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Computer Physics Communications 98 (1996) 45-51

Computer Physics Communications

Elastic net for broken multiple scattered tracks

I. Kisel [1], V. Kovalenko [2]

Joint Institute for Nuclear Research, Dubna, Moscow Region, 141980, Russian Federation

Received 20 March 1996

Abstract

We propose to use an elastic net for fitting charged particle trajectories with multiple scattering in a gas and hard scattering on wires. The advantages of the method are simplicity of the algorithm, fast and stable convergence, and reconstruction efficiency close to 100%. The elastic net is tested on broken multiple scattered tracks simulated for the NEMO-3 spectrometer out of a magnetic field.

1. Introduction

During the last 10-15 years the rapid development of various theories of artificial neural networks was a reflection of an attempt to overcome the gulf between the huge amount of factual material relating to the biological mechanisms of brain operation accumulated in neurophysiology and the inadequate existing mathematical formalism and computational means of technical realization of the formalism. The principal advantages of the brain in fulfilling logical, recognition, and computational functions, using capabilities that are essentially parallel, nonlinear, and nonlocal, did not match the prevailing principle of sequential calculations with orientation of the mathematical formalism toward locality, linearity, and stationarity of the descriptions.

Included among these are problems whose solution is complicated precisely by nonlinearity, nonlocality, discreteness, and, often, nonstationarity of the situation. For instance, we mention here problems of pattern recognition, construction of associative memory and optimization.

Essentially, the theory of artificial neural networks is a part of the general theory of dynamical systems in which particular attention is devoted to the investigation of the complicated collective behavior of a very large number of comparatively simple logical objects.

Here we describe an elastic net for fitting of broken multiple scattered tracks. The algorithm is tested on a simple model of the NEMO-3 spectrometer with drift tubes as the main part of the detector.

2. Elastic net method

The elastic net is a kind of artificial neural network [1] which is used for track recognition in high energy physics [2,3].

Let us demonstrate the elastic net method by a simple example of solving a travelling salesman problem. The travelling salesman problem is a classical problem in the field of combinatorial optimization, concerned with efficient methods for maximizing or minimizing a function of many independent variables.

[1] E-mail: kisel@axnua.jinr.dubna.su
[2] E-mail: koval@nu.jinr.dubna.su

PII S0010-4655(96)00088-4

Given the positions of N cities, what is the shortest closed tour in which each city can be visited once?

All exact methods known for determining an optimal route require a computing effort that increases exponentially with the number of cities, so in practice exact solutions can be attempted only on problems involving a few hundred cities or less. The travelling salesman problem belongs to a large class of NP-complete[3] problems.

One of the most successful approaches to the travelling salesman problem is the elastic net of Durbin and Willshaw [4]. The elastic net can be thought as a number of beads connected by elastic into a ring. The essence of the method is:

Using an iterative procedure, a circular closed path is gradually elongated nonuniformly until it eventually passes sufficiently near to all the cities to define a tour.

A tour can be viewed as a mapping from a circle to the plane so that each city in the plane is mapped to by some point on the circle. We consider mappings from a circular path of points to the plane in which neighboring points on the circle are mapped as close as possible on the plane. This is a special case of the general problem of best preserving neighborhood relations when mapping between different geometrical spaces.

The algorithm is a procedure for the successive recalculation of the positions of a number of points in the plane in which the cities lie. The points describe a closed path which is initially a small circle centered on the center of the distribution of cities and is gradually elongated nonuniformly to pass eventually near all the cities and thus define a tour around them (Fig. 1, see details in the original paper [4]). Each point on the pass moves under the influence of *two types of force* (see Fig. 2):

- one moves it towards those cities to which it is nearest;
- the other pulls it towards its neighbors on the path, acting to minimize the total path length.

By this means, each city becomes associated with a particular section on the path. The tightness of the association is determined by how the force contributed from a city depends on a distance, and the nature of this dependence changes as the algorithm progresses.

[3] Nondeterministic polynomial time complete.

