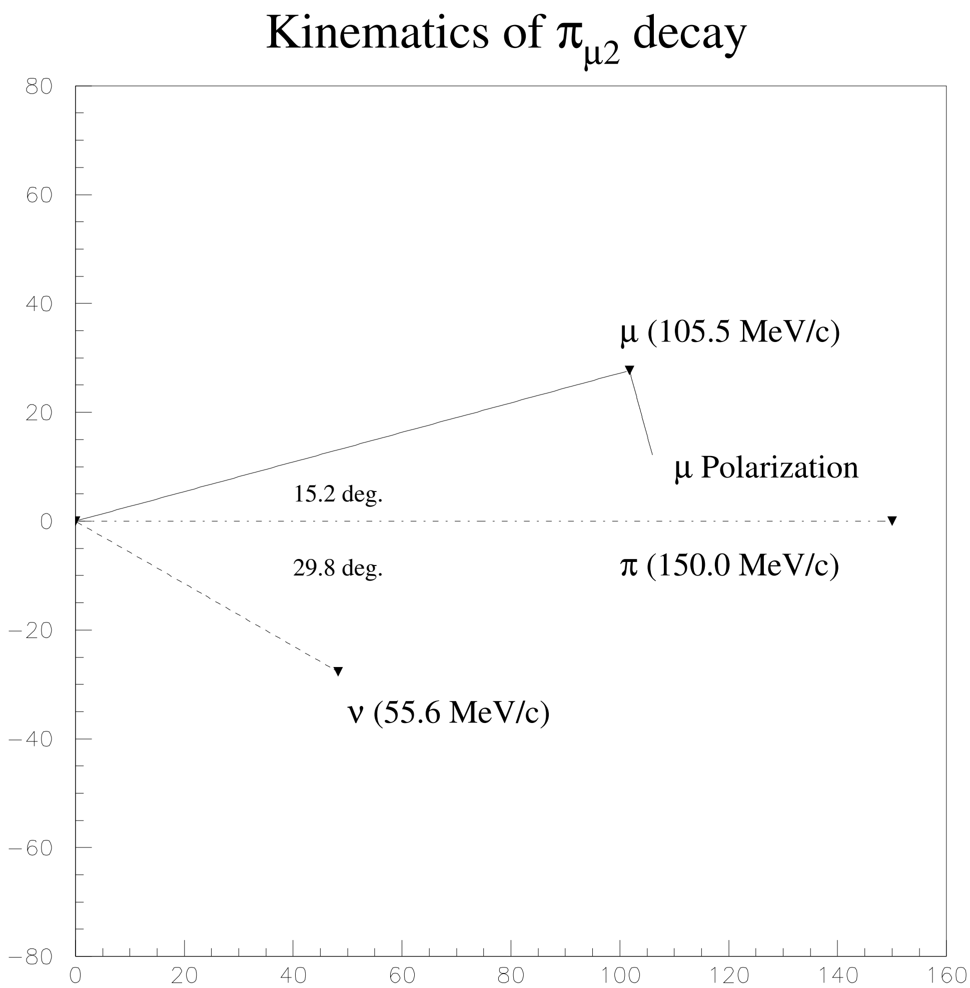

Figure 1: The kinematics for a pion momentum = 150 MeV/c and the muon emitted at the maximum angle. The muon is transversely polarized. Note the direction of the neutrino momentum.

Don't play with arrows! It is dangerous.

arXiv:gr-qc/0209099 v1 26 Sep 2002

PRECESSION OF A PARTICLE WITH ANOMALOUS MAGNETIC MOMENT IN ELECTROMAGNETIC AND GRAVITATIONAL PP-WAVE FIELDS

A. Balakin, V. Kurbanova and W. Zimdahl

While the original BMT equations have been derived for spatially homogeneous EM fields, they are expected to be useful also for inhomogeneous situations in which the corresponding gradients are sufficiently small and the relevant effects are of first order in the spin variable.

