TRIUMF Experiment E614
Technical Note #99

Slow Muon Spin Relaxation in a normal Metal

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A surface muon has energy $E_\mu \simeq 1MeV$ at entrance of E614 stopped target. Possible depolarizing processes during the muon deceleration and thermalization are analyzed in TN-54 (Subsection 3.4). Interactions of thermalized muon in a normal metal are considered below. Goal of the note is analysis possible dependences of slow muon spin relaxation without clarification which interaction defines the obtained muon relaxation rate of $\lambda \simeq 0.00155\mu s^{-1}$ in Al at longitudinal magnetic field $H^L = 2T$.

The analysis done using mostly μ^+SR publications.

1 Interactions with nuclear magnetic moments

Nuclear moments create a magnetic field about a few gauss on localized muon in a metal crystal. The field are random in 3D for different muons because nuclear moments of a normal metal have random directions in different crystal cells. The random fields cause a slow muon relaxation during $1-10\mu s$ depending on a nuclear moment value, size of crystal cell, and place of a localized muon in the cell.

1.1 Transverse magnetic field

Relaxation dependence is close to gaussian [1] for a muon fixed in a cell at transversal magnetic field H_0^T much larger then magnetic fields from nuclear moments:

$$g^T(t) = exp(-\frac{1}{2}\Delta^2 t^2), \tag{1}$$

where Δ^2/γ_μ^2 represents the second moment from the random nuclear fields on a muon:

$$\Delta^2/\gamma_\mu^2 = \langle H_x^2 \rangle = \langle H_y^2 \rangle = \langle H_z^2 \rangle, \tag{2}$$

($\Delta \simeq 0.3\mu s^{-1}$ and $\Delta \simeq 0.25\mu s^{-1}$ for a localized muon in Cu and Al correspondigly). Value $\sqrt{\langle H^2 \rangle}$ equals to $2-3G$ in Al and Cu.

Diffusion motion of muon in a metal creates variable in 3D and time magnetic fields on muon. As result muon relaxation rate drops [2]:

$$g^T(t) = exp[-\Delta^2 \tau_c^2 (e^{-t/\tau_c} - 1 + t/\tau_c)]. \tag{3}$$

where τ_c is hop time of muon in a metal. $g^T(t)$ relaxation becomes of exponential at $t/\tau_c \gg 1$:

$$g^T(t) = exp(-\Delta^2 \tau_c t). \qquad (4)$$

Examples of muon relaxation in Al and Cu in a transversal magnetic field are present in Figs.1-3 [3, 4, 5]. One can see from the figures that muon has maximal relaxation rate in Cu at $T < 100K$ (Fig.3, upper curve for Cu). It means that the muon is localized (no diffusion). Diffusion in Cu begins at $T > 100K$. Muon can be localized at $T < 10K$ in $Al + 0.42\% Cu$ alloy (Fig.3) and at $T < 0.1K$ in alloyes with admixture $\sim 100ppm$ of a different metal. One can see from Fig.3 that muon relaxation rate in Al is not 0 even at $T \simeq 800K$ (Fig.3). This high temperature relaxation tale can not be explained by a diffusion because formula (4). We have to examine different reasons for explanation of this effect.

1.2 Longitudinal magnetic field

Longitudinal muon polarization for localized muon at $H \equiv 0$ can be describe [6] by formula

$$P^L(t) = \frac{1}{3} + \frac{2}{3}(1 - \Delta^2 t^2) exp(-\frac{1}{2}\Delta^2 t^2), \qquad H_0 \equiv 0 \qquad (5)$$

Longitudinal muon polarization for localized muon in a longitudinal magnetic field has been calculated in [7]:

$$P^L(t) = 1 - \frac{2\Delta^2}{\omega_0^2}[1 - exp(-\frac{1}{2}\Delta^2 t^2)\cos\omega_0 t] + \frac{2\Delta^4}{\omega_0^3}\int_0^t exp(-\frac{1}{2}\Delta^2\tau^2)\sin\omega_0\tau d\tau, \qquad H_0^L \neq 0 \quad (6)$$

Where $\omega_0 = \gamma_\mu H_0^L$, H_0^L is longitudinal magnetic field. The formulas (5.6) have been confirmed in many articles (see for example Figs.4-6 from references [7, 8]). $\omega_0 = 2\pi \cdot 0.27 ns^{-1}$ at $H = 2T$. It corresponds to muon precession period of $T_\mu = 3.7ns$.