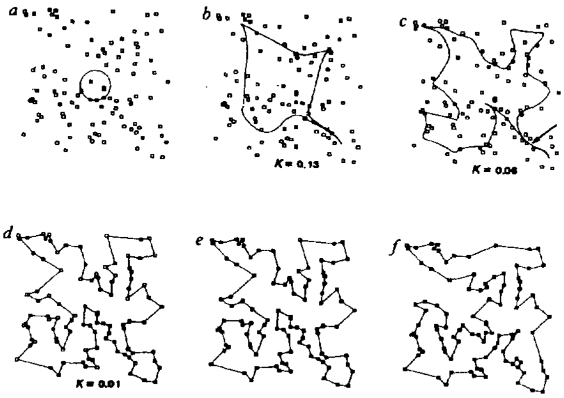

Fig. 1. Example of the progress of the elastic net method for 100 cities.

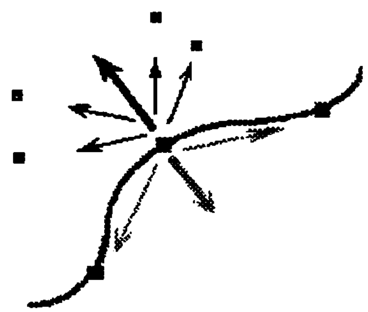

Fig. 2. Two types of force in the TSP problem.

Initially all cities have roughly equal influence on each point on the path. Subsequently, longer distance become less favored, and each city gradually becomes more specific for the points on the path closest to it.

3. Elastic net for tracking

In this section we will formulate the elastic net method for tracking purposes.

Here we will consider only single straight tracks out of a magnetic field (Fig. 3). There are no noise, which does not need track searching, and no missing wires, which slightly simplifies the algorithm.

It is obvious to

Define a track with multiple scattering as the smoothest line touching all circles around fired wires and crossing all layers.

Let us try to construct the elastic net as a line which is deformed, as in the previous example, under the effect of *two types of force* (Fig. 4):

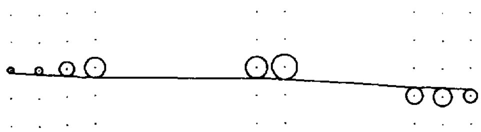

Fig. 3. An example of a track with multiple scattering.

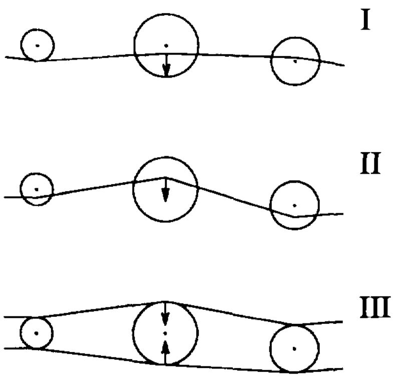

Fig. 4. Three types of force in the tracking problem.

- one pulls it to the edges of circles;
- the other smoothes out the line.

Obviously, our previous requirement of absence of noise points is a fiction because of left-right ambiguity of drift tubes. Knowing approximately the direction of a track, we do not know whether it goes at the left or right side to a circle. So at each fired wire we have two possible touching points of a track, one of which is wrong. As a result we have 100% noise! In this case the task can be considered as a problem of minimization of a function of many variables with many local minima. All known standard methods of minimization [5] start with a point on the function surface and minimize the function in a local area around it at each step of the iterational procedure. In such a way there is a significantly nonzero probability to stop at a local minimum. It is possible to build methods against local minima to save the standard methods, but to do so increases the complexity of our program and makes it more nervous. The only reliable way is to start with a good initial approximation, which cannot be easily found in our case. So the use of the standard minimization methods is not suitable.

Contrary to the standard methods, we propose to start with two points on opposite sites of the global minimum and covering all possible area of the physical region of the parameters to be found. These points are not independent but attract each other. So the points will pull each other from all local minima until the global minimum will be reached. Graphically, this method is represented in Fig. 5.

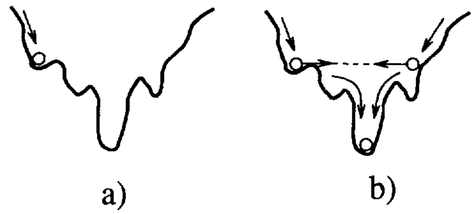

Fig. 5. Global minimum search by (a) a standard method and (b) the elastic net method.

According to this idea we start with two bounded tracks which restrict the geometrical area of the real track. One of them touches the circles at upper sides and the other at lower sides. Then we introduce a *third type of force* (Fig. 4), which is

- attraction between these bounded tracks to squeeze a geometrical area to the real track.

