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Robust estimates of track parameters and spatial resolution for CMS muon chambers

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Abstract

A robust technique with a sub-optimal weight function (M-estimate) was applied to investigate track fitting in cathode strip chambers (CSCs) and determine the CSC spatial resolution. The comparative analysis with the conventional least squares method was made on simulated data and experimental data from the Dubna CSC prototype. The results obtained definitely prove a necessity of using robust track fitting for a reliable estimation of muon chamber spatial resolution. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The muon system is an important part of the Compact Muon Solenoid (CMS) detector [1] which will be installed in the LHC at CERN. The muon system should provide a high spatial resolution under conditions of heavy background. Cathode strip chambers (CSCs), i.e. six-layer multiwire proportional chambers with a strip cathode readout, are used as muon detectors in the forward region of CMS. The required azimuthal spatial resolution (σ_m) for CMS muon endcap CSCs is of the order of a hundred μm. About 10–20% of the muon hits in a CSC will be contaminated by different sources, but we consider here two of the most essential of them: (i) secondary electromagnetic (e.m.) particles (γ and e^-/e^+) entering a muon detector from a calorimeter with a muon and (ii) δ-electrons produced by a muon passing through the material of a muon detector. As a result, the error distribution differs from the normal (Gaussian) distribution and tends to have long non-Gaussian “tails”. As is well known [2], conventional least squares (LSQ) estimates lose their optimal properties in such cases. On the contrary, robust M-estimates [2] are much less sensitive to data contamination. Thus, the aims of our work are: (1) to make a mathematical model of a muon detector with noise taking into account e.m. secondaries and δ-electrons stochastically distributed along the muon track; (2) to apply a robust approach for track fitting under conditions of heavy background; (3) to make a comparative analysis of track parameters obtained by a robust technique and by the LSQ method; (4) to estimate the spatial resolution of CSC prototype in the most reliable way.

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2. Mathematical inference

Let us consider a linear regression dependence,

$$x_i = \sum_{j=1}^{p} \phi_j(z_i) \cdot \theta_j + e_i, \quad i = 1, \ldots, N, \tag{1}$$

where $\phi_j(z)$ are known p linearly independent geometric functions (e.g., $1, z, z^2, \ldots, z^{p-1}$); z_i, x_i, e_i are the coordinate, the response (measurement) and the accidental measurement error of the ith detector, respectively; θ_j are unknown regression parameters ($j = 1, \ldots, p$) which should be estimated by use of the data sample; N is the number of detectors used for fitting. We use the so-called gross-error model [2] of a contaminated distribution of measurement errors e_i,

$$f(e) = (1-\epsilon) \cdot g(e) + \epsilon \cdot h(e), \tag{2}$$

where ϵ is a parameter of contamination; g is the Gauss distribution and h is some long-tailed noise distribution. Using the maximum likelihood method ($L = \prod_{i=1}^{N} f(e_i) \to \max$) we obtain weighted least squares equations,

$$\sum_{i}^{N} w_i \cdot \left[x_i - \sum_{j'}^{p} \phi_{j'}(z_i) \cdot \theta_{j'} \right] \cdot \phi_j(z_i) = 0, \quad j = 1, \ldots, p, \tag{3}$$

with some optimal weights w_i dependent on relations of g and h distributions [3],

$$w_i = \frac{g(e_i)}{g(e_i) + \frac{\epsilon}{1-\epsilon} \cdot h(e_i)}. \tag{4}$$

These weights are nonlinear functions of parameters in questions. Therefore, an iterative procedure was elaborated (see, e.g., [3,4]) in which the weights are recalculated on each iteration in accordance to the new values of parameters. Further we call this procedure as reweighted LSQ. Besides, a polynomial expansion of these optimal weights up to fourth order was proposed in [4]. This leads to the approximation

$$w(e_i) = \begin{cases} \left(1 - \dfrac{e_i^2}{\sigma^2 \cdot c_T^2}\right)^2, & e_i^2 \leqslant c_T^2 \cdot \sigma^2, \\ 0, & e_i^2 > c_T^2 \cdot \sigma^2, \end{cases} \tag{5}$$

which in fact are the famous Tukey's bi-weights [5] and are easier to calculate than optimal ones.

The parameter σ should be also recalculated on each kth iteration as it was recommended in [4],

$$\sigma^{(k)^2} = \sum_i w_i^{(k-1)} (e_i^{(k-1)})^2 \Big/ \sum_i w_i^{(k-1)}. \tag{6}$$

We choose cutting parameter $c_T = 4$. In this way the measurements with errors greater than 4σ (in accordance with formula (5)) have zero weights and are excluded from the fitting procedure.

Our iterative reweighted LSQ-procedure (3) for robust parameters estimation includes some additional ideas:

(a) We vary the polynomial power in (5) as a function of the iteration number.

(b) Before zero-iteration we apply a special “base-line” procedure for selecting initial weights and assign weights lower than 1 to the outliers.

