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Muon decay to one loop order in the left–right symmetric model

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Abstract

One loop corrections to the muon decay are studied in a popular and self-consistent version of the left–right symmetric model. It is shown quantitatively, that the corrections do not split into those that come from the Standard Model sector, and some decoupling terms. For a heavy Spontaneous Symmetry Breaking (SSB) scale of the order of at least 1 TeV, the contributions from the top quark have a logarithmic behaviour and there is a strong quadratic dependence on the heavy Higgs scalar masses. The dependence on the light Higgs boson mass is small. The heavy neutrinos are shown to play an important role, although secondary in comparison with the heavy scalar particles as long as the heavy neutrinos' Majorana Yukawa coupling matrix h_M obeys unitarity bounds.

1. Introduction

Embedding the Standard Model (SM) into a larger gauge group increases the number of degrees of freedom. For the Left–Right Symmetric Models (LRSM) based on the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group [1,2] these are connected with new fields and interactions. The model is complex with extra particles of different types. New neutral leptons, charged and neutral gauge bosons, neutral and charged Higgs scalars appear. There are many different versions of the LR models with equal or different left and right gauge couplings $g_{L,R}$, and specific Higgs sector representations. This robust structure is a challenge and a good theoretical laboratory for testing many phenomena beyond the SM.

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The purpose of the present work is to study numerically one loop corrections to muon decay which come from the extended gauge sector of the LRSM. Since the history of the LRSM is already quite long, there have been some interesting attempts to study radiative corrections within its framework [3–5]. To our knowledge however, there has never been a complete calculation performed. We start our systematic one loop level study of the model from a low-energy muon decay calculation. The subject has already been explored qualitatively in [6]. The main result of this paper was to show that the quadratic top mass dependence of the oblique corrections to Δr is lost. In the SM these corrections come from constraints imposed by the SSB sector on the Weinberg angle counter-term. Here similar constraints connect the counter-term with the heavy SSB scale. As a result the top quark is effectively massless. By the same the SM one loop corrections do not constitute a subset of the full contributions. Therefore in general, it is not true that one can properly fit New Physics Models (NPM) by taking one loop SM corrections modified with tree level NPM couplings. These issues are further explored in [7].

The question which we wish to answer is the following: can we or can we not accommodate the present experimental life-time of the muon within a model that has a minimal Higgs sector structure supported by phenomenology and the smallest possible number of unknown free parameters? It is common wisdom that when there are many free parameters any data can be fitted. We show however, that this induces a strong correlation between the heavy parameters. In fact a full decoupling is not observed, and if the additional masses tend to infinity independently, a huge correction results, which is incompatible with data.

We first discuss assumptions on model's structure and parameters. The renormalization scheme is then introduced and the corrections to muon decay enumerated. Numerical estimates follow with a study of the dependence on the heavy masses and the heavy symmetry breaking scale. Conclusions together with an outlook close the paper. Appendix A gathers our notational conventions and main components of the model.

2. Structure and parameters of the model

As noticed in the Introduction, there are many versions of LRSM. A complete analysis at the one loop level requires the model to be fixed. We choose to explore the most popular version of the model with a Higgs representation with a bidoublet Φ and two (left and right) triplets $\Delta_{L,R}$ [8]. We also assume that the VEV of the left-handed triplet Δ_L vanishes, $\langle \Delta_L \rangle = 0$ and the CP symmetry can be violated by complex phases in the quark and lepton mixing matrices. Left and right gauge couplings are chosen to be equal, $g_L = g_R$. We call this model, the Minimal Left–Right Symmetric Model (MLRSM). The necessary definitions can be found in Appendix A (for details, see [6,8–10]).

We also take advantage of several approximations which come from phenomenological studies.

1. *Mixing of fermions.* As usual in one loop analyses, we neglect quark mixings. The case of neutrino mixings deserves however additional comments. The effective light neutrino mass matrix $M = -M_D M_R^{-1} M_D^{\mathrm{T}}$ (Eq. (A.22)) yields three light Majorana

neutrinos which are predominantly composed of the usual "active" neutrinos ν_L with a very small $O(1/m_N)$ admixture of "sterile" neutrinos $\nu_L^c = \nu_R$. The diagonalization of M_R (Eq. (A.24)) produces 3 heavy Majorana neutrinos which are mainly composed of ν_R. In order to get light neutrino masses at the eV scale (as concluded from experimental data [11,12]) and without extra relations between M_D and M_R matrices, m_N must be large $m_N > 10^9$ (10^{13}) GeV for the lepton (quark) see-saw mechanism. However, we would also like to explore lower scales with v_R of the order of TeV. A crude light–heavy (LH) mixing estimate $O(1/m_N)$ would give in this case larger couplings. However, they would lead to a problem with obtaining the light neutrino spectrum, namely, from Eq. (A.22) their masses would be much above the eV scale. A fine-tuning of M_D and/or M_R parameters or additional discrete symmetries must be applied for the full neutrino mass matrix to get the proper light neutrino spectrum. Therefore it has been argued in [13] that it is not natural to obtain large LH mixings for heavy neutrinos at the TeV scale. In accordance with these arguments we assume here that the light and heavy neutrino sectors are disconnected (negligible mixings). In this way, W_1 couples only to light neutrinos, while W_2 couples to the heavy ones. Z_1 and Z_2 turn out to couple to both of them [9,10]. This generates automatically an extended flavour symmetry, where transitions outside of a family composed from a lepton, a light and a heavy neutrino are forbidden.
As lower limit on the heavy neutrino masses we use the direct experimental limit from the lack of $Z \to \nu N$ decay, which is $m_N \geqslant M_Z$.