This method allows one to find an optimal trajectory which corresponds to our model of a track.

The elastic net can be simply modified to be able to reconstruct broken tracks. We have only to switch off track smoothing at a break point, which has to be found during preliminary analysis.

4. Testing of the elastic net for the NEMO-3 spectrometer

The goal of the NEMO collaboration [6,7] is to study $\beta\beta2\nu$ and $\beta\beta0\nu$ decay of ^{100}Mo and other nuclei to probe the effective Majorana neutrino mass down to 0.1 eV.

The NEMO-3 spectrometer (Fig. 6) has been designed in 1993 for the investigation of a 10 kg enriched molybdenum source in order to reach a sensitivity limit of 10^{25} years for the half-life of the $\beta\beta0\nu$ mode. The spectrometer is planned to start its operation in 1996.

The tracking part of the NEMO-3 detector consists of octagonal Geiger-tubes, each with a central wire surrounded by 8 ground wires. The tubes are 32 mm in diameter and 2.7 m in length. The diameter of the central wire is 100 μm. The ground wires are 30 μm in diameter. The octagonal tubes are slightly deformed

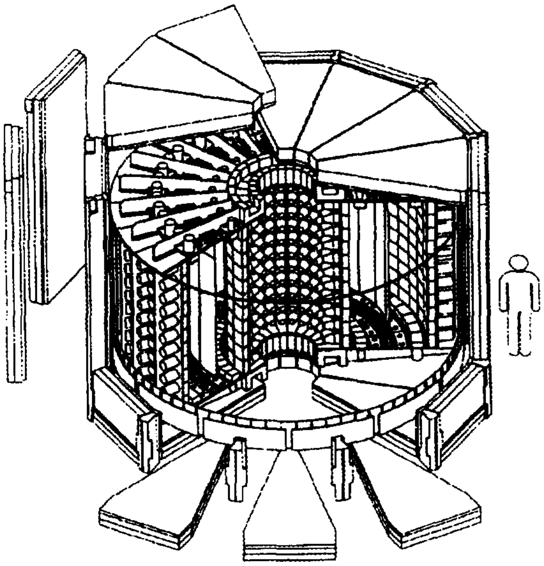

Fig. 6. General view of the NEMO-3 spectrometer.

to fit a circular geometry, and form a four-two-three sequence of layers. Four layers near the central foil give a sufficient number of points to get a precise vertex position. The layers in the middle and close to the scintillators allow the measurement of the trajectory curvature in a magnetic field. The expected point measurement resolutions are $\sigma_T \approx 0.5$ mm and $\sigma_L \approx 5$ mm. The peculiarity of the NEMO experiment is the presence of very low energy electrons, so we must take into account quite large multiple scattering effects in the helium gas as well as hard interactions with different wires.

The elastic net method was tested [8] on a simple model of the NEMO-3 detector. We consider 9 plane layers of tubes which are placed according to the geometrical parameters of the detector. Fired tubes are simulated as described below.

At the first stage we simulate a particle with a random coordinate and direction in the first layer. Its path is extrapolated to the next layer and simulates multiple scattering by random changing direction in a $\pm 2.5^{\circ}$ region. Then we find the distance from this crossing point to the nearest central anode wire that determines the radius of a circle around the wire. This procedure is repeated for all layers. The simulated information is used as an input data for the elastic net method.

The algorithm is

```
repeat
  for i := 1 to Number_of_Planes do
  begin      { at i-th plane: }
(1) Calculate distance from both tracks
    to a fired wire.
(2) Calculate bends of tracks.
(3) Calculate distance between both
    tracks.
(4) Summarize calculated values with
    weights to determine new crossing
    points of tracks with the plane.
  end;
until ( True_Track_is_Found );
```

An example of evolution of the elastic net for a multiple scattered track is presented in Fig. 7. Layers are numbered from left to right and iterations go from top to bottom. There are two starting tracks - upper and down tracks. One can see smoothing of the upper track in the first layer after the first iteration, but then this track becomes almost linear in the left group of the layers and is stable being attracted to neighboring edges of the circles. It is in a local minimum and moves down only under pressure of the third type of force - attraction to the lower track. The middle part of the upper track is smoothed at the beginning and then evolves mainly due to attraction to the lower track. The right part of the tracks is in equilibrium in the middle of the evolution and goes to the global minimum only due to smoothing.