Let me analyze $P_z^L(t)$ function at $B^L = 2T$ for a localized muon. According to (6) we will see Larmor precession with muon period of $T_\mu = 3.7ns$ with gaussian relaxation time $\tau = 1/\Delta \simeq 3 - 4\mu s$. Amplitude of the precession is $A = 2\Delta^2/\omega_0^2 \simeq 10^{-7}$ at $H^L = 2T$. Value $1 - P_z^L(t = \infty) \simeq 2\Delta^2/\omega_0^2 \simeq 10^{-7}$, therefore interaction of a localized muon spin with nuclear moments gives the worthless contribution to muon polarization behavior. One can see from Figs.1-3 that muon diffuses through a pure Al crystal. Relaxation dependence is exponential even at a slow diffusion according to Ref.[7]. High temperature tail of muon relaxation in Fig.3 and practically therefore a constant diffusion rate can be explained by a dislocation deceleration of muon diffusion rate. Muon relaxation has exponential form again. Second explanation of the high temperature tale is a capture of muon by defects in a metal. It means that a low percentage of muons remains localized even at a high temperature. The muon relaxation will have gaussian form, but amplitude of this part of muons will be $\ll A = 10^{-7}$.

Finally interactions on muon spin with nuclear moments can produce a remarkable exponential relaxation only.

2 Interactions of muon spin with conductive electrons (Korringa relaxation)

In metals the μ^+ is thermalized in a quasi free state because the conductive electron concentration $n \approx 10^{23}\text{cm}^{-3}$ in normal metals effectively screens the μ^+ from interactions with individual electrons. Muon forms so called Kondo impurity (see, for example [9]) in a metal with characteristic temperature T_K. Conductive electrons completely screen the muon charge at $T < T_K$ and muon depolarization rate is zero. A muon relaxation is possible at $T > T_K$. The effects have been analyzed in Ref.[1], p.355. Exponential relaxation of muon polarization is result of the interactions. The relaxation in a longitudinal magnetic field has been analyzed in [10]. Experimental data of Korringa relaxation in several metals present in Fig.7 [11]. One can see that "Relaxation ratio" values in Fig.7 are close to $\lambda = 0.00155\mu s^{-1}$ obtained by Blair for exponential fit. Longitudinal magnetic field have been chosen in [11] to suppress contribution of interaction with nuclear magnetic moments and another possible effects. A relaxation rate in Ref.[11] was independent of field up to several kilogauss. Simple speculations demonstrate [12] that even the longitudinal field $H^L = 2T$ cannot "hold" the spins of quasi free muons in a metal. The energy difference between states in which the muon spin is parallel or antiparallel to the $H^L = 2T$ field is only $1.1 \cdot 10^{-6} eV$, while the room-temperature thermal energy is $kT = 2.6 \cdot 10^{-2} eV$.

3 Interactions with paramagnetic admixtures

A high purity aluminum of our stopped target has impurities (ppm): Cu - 0.3, Fe - 0.3, Mg - 1.2, Si - 0.8. Their total concentration is $2.6 \cdot 10^{-6}$. It means that an average distance between the impurities is about 100 crystal cell of a metal. That excludes any noticeable contribution of the impurities to muon relaxation. Besides that interaction with fixed paramagnetic impurities causes an exponential relaxation according to [1], p.378. Diffusion of muons through a metal preserves the exponential dependence.

4 Muon spin interactions with terminal spur of its track

High-energy muon creates defects in a matter. The paramagnetic defects can produce sufficient magnetic fields on the stopped muon. Electron configuration recovers in $\sim 10^{-11} sec$ in a metal according to Ref.[13]. Therefore it can cause a fast muon relaxation only. Muon also displaces atoms in a crystal cell. The displaced atom configuration remains either constant or changes very slowly. Average distance between the stopped muon and the last atom displacement produced in graphite, for example, is $9000\mathring{A}$ according to reference [14]. We believe a contribution of the effect on P_μ value is negligible.