3. Monte Carlo model and results

We build a Monte Carlo (M.C.) mathematical model of a linear regression $x = az + b$ for a straight line muon track passing through 6 equidistant CSC layers. Charges induced on each of the cathode planes are space-distributed

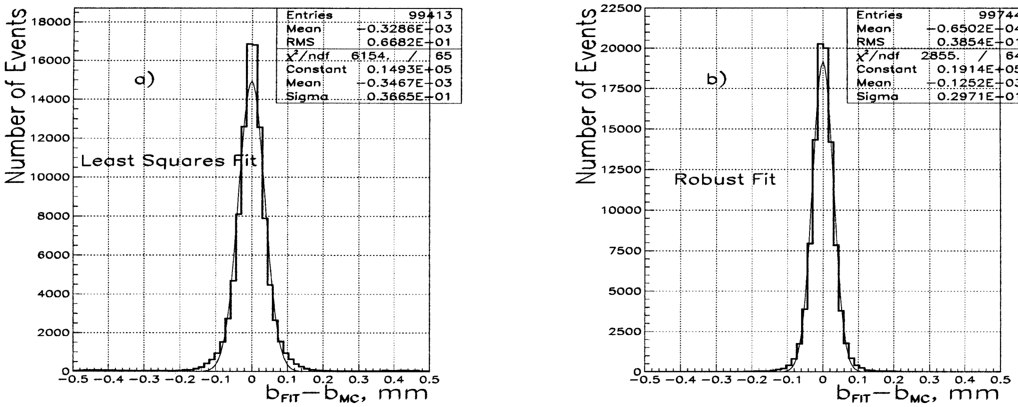

Fig. 1. The distribution of deviations of intercept parameters b_{FIT} from original M.C. parameters b_{MC} (fitted by a Gaussian).

in accordance with the Gatti formula [6]. The total charge on each strip $q_j^{(0)}$ is smeared (Δq_j) by adding a normal noise (a readout electronics noise) with the given σ_{noise} (i.e. $q_j^{(0)} \to q_j = q_j^{(0)} + \Delta q_j$). To simulate contamination we include δ-electrons and e.m. accompaniment stochastically distributed along the muon track. The contamination parameter ϵ, the number of δ-electrons in each layer and the distance between the muon and the δ-electron are parametrised on the basis of a previous GEANT simulation [7] of muons passing through all detectors.

As we can see from Fig. 1, the distribution of intercept parameter deviations for LSQ fitting (Fig. 1(a)) has longer tails than the distribution for a robust fit. Track parameters obtained by the robust method (Fig. 1(b)) have a value of root mean square (RMS) 1.7 times better than parameters obtained by the LSQ method. For the slope parameter a we obtained a similar result.

It is well known from mathematical statistics (see, e.g., [8,9]) that any parameter estimation can be qualified by a confidence level (CL). In accordance with 95% CL for Student distribution (6-point tracks, i.e. 4 degrees of freedom, with the confidence coefficient $t_{95\%} = 2.78$) we obtain a percentage of events, for which at least one of parameters (a, b) lies outside of the 95% confidence interval, amounts to 4.9% for robust track fitting and to 22.7% for LSQ fitting (the latter ones are quite suspicious to be used).

In order to estimate spatial resolution one should calculate normalised (due to track extrapolation) residuals for simulated events. Applying the procedure [1] based on the LSQ method we obtain a distribution of residuals which differs from Gaussian and has long tails (see Fig. 2(a)). We can conclude that the RMS is significantly greater (approximately 1.6 times) than the model device resolution $\sigma_m \approx 60$ μm. In contrast, the distribution of residuals obtained by robust track fitting (Fig. 2(b)) gives a distribution very close to a Gaussian (RMS $\approx \sigma$) and a good measurement of the chamber resolution.

After testing our elaborated approach to estimate a realistic device resolution on simulated data, we apply this technique to experimental data from the detector prototype [10]. These data were obtained from the full-scale Dubna CSC prototype at the Integrated Test setup in H2 beam line of the CERN SPS accelerator. Muon hits were heavily contaminated by δ-rays and e.m. secondaries. Residuals obtained by both robust and LSQ track fitting on experimental data are shown in Fig. 3. We can see non-Gaussian tails in the distribution of residuals for LSQ fitting and the distribution of residuals for robust fitting which is very close to Gaussian. So we can make a definite conclusion about our proposed robust technique that this approach can be used for a reliable estimation of CSC spatial resolution.

[1] This procedure is described in [10] and in addition to track fitting it includes checking on the goodness-of-fit criterion for the residual sum of squares. If a track does not satisfy this criterion, the most distant point from the fitted line is rejected and the track is refitted.