The elastic net for broken tracks was also tested on the same model of the NEMO-3 detector as in the previous case, but tracks changed direction randomly in the $\pm 25.0^{\circ}$ region at one of the trajectory points.

An example of evolution of the elastic net for a broken track is presented in Fig. 8. The simulated track had a break in the third layer. One can mention that the algorithm finds a wrong solution in the first layer but then corrects it. Other parts of the evolution are obvious.

The test demonstrates good performance of the method and fast convergence to the simulated tracks with few iterations (see Fig. 9). Each iteration is very fast, as one can see from the algorithm presented above.

The reconstruction efficiency is close to 100%. This is clear from Fig. 10, where distances between the simulated and found tracks in each layer are presented

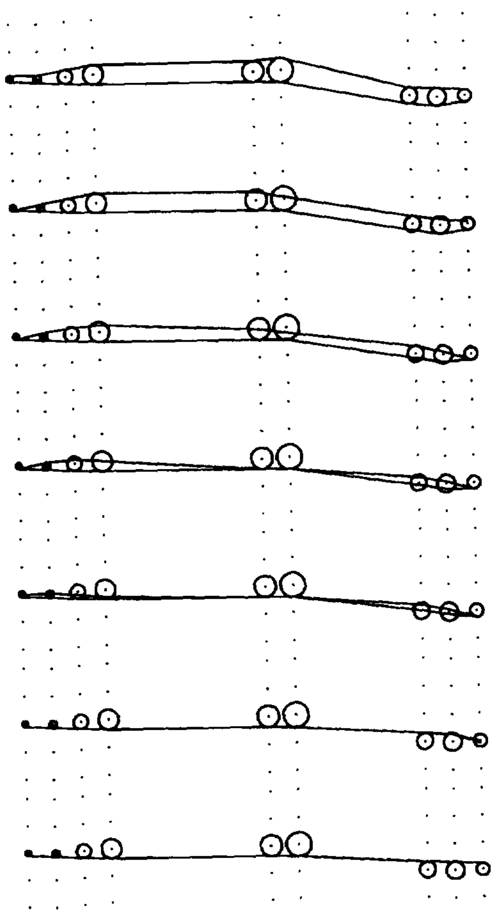

Fig. 7. Example of the elastic net evolution for a track with multiple scattering.

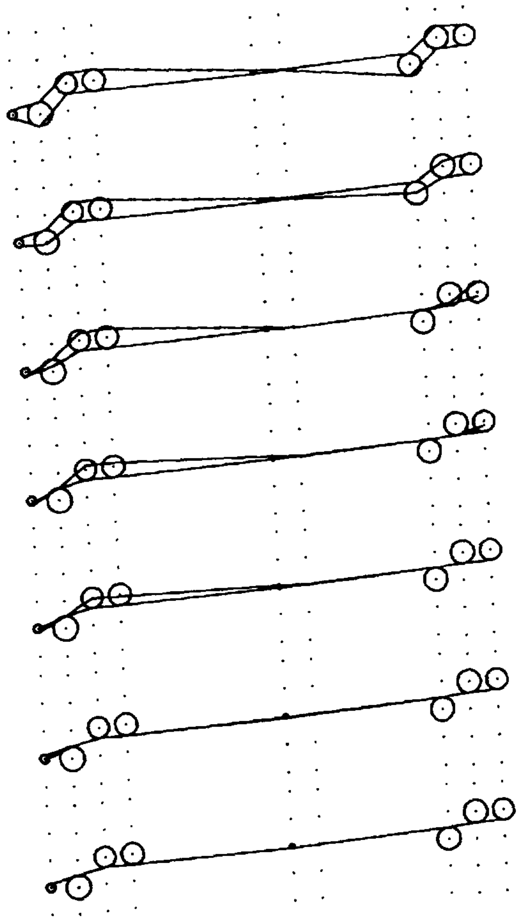

Fig. 8. Example of the elastic net evolution for a track scattered on a wire.

in the logarithmic scale. One can see that the found track coincides with the simulated one almost in all cases. Small differences between tracks can be at wires with small radii. In such cases the algorithm can find a smoother track than the simulated one. This follows from Fig. 11. One can see random distribution of angles between segments of simulated tracks in the left picture but the right distribution shows a small peak at the zero angle, which means that in some cases the found track is smoother than the simulated one.