2. *Mixing of charged gauge bosons.* In principle the model allows for mixing of charged gauge bosons. However, experimental data analyses give the following conservative upper bound on the mixing angle [14,15]

$$|\xi| \leqslant 0.013 \text{ rad.} \tag{1}$$

The tree level contribution to Δr coming from the mixing is proportional to

$$\sin^2\xi \frac{M_{W_1}^2}{M_{W_2}^2}. \tag{2}$$

Even if the second charged gauge boson had a mass of the order of the SM W, this number would be negligible compared to the experimental value which is of the order of 3%.
We therefore put $\xi = 0$, which also means that $\kappa_2 = 0$. There are several advantages of this approximation. First there is no need to renormalize the mixing of the gauge bosons. It turns also out, that together with the previous approximation on lepton sector mixings, there is no need to renormalize the a priori possible mixing between the light and the heavy neutrino within a family. At last, the QED contributions to the process form a self-contained class as in the SM (see Section 4).
This model has the nice feature that all the constraints on the right handed sector come uniquely from one loop corrections. The tree level W_1 exchange diagram is not sensitive to the additional gauge structure anymore [16].

3. *Yukawa couplings to charged Higgs scalars.* The approximations from the preceding two points, leave still a possibility for muon decay through one of the charged Higgs scalars H_1^+. It turns out, that the experimental data on polarized muon decay asymmetries are compatible even with a decay through scalar currents only. However, inverse muon decay

bounds these contributions to be at most one order of magnitude smaller than the SM left-handed current decay. We assume in this work that these diagrams are either negligible or require only to be included at the tree level in which case, the space left for Δr in Eq. (18) would be respectively smaller.

3. Renormalization in one loop order

As the basic set of input parameters we choose the electromagnetic coupling constant and the masses of the four gauge bosons, Higgs scalars and fermions. It turns out that as long as no corrections need to be included to tree level Higgs scalar exchange diagrams, the on-shell conditions of gauge bosons only suffice to fix all of the necessary counter-terms. Moreover, decoupling effects should be automatically included.

In the present approximation, where we neglect charged gauge boson mixing, only the Weinberg angle requires renormalization. We recall here its counter-term [6]

$$\begin{aligned}\delta s_W^2 = 2c_W^2 &\frac{(\delta M_{Z_2}^2+\delta M_{Z_1}^2)-(\delta M_{W_2}^2+\delta M_{W_1}^2)}{(M_{Z_2}^2+M_{Z_1}^2)-(M_{W_2}^2+M_{W_1}^2)}\\ &+\frac{1}{2}\frac{(M_{W_2}^2+M_{W_1}^2)(\delta M_{Z_2}^2+\delta M_{Z_1}^2)+(M_{Z_2}^2+M_{Z_1}^2)(\delta M_{W_2}^2+\delta M_{W_1}^2)}{((M_{Z_2}^2+M_{Z_1}^2)-(M_{W_2}^2+M_{W_1}^2))^2}\\ &-\frac{1}{2}\frac{(2M_{Z_1}^2+M_{Z_2}^2)\delta M_{Z_1}^2+(2M_{Z_2}^2+M_{Z_1}^2)\delta M_{Z_2}^2}{((M_{Z_2}^2+M_{Z_1}^2)-(M_{W_2}^2+M_{W_1}^2))^2}.\end{aligned} \tag{3}$$

As discussed in Section 2, no fermion mixing renormalization is needed, and the hard corrections (factorized weak contributions) are properly included through the simple fermion counter-terms

$$\delta Z_{L,R}^{l,\nu} = \Sigma_{\gamma L,R}^{l,\nu}, \tag{4}$$

where we used the following decomposition of fermion self-energy

$$\Sigma = \hat{p} P_L \Sigma_{\gamma L} + \hat{p} P_R \Sigma_{\gamma R} + P_L \Sigma_L + P_R \Sigma_R. \tag{5}$$

An interesting problem is connected to charge universality and renormalization of the electromagnetic coupling. We wish here to show that charge universality follows simply from Ward identities and the constructive proof gives also the correct counter-term. To one loop the potentially problematic contributions come from diagrams involving a heavy neutrino and the traditional approach gives a result independent of these masses only after summation of vertex and external line contributions [6].

Let us start from the following relation, which comes from charge assignments within a fermion doublet and the definition of the physical fields

$$\begin{pmatrix} 0 \\ 0 \\ \langle (l_L^0 \overline{l_L^0} + \nu_L^0 \overline{\nu_L^0}) B^{0\mu} \rangle_{\text{amp}} \end{pmatrix} = \left(U_0^{\mathrm{T}}\right)^{-1} \begin{pmatrix} \langle (l_L^0 \overline{l_L^0} + \nu_L^0 \overline{\nu_L^0}) Z_1^{\mu} \rangle_{\text{amp}} \\ \langle (l_L^0 \overline{l_L^0} + \nu_L^0 \overline{\nu_L^0}) Z_2^{\mu} \rangle_{\text{amp}} \\ \langle l_L^0 \overline{l_L^0} A^{\mu} \rangle_{\text{amp}} \end{pmatrix}, \tag{6}$$

where U_0 is the bare neutral sector mixing matrix Eq. (A.12) multiplied by the renormalization constants of the physical fields