5 Conclusion

• Interaction of muon magnetic moment with nuclear magnetic moments of a metal create a non-exponential relaxation of P_μ [1] for a localized muon in a crystal cell. This non-exponential relaxation can decrease P_μ value on $\simeq 10^{-7}$ at longitudinal magnetic field $H^L = 2T$ [7]. Diffusion of muon through a crystal produces an exponential relaxation [7].

• Interactions of muon spin with conductive electrons is described by an exponential function [10].
• Interactions with paramagnetic admixtures creates an exponential relaxation according to Ref.[1],p.378.
• Terminal spur of muon track in a metal causes a negligible changing of P_μ [13, 14].

•• Finally: only an exponential relaxation of muon spin are possible in a normal metal at $H^L = 2T$ if one measures muon polarization with accuracy of $\Delta P_\mu > 10^{-5}$.

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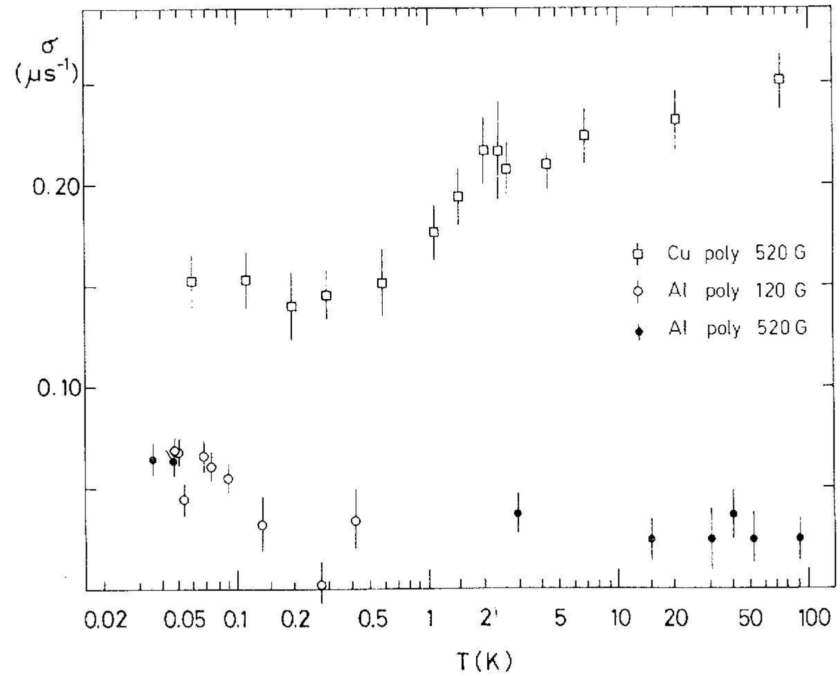

FIG. 1. Damping parameter σ for polycrystalline Al and Cu as function of temperature.

Figure 1: Figure from reference [3]. Gaussian function describes a muon relaxation.