Questions:

1. **What is meant by "sufficiently small"?**
2. **What are the "higher orders in the spin variable"?**

QUESTION 1

From the paper "Polarization Precession"
by Kirk T. McDonald, California Institute of Technology
(January 14, 1970)
No reference given but can be found on SPIRES.

In an inhomogeneous magnetic field, there is an additional term $\nabla(\vec{\mu} \cdot \vec{B})$.

To be compared to the usual term $e\vec{\beta} \times \vec{B}$

In the particular case $\vec{\mu} // \vec{B}$ and $\vec{\mu} \perp \vec{\beta}$ the ratio is:

$$R = \frac{\mu}{e\beta} \frac{\nabla B}{B} = \frac{\hbar}{mc\beta} \frac{\nabla B}{B} = \frac{\lambda}{2\pi} \frac{1}{\beta} \frac{\nabla B}{B} \tag{1}$$

For the muon:

$$\frac{\lambda}{2\pi} = 1.86 \times 10^{-13} \text{ cm} \quad \text{and} \quad \beta = 0.19 \tag{2}$$

Finally:

$$R \sim 10^{-12} \frac{\nabla B}{B} \text{cm} \tag{3}$$

Even with a gradient:

$$\frac{\nabla B}{B} = 1.0 \text{ cm}^{-1} \tag{4}$$

$$R \sim 10^{-12} \qquad \text{(absolutely negligible)} \tag{5}$$

QUESTION 2

arXiv:gf-qc/9809069 v1 24 Sep 1998

EQUATIONS OF MOTION OF SPINNING RELATIVISTIC PARTICLE IN EXTERNAL FIELDS

I.B. Khriplovich and A.A. Pomeransky

Higher-order effects:

$$L_{1s} = \frac{e}{2m}\,\vec{s}\cdot\Big\{\,(g-2)\,\Big[\,\vec{B}-\frac{\gamma}{\gamma+1}\vec{v}\,(\vec{v}\cdot\vec{B})-\vec{v}\times\vec{E}\,\Big]+2\,\Big[\,\frac{1}{\gamma}\,\vec{B}-\frac{1}{\gamma+1}\,\vec{v}\times\vec{E}\Big]\Big\}$$

$$L_{2s} = \frac{e}{2m^2}\,\frac{\gamma}{\gamma+1}\,(\vec{s}\cdot[\vec{v}\times\vec{\nabla})])$$

$$\Big[\Big(g-1+\frac{1}{\gamma}\Big)(\vec{s}\cdot\vec{B})-(g-1)\frac{\gamma}{\gamma+1}(\vec{s}\cdot\vec{v})(\vec{v}\cdot\vec{B})-\Big(g-\frac{\gamma}{\gamma+1}\Big)(\vec{s}\cdot[\vec{v}\times\vec{E}])\Big]$$

$$\frac{L_{2s}}{L_{1s}} \sim \frac{\nabla}{m}$$

Restoring the constants $\hbar$ and c:

$$\frac{L_{2s}}{L_{1s}} \sim \nabla\frac{\hbar}{mc} = \nabla\lambda_C$$

Same conclusion as before.

Equations of Motion of Spin in Electromagnetic Field

I.B. Khriplovich

Part of a note received by email

The covariant equation for motion of spin is:

$$\frac{dS_\mu}{d\tau} = \frac{eg}{2m}\, F_{\mu\nu}\, S_\nu - \frac{e}{2m}\,(g-2)\, u_\mu (F_{\nu\lambda} u_\nu S_\lambda) \qquad (6)$$

(*Ya.I. Frenkel, 1926; V. Bargman, L. Michel, V. Telegdi, 1959*).

Let us discuss the limits of applicability for this equation.