$$U_0 = \begin{pmatrix} c_W^0 c^0 & c_W^0 s^0 & s_W^0 \\ -s_W^0 s_M^0 c^0 - c_M^0 s^0 & -s_W^0 s_M^0 s^0 + c_M^0 c^0 & c_W^0 s_M^0 \\ -s_W^0 c_M^0 c^0 + s_M^0 s^0 & -s_W^0 c_M^0 s^0 - s_M^0 c^0 & c_W^0 c_M^0 \end{pmatrix} \begin{pmatrix} Z^{\frac{1}{2}}_{Z_1 Z_1} & Z^{\frac{1}{2}}_{Z_1 Z_2} & Z^{\frac{1}{2}}_{Z_1 \gamma} \\ Z^{\frac{1}{2}}_{Z_2 Z_1} & Z^{\frac{1}{2}}_{Z_2 Z_2} & Z^{\frac{1}{2}}_{Z_2 \gamma} \\ Z^{\frac{1}{2}}_{\gamma Z_1} & Z^{\frac{1}{2}}_{\gamma Z_2} & Z^{\frac{1}{2}}_{\gamma \gamma} \end{pmatrix} \tag{7}$$

and $\langle \cdots \rangle_{\mathrm{amp}}$ is a shorthand for amputated Green functions. From this we obtain

$$\langle l_L^0 \overline{l_L^0} A^\mu \rangle_{\mathrm{amp}} = \left(U_0^{\mathrm{T}}\right)_{33} \left\langle \left(l_L^0 \overline{l_L^0} + \nu_L^0 \overline{\nu_L^0}\right) B^{0\mu} \right\rangle_{\mathrm{amp}}. \tag{8}$$

After taking the divergence of the current, we can use the $U(1)$ Ward identity for the B field and the on-shell renormalization conditions on the fermion propagators and the electromagnetic vertex, which leads to the following identity

$$e = \frac{e_0}{\sqrt{\cos 2\Theta_W^0}} \left(U_0^{\mathrm{T}}\right)_{33}, \tag{9}$$

which can be put into the following form

$$\frac{e_0}{e} \left(Z^{\frac{1}{2}}_{\gamma\gamma} + \frac{\sin\phi^0 \tan\Theta_W^0 - \cos\phi^0 \sqrt{\cos 2\Theta_W^0}\, \tan\Theta_W^0}{\sqrt{\cos 2\Theta_W^0}} Z^{\frac{1}{2}}_{Z_1\gamma} - \frac{\cos\phi^0 \tan\Theta_W^0 + \sin\phi^0 \sqrt{\cos 2\Theta_W^0}\, \tan\Theta_W^0}{\sqrt{\cos 2\Theta_W^0}} Z^{\frac{1}{2}}_{Z_2\gamma} \right) = 1. \tag{10}$$

Since none of the above renormalization constants depends on the initial fermion species, we have obtained the required charge universality. At the same time we can expand this relation to first order to yield the electromagnetic coupling renormalization counter-term

$$\frac{\delta e}{e} = -\frac{1}{2}\delta Z_{\gamma\gamma} - \frac{\sin\phi \tan\Theta_W - \cos\phi \sqrt{\cos 2\Theta_W}\, \tan\Theta_W}{\sqrt{\cos 2\Theta_W}} Z^{\frac{1}{2}}_{Z_1\gamma} + \frac{\cos\phi \tan\Theta_W + \sin\phi \sqrt{\cos 2\Theta_W}\, \tan\Theta_W}{\sqrt{\cos 2\Theta_W}} Z^{\frac{1}{2}}_{Z_2\gamma}. \tag{11}$$

We have checked by explicit calculation that the above formula gives the same value as the usual approach. We would like to stress that to our knowledge, such a formula for LR models has never been derived, although similar methods have been used in SM analyses [17].

4. Structure of corrections to muon decay

The muon life-time is parametrized through the Fermi coupling constant, the mass of the muon and the QED corrections to the four-fermion interaction Δq, which are presently known up to second order in the fine structure constant [18]

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1 + \Delta q). \tag{12}$$

The Fermi constant on the other hand is related to the tree level SM coupling of the charged W boson to fermions through

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1 + \Delta r), \tag{13}$$

where Δr are higher order corrections to which we already made reference.

There are two problems with these formulas, when moving from the SM to other interactions. Let us first consider Eq. (12). It is based on the assumption, that the basic process is described by the four-fermion interaction, which in the charge conserving form is of a pure $V - A$ type. In fact, as long as the interaction has only an admixture of vector and axial currents, the QED corrections are finite and gauge invariant, hence meaningful. Notice however, that if the process is induced also by right-handed currents, then after moving to the charge conserving form of the interaction (Fierz transformation), there appear also scalar and tensor interactions, which are known not to have a finite QED correction. The same problem occurs if we add charged scalar particles to the list. There are two possibilities to remedy the situation, the first being of course calculating by any means the process in the full model and resign from the separation of QED corrections. The second possibility is somewhat simpler. If the tree level corrections from right-handed and/or scalar interactions are of the same size as the one loop corrections to the basic diagrams, we can simply ignore one loop contributions to these additional currents and consider Eq. (12) as approximate and valid to one loop order only.

The second problem we have to face is the fact that the tree level coupling to the light charged gauge boson can be different from the SM one. This concerns mainly the sine of the Weinberg angle s_W. In fact this happens to be the case of the considered model, where due to constraints if we fix the mass of the two light gauge bosons, then s_W is given by a function of the heavy SSB scale v_R. This dependence is depicted in Fig. 1. For small values of v_R, the difference from the SM value is large. We choose here to include the change of s_W from the SM to the LRSM in Δr.