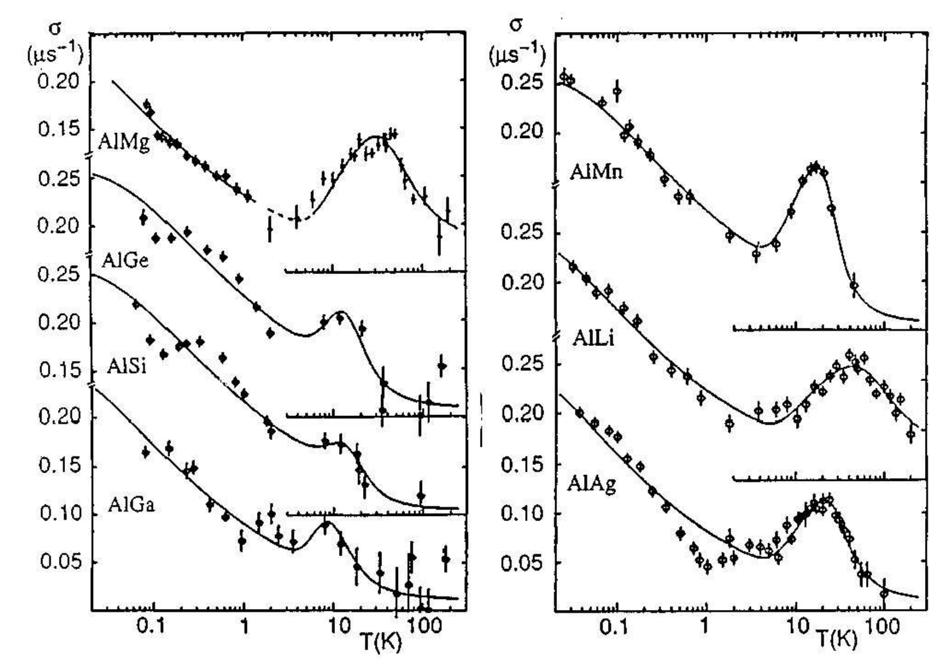

FIG. 11. Temperature dependences of Gaussian relaxation rate of the μ^+ polarization in aluminum samples doped with different impurities (Hartmann *et al.*, 1988).

Figure 2: Figure from reference [4]. Impurity concentration in Al was (*ppm*): Mg - 92, Ge - 177, Si - 110, Ga - 163, Mn - 92, Li - 75, Ag - 117. Gaussian function describes a muon relaxation.

