Validating the Simulation and Response Function in **TWIST**

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- Overview of **TWIST**'s simulation verification needs and methods.
- Downstream Materials study.
- Energy loss validation.
- Multiple scattering validation.
- Conclusion.
TWIST Analysis

• Muon decay parameters will be measured by comparing data to reconstructed simulation (presently GEANT3).
  – Simulation output will be reconstructed with the same software used to analyze data.

• We need to know that our simulation is not introducing biases that can affect the measurements of the muon decay parameters.
  – Need to test the simulation independently of decay parameters.
Verification Scheme

- Take data under special conditions.
- Run Monte Carlo with the same conditions.
- Analyze both with the same analysis software.
- Check that the effects of the changed conditions appear the same in data and MC.
- Determine how well we need the simulation to reproduce the data.
  - Usually by comparing relative Michel Fit results.
Verification Studies

- Material outside the detector
- $p_{\text{max}}$ vs angle
- $\chi^2$ and confidence level distributions
- hits per plane
- muon stopping distribution
- delta production cross-section
- energy loss
- multiple scattering
- ...and more...
Downstream Materials Study

- Test of response function's sensitivity to material outside the detector.
- Measure how Geant reacts to a plate of material placed downstream; compare with data.
- Use results to estimate sensitivity to other material, e.g. the upstream beam package.
Effect of Downstream Aluminum

Data

Geant

\[
\chi^2 / \text{ndf} = 178.3 / 90
\]

Prob = 1.863e-08

\[
\text{p0} = 0.9994 \pm 0.001511
\]

\[
\chi^2 / \text{ndf} = 111.1 / 91
\]

Prob = 0.07438

\[
\text{p0} = 0.9997 \pm 0.001423
\]
### DS Aluminum and Michel Fits

Michel Fit Results (x10^{-3}):

<table>
<thead>
<tr>
<th></th>
<th>d(rho)</th>
<th>d(eta)</th>
<th>d(xi)</th>
<th>d(delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC to MC:</td>
<td>-2.3±2.0</td>
<td>-0.15±0.11</td>
<td>3.5±2.7</td>
<td>-5.0±1.4</td>
</tr>
<tr>
<td>Data to Data:</td>
<td>-10.2±3.2</td>
<td>-0.64±0.18</td>
<td>-10.4±4.1</td>
<td>-5.3±2.3</td>
</tr>
<tr>
<td>Difference:</td>
<td>8±4</td>
<td>0.5±0.2</td>
<td>14±5</td>
<td>0.3±2.7</td>
</tr>
<tr>
<td>Std. Devs:</td>
<td>2</td>
<td>2.3</td>
<td>2.8</td>
<td>0.1</td>
</tr>
</tbody>
</table>

- DS Aluminum plate does affect fitted Michel parameters.
- Effect is **different** between data and Geant.
  - Must reduce discrepancy as much as possible.
  - Estimate how this corresponds to US material discrepancy (i.e. systematic).
DS Al and Delta Particles

• Analysis code identified more "delta" particles per event when cross-section was doubled.

• Fraction of events with identified "deltas":

<table>
<thead>
<tr>
<th></th>
<th>Std</th>
<th>DS Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geant, Standard</td>
<td>4.1%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Geant, 2xDeltas</td>
<td>6.8%</td>
<td>7.4%</td>
</tr>
<tr>
<td>Data</td>
<td>3.9%</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

• Simulation produces (roughly) the same rate of delta particles as seen in data.

• "Downstream Aluminum Discrepancy" not due to mis-simulation of delta particles.
Detector-Spanning Positrons

- Stop muons in Dense Stack.
- Measure positron track in each detector half.
- Determine Response Function.
Double Gaussian fits

- Fit function:
\[
N_1 \exp \left( - \left( \frac{x - \mu}{2\sigma_1} \right)^2 \right) + \left( \frac{N_2}{N_1} \right) \exp \left( - \left( \frac{x - \mu}{2\sigma_2} \right)^2 \right)
\]

- Fit Parameters:
\[
N_1, \left( \frac{N_2}{N_1} \right), \sigma_1, \sigma_2, \mu
\]

- Weighted Width:
\[
\frac{N_1 \sigma_1 + N_2 \sigma_2}{N_1 + N_2}
\]
Energy Loss

- **Mean:**
  - Geant: $-161.9 \pm 0.7$ keV
  - Data: $-161.4 \pm 0.5$ keV
  - Diff: $0.5 \pm 0.9$ keV

- **Weighted Width:**
  - Geant: $306 \pm 2$ keV
  - Data: $314 \pm 2$ keV
  - Diff: $8 \pm 3$ keV
Energy Loss Requirements

- Need error/bias in energy loss to be small compared to energy reconstruction requirements.

- Example: Energy loss for 35MeV/c e+ is about 130keV. \((0.130)/(35)=0.004\).
  - So for 1e-3 measurement, energy loss accuracy of a few percent (i.e. a few keV) should be sufficient (assuming uncertainties in decay parameters are linear in momentum uncertainty).
Multiple Scattering

- **Mean:**
  - Geant: $-0.47\pm0.04$ mr
  - Data: $-1.61\pm0.03$ mr
  - Diff: $1.14\pm0.05$ mr

- **Weighted Width:**
  - Geant: $16.9\pm0.6$ mr
  - Data: $16.3\pm0.5$ mr
  - Diff: $0.6\pm0.8$ mr
Multiple Scattering Requirements

- Need error/bias in multiple scattering to be small compared to angle reconstruction requirements.
- Example: Say typical scattering is 50 mrad (~2xHWHM) for track at about 0.8 rad (π/4). 
  \[(0.05/0.80)=0.07.\]
  - So for a 1e-3 measurement, multiple scattering accuracy of a few percent (i.e. around a milliradian) should be sufficient (assuming uncertainties in decay parameters are linear in angle uncertainty).
Conclusions

- Verifying that response function is correctly simulated is vital to TWIST.

- High-precision studies of GEANT3 are underway.
  - Studies must be independent of muon decay parameters.

- Simulation agrees strongly with real data.
  - Once discrepancies are understood, this knowledge will be incorporated into our simulation.