Δr can now be obtained from the formula

$$\Delta r = \frac{(s_W^2)_{\mathrm{SM}}}{(s_W^2)_{\mathrm{LRSM}}}\left(\frac{-\Pi_W^{\mathrm{T}}(0) - \delta M_W^2}{M_W^2} + 2\frac{\delta e}{e} - \frac{\delta s_W^2}{s_W^2} + \delta_V + \delta_B\right) - \frac{(s_W^2)_{\mathrm{LRSM}} - (s_W^2)_{\mathrm{SM}}}{(s_W^2)_{\mathrm{LRSM}}}, \tag{14}$$

where δ_V denotes the vertex corrections, which consist of the proper one loop vertex diagrams and the incomplete counter-term made only of the fermion wave function renormalization constants

$$\delta_V = \frac{\sqrt{2}s_W}{e}(\Lambda_{e\nu_e W} + \Lambda_{\mu\nu_\mu W}) + \frac{1}{2}\left(\delta Z_L^e + \delta Z_L^{\nu_e} + \delta Z_L^\mu + \delta Z_L^{\nu_\mu}\right), \tag{15}$$

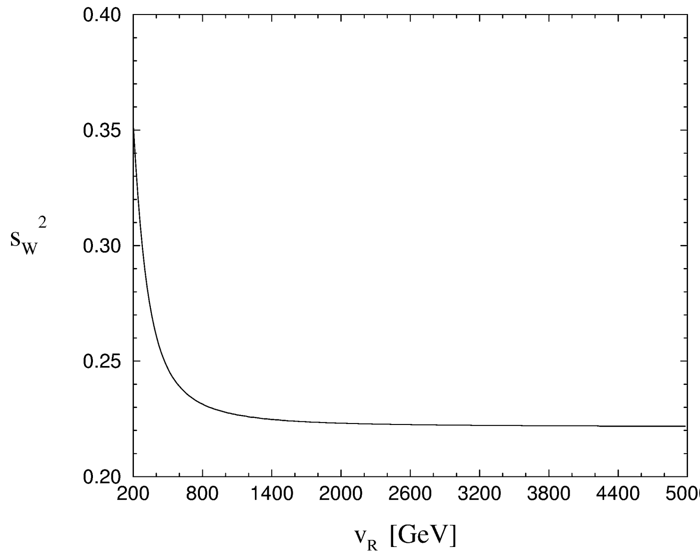

Fig. 1. $\sin^2 \Theta_W$ as function of v_R with M_{W_1} and M_{Z_1} as in Eq. (17).

with Λ being the coefficient in front of the operator $\gamma^\mu P_L$, and δ_B represents the box contributions. The last term comes of course from the "renormalization" of the Weinberg angle between the two models with $(s_W^2)_{\text{SM}} = 1 - \frac{M_{W_1}^2}{M_{Z_1}^2}$ and $(s_W^2)_{\text{LRSM}}$ as obtained by solving Eqs. (A.9), (A.10).

The strong dependence on the light fermion masses in δe is avoided as usual by a shift up to the Z_1 mass, and insertion of the running of the fine structure constant, for which we take [20]

$$\Delta\alpha(M_{Z_1}) = 0.059394 \pm 0.000395. \tag{16}$$

The factorization of the QED corrections is obtained with the Sirlin's method [19], which amounts to rejecting the infrared divergent box diagram and replacing the photon vanishing mass by the W_1 mass in the infrared divergent lepton wave function renormalization constants.

5. Quantitative results

The evaluation of one loop corrections within the LRSM is a task of moderate size as far as the number of diagrams is concerned. In fact approximately 600 had to be calculated already after our simplifying assumptions. It would not be possible to perform this work without using an automated system. For the generation of diagrams we used the C++ library `DiaGen` [21], which currently contains a topology generator, with several tools to analyse the properties of the created objects, and a diagram generator with support for

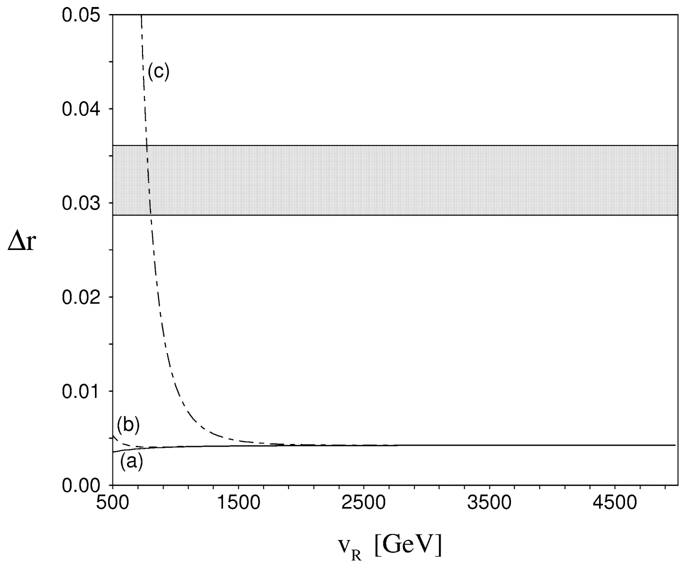

Fig. 2. Contribution of box diagrams to Δr. With increasing v_R heavy particles decouple and the lines aim at the SM contribution. The (a) line is for (three heavy neutrinos) $m_N = 100$ GeV; (b) is for $m_N = 500$ GeV; (c) is for $m_N = 2$ TeV. Higgs particle masses obey Eqs. (A.3), (A.4). The gray area shows the experimentally allowed values of Δr.