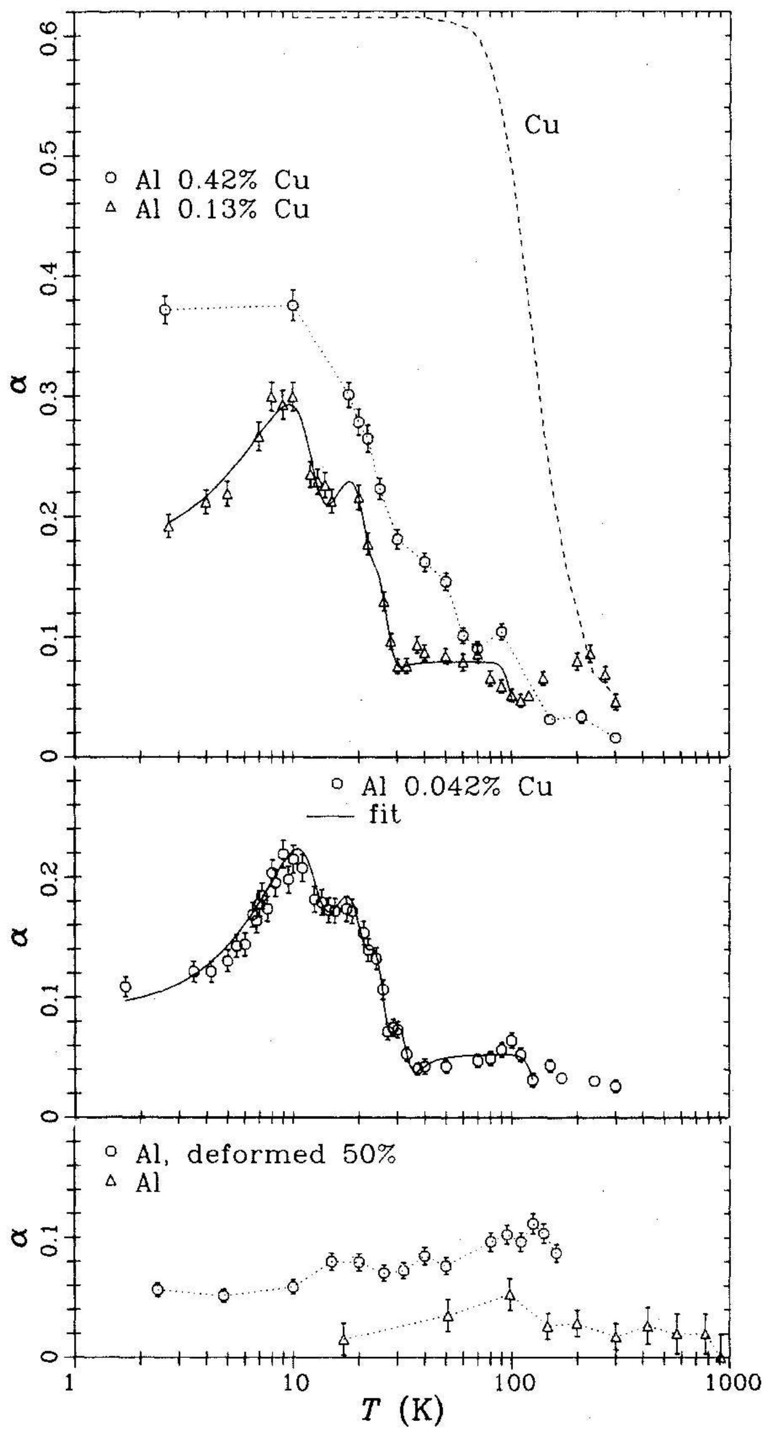

FIG. 1. Temperature dependence of the depolarization parameter α, measured with external fields of 500 Oe for the 0.042% and 0.13% alloys and deformed Al, and 330 Oe for the 0.42% alloy. Data for Cu (Ref. 10, 30 Oe) and Al (Ref. 8, 75 Oe) are uncorrected for weak field perturbations (Ref. 11). The fits are Eq. (5) with the parameters in Table I.

Figure 3: Figure from reference [5]. Fit with gaussian function (1) with $\Delta = s\sqrt{\alpha}$ has been used. $s^{-1} = 2.2\mu s$ is muon lifetime.