Of course, typical distances at which the trajectory changes (for instance, the Larmor radius in a magnetic field) should be large as compared to the *de Broglie* wave length $\hbar/p$ of the elementary particle. Then, the external field itself should not change essentially at the distances on the order of both the *de Broglie* wave length $\hbar/p$ and the *Compton* wave length $\hbar/(mc)$ of the particle. In particular, if the last condition does not hold, the scatter of velocities in the rest frame is not small compared to c, and one cannot use in this frame the nonrelativistic formulae.

$$\frac{dS_\mu}{d\tau} = \frac{eg}{2m}\, F_{\mu\nu}\, S_\nu - \frac{e}{2m}\,(g-2)\, u_\mu (F_{\nu\lambda} u_\nu S_\lambda) \tag{7}$$

Besides, if the external field changes rapidly, the motion of spin will be influenced by interaction of higher multipoles of the particle with field gradients. For a particle of spin 1/2 higher multipoles are absent, and the gradient-dependent terms are due to finite form factors of the particle. These effects start here at least in second order in field gradients and usually are negligible.

At last, in equation (7) we confine to effects of first order in the external field. This approximation relies in fact on the implicit assumption that the first-order interaction with the external field is less than the excitation energy of the spinning system. Usually this assumption is true and the first order equation (7) is valid.

Don't take this too seriously

A charged pion (spinless particle) is put in a magnetic field:

$$B_x = B_y = 0 \qquad B_z = B$$

with an initial velocity:

$$v_x = v_y = 0 \qquad v_z = v_0$$

Its trajectory will be linear along the z axis, with:

$$x = y = 0 \qquad p_x = p_y = 0$$

According to quantum mechanics it is impossible to have simultaneously:

$$x = 0 \quad p_x = 0 \qquad \text{and} \qquad y = 0 \quad p_y = 0$$

since:

$$\Delta x \cdot \Delta p_x \sim \hbar \quad \text{and} \quad \Delta y \cdot \Delta p_y \sim \hbar$$

Therefore the initial velocity must have a transverse component. The trajectory of the pion will be an helix. Seen from a distance the pion will precess around the z axis.

If the magnetic field is sufficiently high ($\sim 10^{12}$ T), the radius of the helix will be smaller than the size of the pion (its Compton wave length). Seen from a distance, the pion will spin.

A NEW EXPERIMENT TO MEASURE THE MUON ELECTRIC DIPOLE MOMENT, EDM Collaboration (J.P. Miller et al.), AIP Conf. Proc. 698, 196-199, 2004; and "New York 2003, Intersections of particle and nuclear physics", 196-199

**SPINNING RELATIVISTIC PARTICLES IN EXTERNAL FIELDS.
A.A. Pomeransky, R.A. Senkov, I.B. Khriplovich (Novosibirsk, IYF), Phys. Usp. 43, 1055-1066, 2000, Usp. Fiz. Nauk 43, 1129-1141, 2000**

**EQUATIONS OF MOTION OF SPINNING RELATIVISTIC PARTICLES.
I.B. Khriplovich (Novosibirsk, IYF), Proceedings of International Conference on Quantization, Gauge Theory and Strings: Conference Dedicated to the Memory of Professor Efim Fradkin, Moscow, Russia, 5-10 June 2000; "Moscow 2000, Quantization, gauge theory, and strings", vol. 2, 36-44; e-Print Archive: hep-th/0009218**

**EQUATIONS OF MOTION OF SPINNING RELATIVISTIC PARTICLE IN EXTERNAL FIELDS.
I.B. Khriplovich, A.A. Pomeransky (Novosibirsk, IYF & Novosibirsk State U.), Talk given at 1st ICRA Network Workshop and the 3rd William Fairbank Meeting: The Lense-Thirring Effect, Rome, Italy, 29 June - 4 July 1998.
Surveys High Energy Phys. 14, 145-173, 1999; e-Print Archive: gr-qc/9809069**

**EQUATIONS OF MOTION OF SPINNING RELATIVISTIC PARTICLE IN EXTERNAL FIELDS.
I.B. Khriplovich, A.A. Pomeransky (Novosibirsk, IYF); J. Exp. Theor. Phys. 86, 839-849, 1998; Zh. Eksp. Teor. Fiz. 113, 1537-1557, 1998, e-Print Archive: gr-qc/9710098**