Majorana fields. The output has then been algebraically simplified with `FORM` [22], and at last numerically evaluated with the help of the `FF` [23] based package `LoopTools` [24].

As discussed in the previous section we parametrized the muon lifetime corrections coming from the LRSM through Δr, which is defined analogously as in the SM. With the present values of the coupling constants and masses [15]

$$G_F = 1.16639(1) \times 10^{-5}\ \text{GeV}^{-2}, \qquad 1/\alpha = 137.0359976 \pm 0.00000050,$$
$$M_{W_1} = 80.451 \pm 0.033\ \text{GeV}, \qquad M_{Z_1} = 91.1875 \pm 0.0021\ \text{GeV}, \tag{17}$$

the value of Δr with error is

$$\Delta r = 0.032 \pm 0.004. \tag{18}$$

We depicted this experimentally allowed range by a shaded region on the relevant figures.

As noted already in a previous work [6], we should not expect decoupling in the sense that for large v_R and large masses of the additional particles the SM result for Δr would be obtained. Some type of decoupling is however observable. For example if we take the box diagrams in the 't Hooft–Feynman gauge, then the result tends to the SM one as depicted in Fig. 2. It is worth noting however the effect of taking heavy neutrinos with a low v_R as for the (c) curve, where the contribution blows up. This is simply a consequence of the fact, that the ratio of a neutrino mass and the heavy SSB scale is proportional to the Yukawa coupling h_M (Eq. (A.17)) and the respective diagrams are proportional to at least the square of these couplings. Obviously, if the Yukawa couplings start to be larger than one then the perturbative expansion must break down.

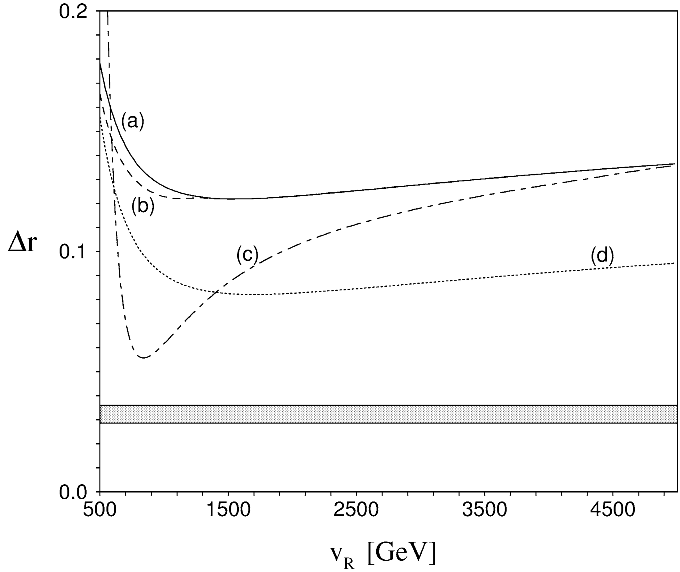

Fig. 3. Δr as function of v_R for different heavy neutrino masses. Higgs masses are chosen according to Eqs. (A.3), (A.4). The (a) line is for (three heavy neutrinos) $m_N = 100$ GeV; (b) is for $m_N = 500$ GeV; (c) is for $m_N = 2$ TeV. Line (d) shows the results when heavy neutrino masses follow from $h_M = 1$ (see Eq. (19)). The gray area shows the experimentally allowed values of Δr.

An interesting effect is obtained, if we take the masses of the Higgses to follow some simple pattern as in Appendix A Eqs. (A.3) and (A.4). The respective Δr is shown for several heavy neutrino masses in Fig. 3. With growing v_R the value grows strongly away from the allowed range and these parameters must be rejected. Although heavy neutrinos lower down Δr, we cannot obtain a reasonable value even if their masses are at the edge of the perturbatively range. The line (d) realizes this situation with the largest possible heavy neutrino mass as a function of v_R (Eq. (A.17))

$$m_N = \sqrt{2}\, v_R. \tag{19}$$

If we now assume for simplicity that all of the Higgs scalar masses are equal, apart from the SM Higgs boson, then we obtain the strong dependence as depicted in Fig. 4. If all the scalars are approximately two times heavier than v_R (for large Higgs masses), the experimental value for the muon decay life-time can be accommodated. Let us note at this point that large Higgs masses, at least of the order of a few TeV are needed because of FCNC [4]. It is obvious from Fig. 4 that Higgs scalars, heavy neutrinos and additional gauge boson masses are very much fine-tuned to be within the SM gray area, e.g., the line (a′) with $m_H = 1$ TeV and $m_N = 100$ GeV gives $v_R \simeq 800$ GeV, which fixes M_{W_2} and M_{Z_2} to the values as in Fig. 5.

It is interesting, that for a larger SSB breaking scale, the variation of Δr with the SM Higgs scalar mass is negligible. This is shown in Fig. 6 for $v_R = 2390$ GeV and neutrino and heavy scalar masses chosen to fit the experimental value.