5. Conclusion

The results of testing on simulated NEMO-3 tracks demonstrate reliable performance of the elastic net method in the case of multiple scattering in a gas and hard interactions with different wires. The advantages of the method are simplicity of the algorithm, fast and stable convergence, and high reconstruction efficiency.

Acknowledgements

We want to thank all members of the NEMO collaboration for their interest in this work. We are especially grateful to F. Laplanche for helpful discussions.

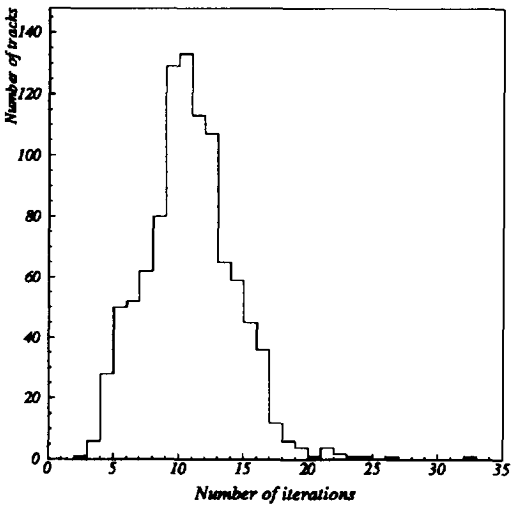

Fig. 9. Number of iterations for convergence.

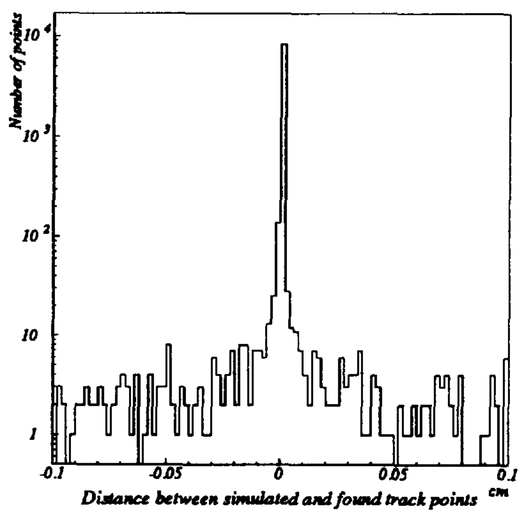

Fig. 10. Distance between simulated and found tracks on a logarithmic scale.

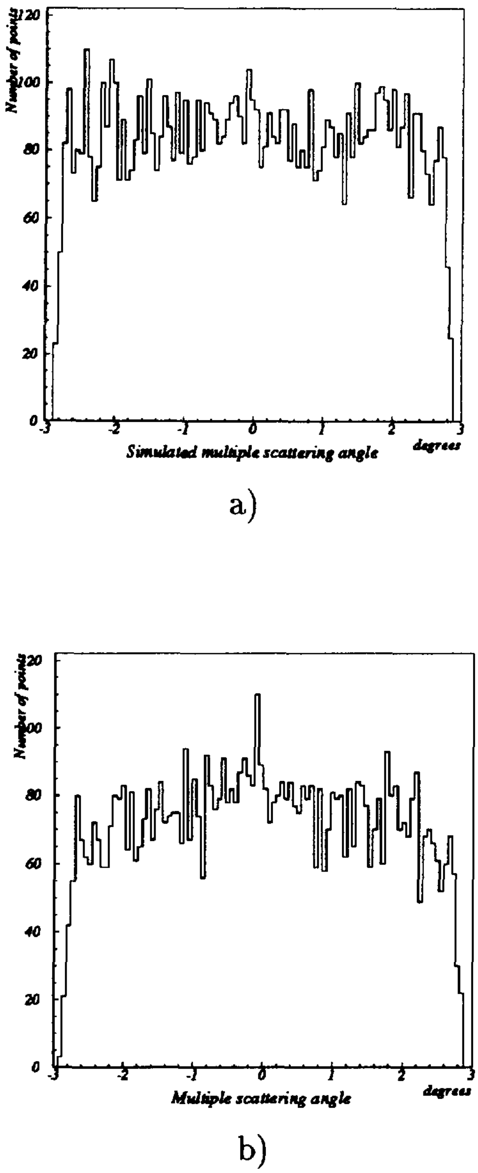

Fig. 11. Angles between segments of (a) simulated and (b) found tracks.

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