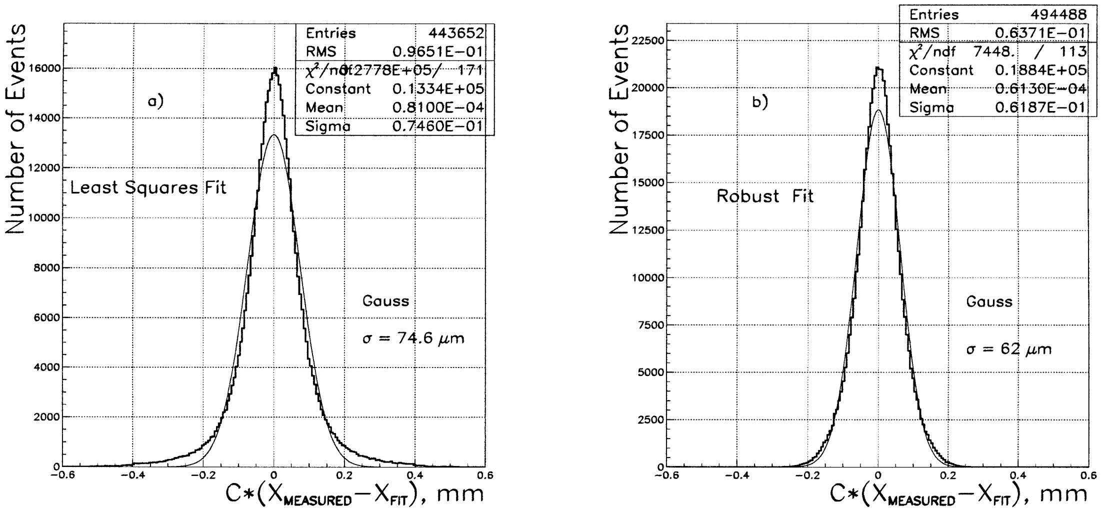

Fig. 2. Distribution of normalised residuals between measured and fitted values of x_i for LSQ and robust methods on simulated data (fitted by a Gaussian).

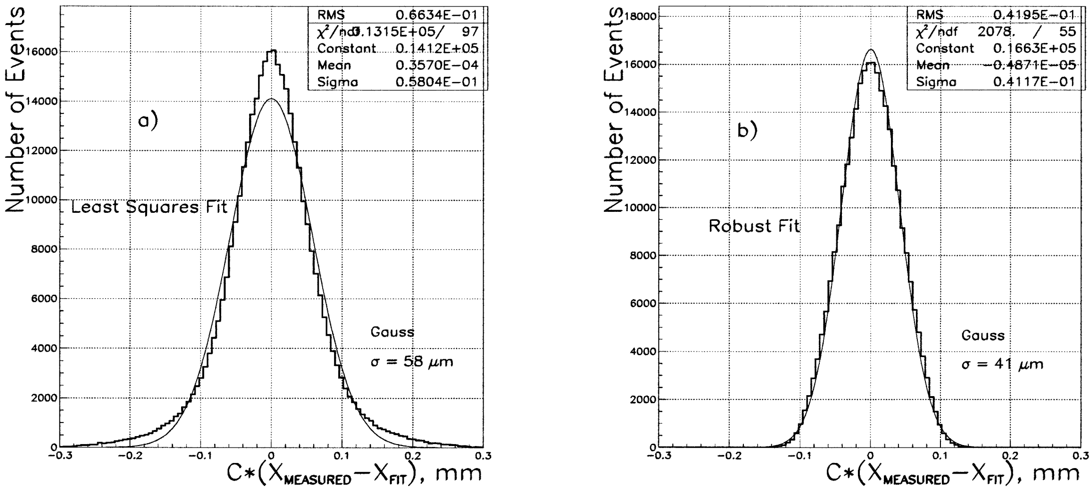

Fig. 3. Distribution of normalised residuals between measured and fitted values of x_i for LSQ and robust methods on Dubna CSC prototype experimental data (fitted by a Gaussian).

4. Conclusions

(1) A robust track fitting approach with use of sub-optimal weights is proposed instead of the conventional LSQ method for processing of muon chamber data with high background.
(2) A mathematical model of a CSC including realistic background was developed to test the track fitting procedure.
(3) Tests on simulated data show that the track parameters obtained by the robust method have up to 1.7 times better RMS than the parameters obtained by the LSQ method.

(4) Moreover, we should point out that the part of events with robust estimated parameters, which lie out of the 95% confidence interval, are corresponding to the remaining 5%. However, for LSQ fitting this part is exceeded 20%. Therefore, one can conclude that the latter part of events with unsatisfactory parameter values is not possible to apply for calculation of residuals.
(5) Long non-Gaussian tails in the distributions of LSQ residuals from experimental data lead to ambiguity in the estimation of CSC spatial resolution. In contrast, the distributions of robust residuals are very close to Gaussian.

The results obtained definitely demonstrate the advantages of using robust track fitting for a reliable estimation of muon chamber spatial resolution.

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