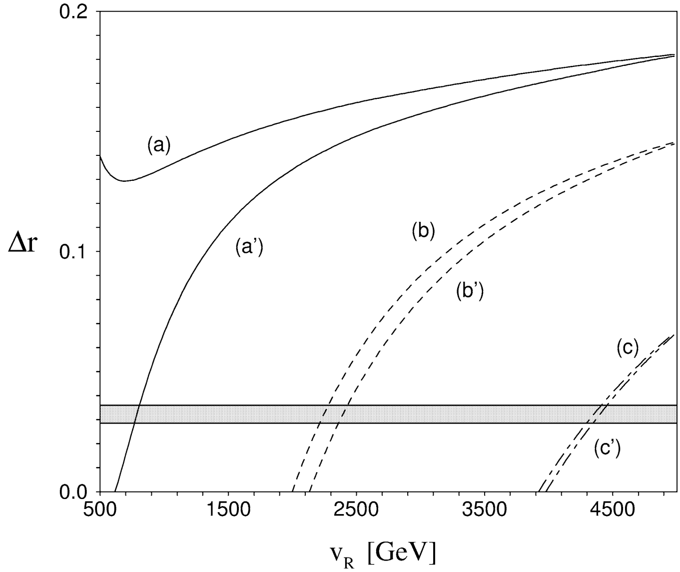

Fig. 4. Δr as function of v_R. Sets with and without primes show results for three heavy neutrino masses with $m_N = 100$ GeV and $m_N = 2$ TeV, respectively. The lines describe different values of Higgs scalar masses: (a) is for all Higgs masses $M_H = 1$ TeV; (b) is for $M_H = 5$ TeV; (c) is for $M_H = 10$ TeV. The gray area shows the experimentally allowed values of Δr.

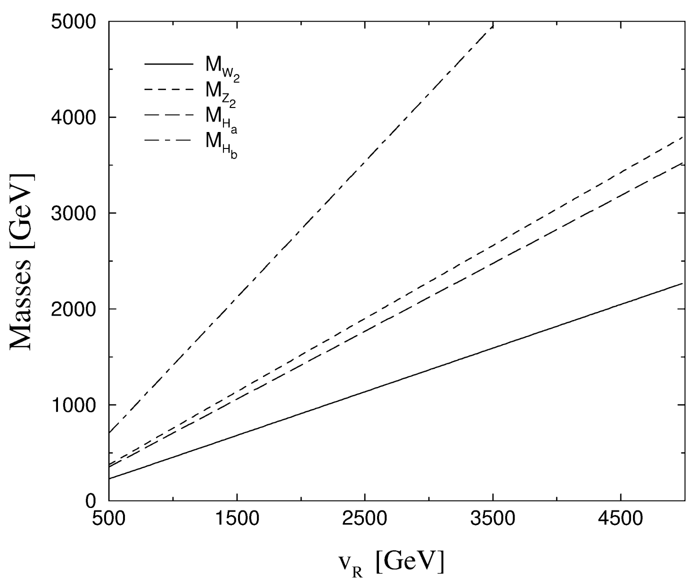

Fig. 5. Masses of additional gauge bosons and two sets of Higgs scalar particles Eqs. (A.3), (A.4) as function of the v_R scale.

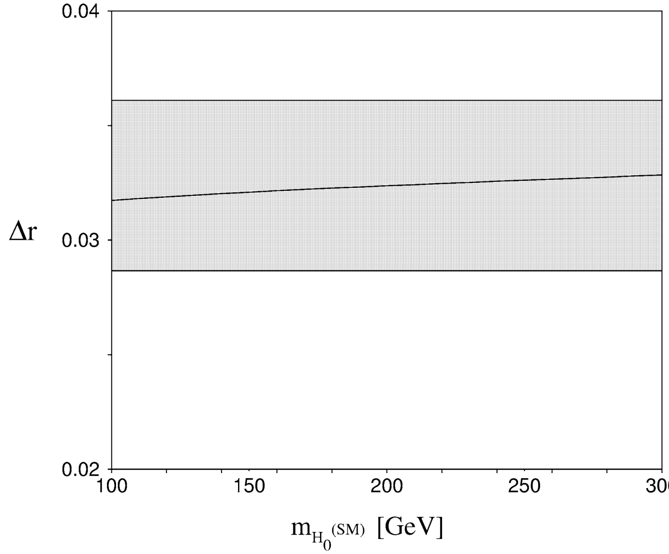

Fig. 6. Δr as function of the lightest Higgs scalar mass $M_{H_0^0}$. The gray area shows the experimentally allowed values of Δr and the heavy particle spectrum is chosen to fit approximately to this region, namely, $v_R = 2390$ GeV, $m_N = 2$ TeV, $m_H = 5$ TeV.

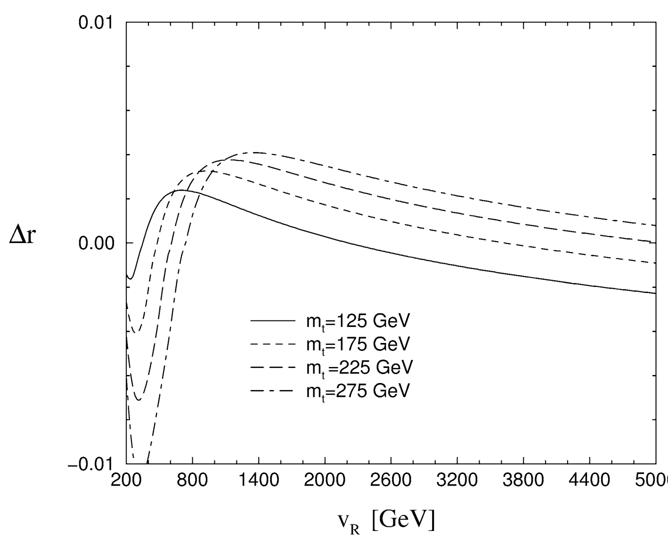

Fig. 7. The contribution of the third quark family to Δr as function of v_R for different top quark masses.