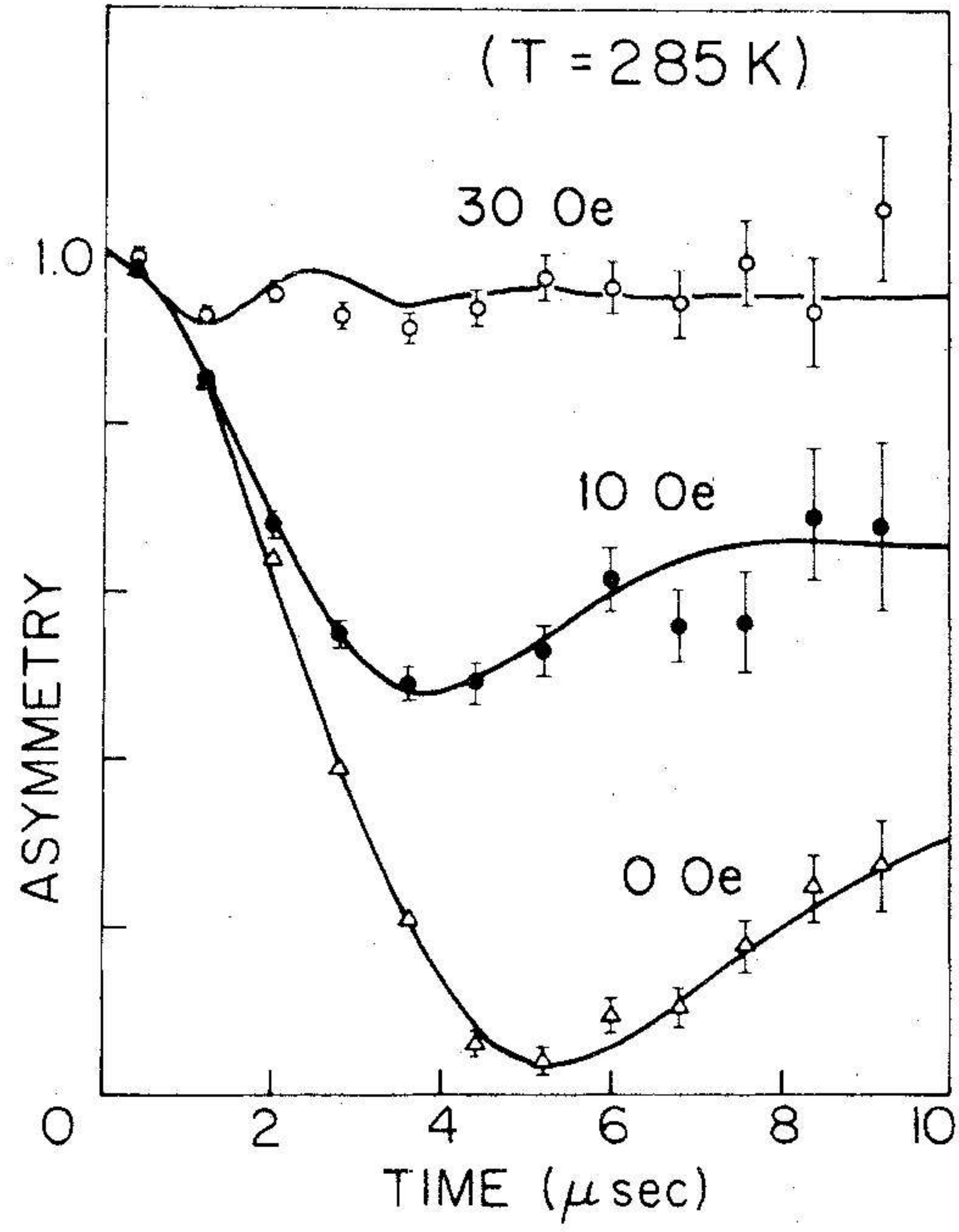

FIG. 2. Observed μ^+ longitudinal relaxation functions in MnSi at room temperature with 0, 10, and 30 Oe external fields. The solid curves are the best fits to Eq. (10).

Figure 4: Figure from reference [7]. No muon diffusion. The solid curves are the best fit to Eq. (6).

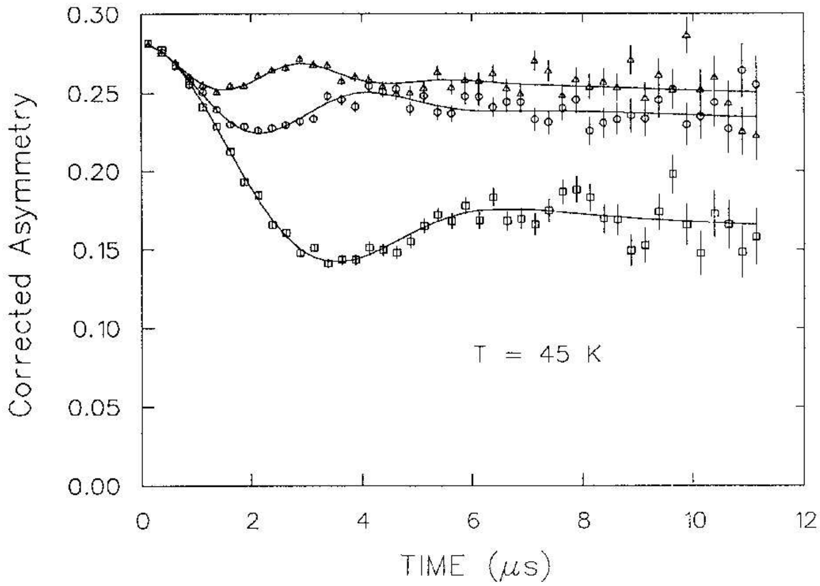

FIG. 7. WLF time-differential μSR spectra for applied fields 8 (squares), 16 (circles), and 24 G (triangles). Muons are nearly static: T=45 K.

Figure 5: Figure from reference [8]. No muon diffusion. The solid curves are the best fit to Eq. (6).