At last let us comment on the dependence on the top quark mass. We show the contributions of the third quark family in Fig. 7. The values of the top mass spread over a vary large range to show the behaviour of the correction. As already foreseen in [6], for low values of v_R the variation is described by a negative quadratic function. However for a v_R as low as 1 TeV, only a positive logarithmic contribution is visible. Notice also, that even in the low v_R range, the top mass squared enters with a smaller coefficient than in the SM.

6. Conclusions

In this paper we have studied the full one loop corrections to the muon decay in a self consistent left–right symmetric model. We have shown quantitatively that the contributions have a different structure from the SM ones and that they cannot be separated into these and some corrections that would vanish with v_R. Moreover, we have shown that the muon decay alone already puts some stringent restrictions on the different heavy particle masses with respect to the heavy SSB breaking scale.

Our analysis should be extended to cover also other low-energy experiments [14]. This should elucidate the question of the contribution of the H_1^+ boson to the muon decay. It will then also be possible to derive bounds on the extra boson masses.

At last let us note that several of the assumptions that we here took could be raised, but it is doubtful that this would change qualitatively the numerical results. On the other hand it would certainly make the analysis more involved, starting from the necessity to renormalize the ξ angle, LH neutrino mixing and ending with problems with the QED contributions.

Acknowledgement

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Appendix A

In this appendix we gather our definitions and give a short account of the particle content of the model.

A.1. Higgs sector

The Higgs sector contains one bidoublet and two triplets

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \qquad \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix}, \tag{A.1}$$

with the allowed Vacuum Expectation Values (VEV)

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \qquad \langle \Delta_{L,R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}, \tag{A.2}$$

of which κ_2 and v_L are assumed to vanish.

The full potential has been studied in [8–10]. Here we only recall the physical spectrum of the particles, which consists of

(i) four neutral scalars with $J^{PC} = 0^{++}$ (H_i^0, $i = 0, 1, 2, 3$);
(ii) two neutral pseudoscalars with $J^{PC} = 0^{+-}$ (A_i^0, $i = 1, 2$);
(iii) two singly charged bosons ($H_i^{\pm}$, $i = 1, 2$), and
(iv) two doubly charged Higgs particles ($\delta_L^{\pm\pm}, \delta_R^{\pm\pm}$).

If $v_R \gg \kappa_1$ and all the parameters of the potential which enter the Higgs masses are taken to be 1 (these are combinations of $\mu_{1,2}, \lambda_{1,\ldots,6}, \rho_{1,\ldots,4}$ defined in [8]), then neglecting terms proportional to the VEV of the SM, the masses satisfy the relations

$$M_{H_a} \equiv M_{H_1^0} = M_{H_3^0} = M_{A_1^0} = M_{A_2^0} = M_{H_1^+} = M_{H_2^+} = M_{\delta_L^{++}} = v_R/\sqrt{2}, \tag{A.3}$$

$$M_{H_b} \equiv M_{H_2^0} = M_{\delta_R^{++}} = \sqrt{2}\, v_R, \tag{A.4}$$

$$M_{H_0^0} = \sqrt{2}\,\kappa_1. \tag{A.5}$$

A.2. Gauge boson sector

Gauge boson masses are generated by the following mass terms (with the assumption of equal couplings for the two $SU(2)$ groups)

$$L_M = \left(W_L^{+\mu}, W_R^{+\mu}\right) M^2_{\text{Charged}} \begin{pmatrix} W^-_{L\mu} \\ W^-_{R\mu} \end{pmatrix} + \text{h.c.} + \frac{1}{2}\left(W^\mu_{3L}, W^\mu_{3R}, B^\mu\right) M^2_{\text{Neutral}} \begin{pmatrix} W_{3L\mu} \\ W_{3R\mu} \\ B_\mu \end{pmatrix}, \tag{A.6}$$

with

$$M^2_{\text{Charged}} = \frac{g^2}{4} \begin{pmatrix} \kappa_+^2 & -2\kappa_1\kappa_2 \\ -2\kappa_1\kappa_2 & \kappa_+^2 + 2v_R^2 \end{pmatrix}, \tag{A.7}$$

and

$$M^2_{\text{Neutral}} = \frac{1}{2} \begin{pmatrix} \frac{g^2}{2}\kappa_+^2 & -\frac{g^2}{2}\kappa_+^2 & 0 \\ -\frac{g^2}{2}\kappa_+^2 & \frac{g^2}{2}\left(\kappa_+^2 + 4v_R^2\right) & -2gg'v_R^2 \\ 0 & -2gg'v_R^2 & 2g'^2 v_R^2 \end{pmatrix}, \tag{A.8}$$

where $\kappa_+ = \sqrt{\kappa_1^2 + \kappa_2^2}$. The masses of the physical gauge bosons are then given by

$$M^2_{W_{1,2}} = \frac{g^2}{4}\left[\kappa_+^2 + v_R^2 \mp \sqrt{v_R^4 + 4\kappa_1^2\kappa_2^2}\,\right], \tag{A.9}$$

$$M^2_{Z_{1,2}} = \frac{1}{4}\Big\{\left[g^2\kappa_+^2 + 2v_R^2\left(g^2 + g'^2\right)\right] \mp \sqrt{\left[g^2\kappa_+^2 + 2v_R^2\left(g^2 + g'^2\right)\right]^2 - 4g^2\left(g^2 + 2g'^2\right)\kappa_+^2 v_R^2}\Big\}. \tag{A.10}$$

The symmetric mass matrices are diagonalized by the orthogonal transformations

$$\begin{pmatrix} W_L^{\pm} \\ W_R^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} = \begin{pmatrix} W_1^{\pm} \\ W_2^{\pm} \end{pmatrix}, \tag{A.11}$$

and

$$\begin{pmatrix} W_{3L} \\ W_{3R} \\ B \end{pmatrix} = \begin{pmatrix} c_W c & c_W s & s_W \\ -s_W s_M c - c_M s & -s_W s_M s + c_M c & c_W s_M \\ -s_W c_M c + s_M s & -s_W c_M s - s_M c & c_W c_M \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix}, \tag{A.12}$$

where

$$g = \frac{e}{\sin\Theta_W}, \qquad g' = \frac{e}{\sqrt{\cos 2\Theta_W}}, \qquad c_W = \cos\Theta_W, \qquad s_W = \sin\Theta_W,$$
$$c_M = \frac{\sqrt{\cos 2\Theta_W}}{\cos\Theta_W}, \qquad s_M = \tan\Theta_W, \qquad s = \sin\phi, \qquad c = \cos\phi.$$

The mixing angles are given by

$$\tan 2\xi = -\frac{2\kappa_1\kappa_2}{v_R^2}, \qquad \sin 2\phi = -\frac{g^2\kappa_+^2\sqrt{\cos 2\Theta_W}}{2\cos^2\Theta_W (M_{Z_2}^2 - M_{Z_1}^2)}. \tag{A.13}$$

Fig. 5 sums up the mass dependence of the additional gauge bosons and two sets of Higgs scalar particles Eqs. (A.3), (A.4) on the v_R scale.

A.3. Neutrinos

The MLRSM naturally contains left as well as right handed neutrino states. We chose the following basis for these fields

$$n_R = \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}, \qquad n_L = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix}, \qquad \nu_{L,R}^c = C\bar{\nu}_{L,R}^{\mathrm{T}}, \tag{A.14}$$

where both $n_{L(R)}$ form 6-dimensional vectors. The neutrino mass matrix takes the form

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^{\mathrm{T}} & M_R \end{pmatrix}, \tag{A.15}$$

$$M_D = \frac{1}{\sqrt{2}}(h_l\kappa_1 + \tilde{h}_l\kappa_2) = M_D^{\dagger}, \tag{A.16}$$

$$M_R = \sqrt{2}\, h_M v_R = M_R^{\mathrm{T}}. \tag{A.17}$$

$h_l, \tilde{h}_l, h_M$ are the respective Yukawa coupling matrices.

M_ν can be diagonalized with the following unitary transformation

$$U = \begin{pmatrix} K_L^{\mathrm{T}} \\ K_R^{\dagger} \end{pmatrix}, \tag{A.18}$$

where the $K_{L,R}$ matrices have dimension 6×3. The *LEP* neutrino counting results show that there must be three light active neutrino states. This means that we must have $M_D \ll M_R$, which after requiring “natural” couplings (order one), turns into $\kappa_{1,2} \ll v_R$.

Let us now introduce 3×3 matrices $U_{Ll(h)}$, $U_{Rl(h)}$ [9]:

$$K_L = \begin{pmatrix} U_{Ll}^{\dagger} \\ U_{Lh}^{\dagger} \end{pmatrix}, \tag{A.19}$$

$$K_R = \begin{pmatrix} U_{Rl}^{\dagger} \\ U_{Rh}^{\dagger} \end{pmatrix}. \tag{A.20}$$

The diagonalization equation assumes the form (m_{diag} and M_{diag} correspond to light and heavy neutrino mass matrices, respectively):

$$\begin{pmatrix} U_{Ll}^{\dagger} & U_{Rl}^{\mathrm{T}} \\ U_{Lh}^{\dagger} & U_{Rh}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^{\mathrm{T}} & M_R \end{pmatrix} \begin{pmatrix} U_{Ll}^{*} & U_{Lh}^{*} \\ U_{Rl} & U_{Rh} \end{pmatrix} = \begin{pmatrix} m_{\text{diag}} & 0 \\ 0 & M_{\text{diag}} \end{pmatrix}. \tag{A.21}$$

This can be written as:

$$U_{Ll}^{\dagger}\left(-M_D M_R^{-1} M_D^{\mathrm{T}}\right) U_{Ll}^{*} \simeq m_{\text{diag}}, \tag{A.22}$$

$$U_{Rl} \simeq -M_R^{-1} M_D^{\mathrm{T}} U_{Ll}^{*}, \tag{A.23}$$

$$U_{Rh}^{\mathrm{T}} M_R U_{Rh} \simeq M_{\text{diag}}, \tag{A.24}$$

$$U_{Lh}^{*} \simeq M_D^{*}\left(M_R^{*}\right)^{-1} U_{Rh}, \tag{A.25}$$

where the unitarity of U, and the large scale difference $M_{\text{diag}} \gg m_{\text{diag}}$, have been used. Two important conclusions can be drawn from it:

- the matrices U_{Ll} and U_{Rh} are approximately unitary;
- the elements of the non-diagonal submatrices U_{Rl} and U_{Lh} are small, of order $\frac{\langle m_D \rangle}{\langle M_R \rangle} \leqslant \frac{O(1\ \text{GeV})}{m_N}$, where $m_N = \langle M_{\text{diag}} \rangle$.

The symbol $\langle \cdots \rangle$ denotes the relevant scale of mass matrices.

Altogether we can write

$$U = \begin{pmatrix} K_L^{\mathrm{T}} \\ K_R^{\dagger} \end{pmatrix} = \begin{pmatrix} O(1) & O(1/m_N) \\ O(1/m_N) & O(1) \end{pmatrix}. \tag{A.26}$$

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