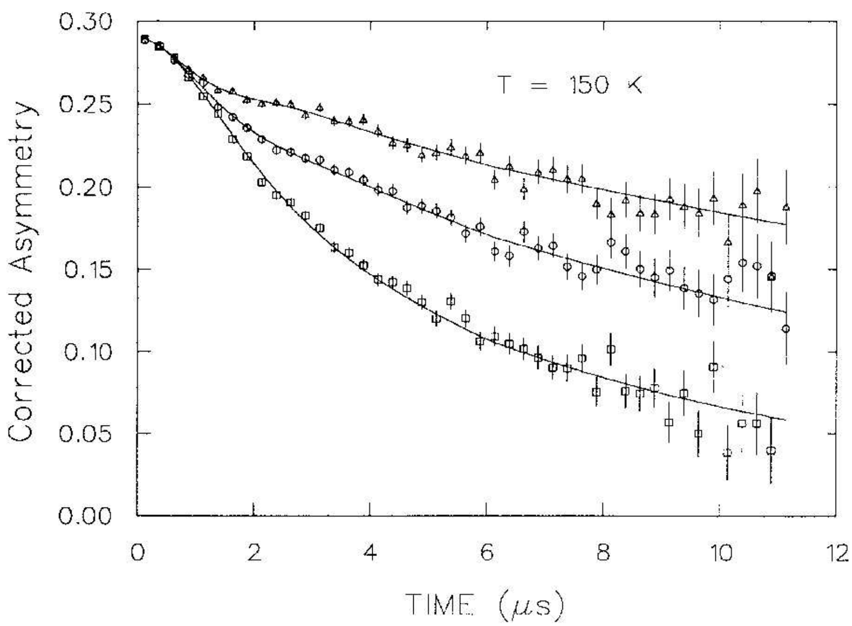

FIG. 8. WLF time-differential μSR spectra for applied fields 8 (squares), 16 (circles), and 24 G (triangles). Muons are diffusing rapidly: T=150 K.

Figure 6: Figure from reference [8]. Fast muon diffusion.

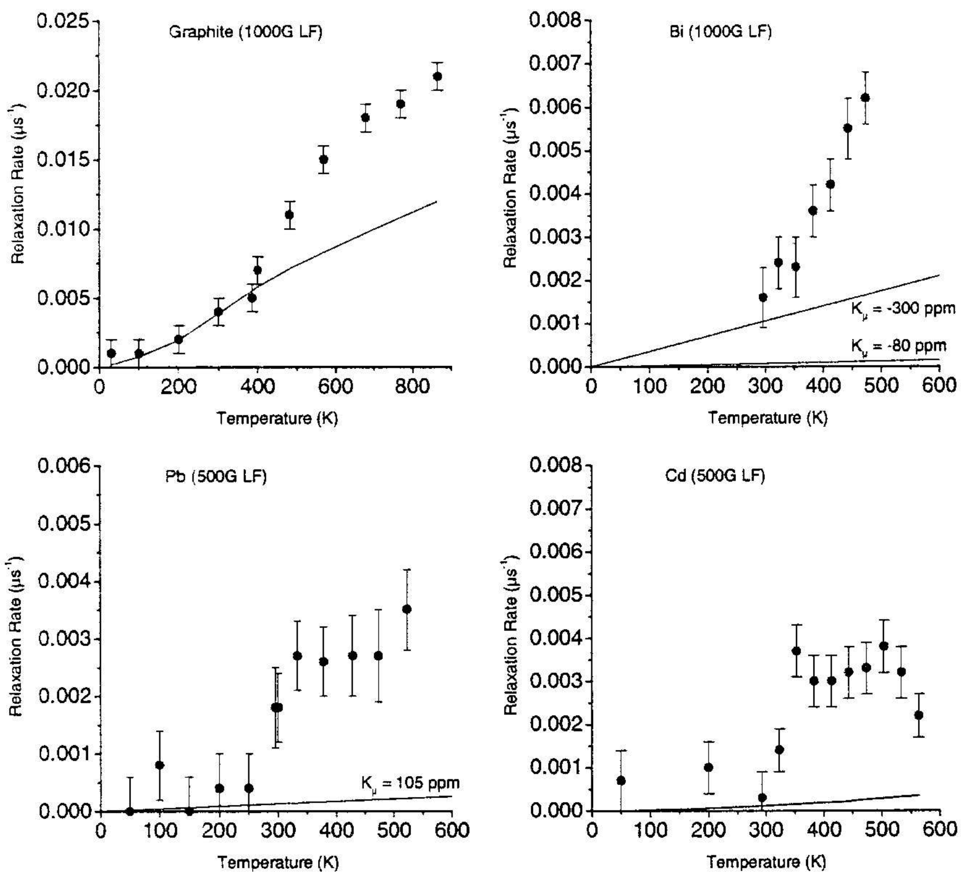

Fig. 1. Temperature dependences of longitudinal relaxation rate for various elements for which K_μ values are known, together with the predictions of the Korringa law.

Figure 7: Figure from reference [10]. An exponential function describes a muon relaxation.