

Positron energy calibration in the *TWIST* experiment.

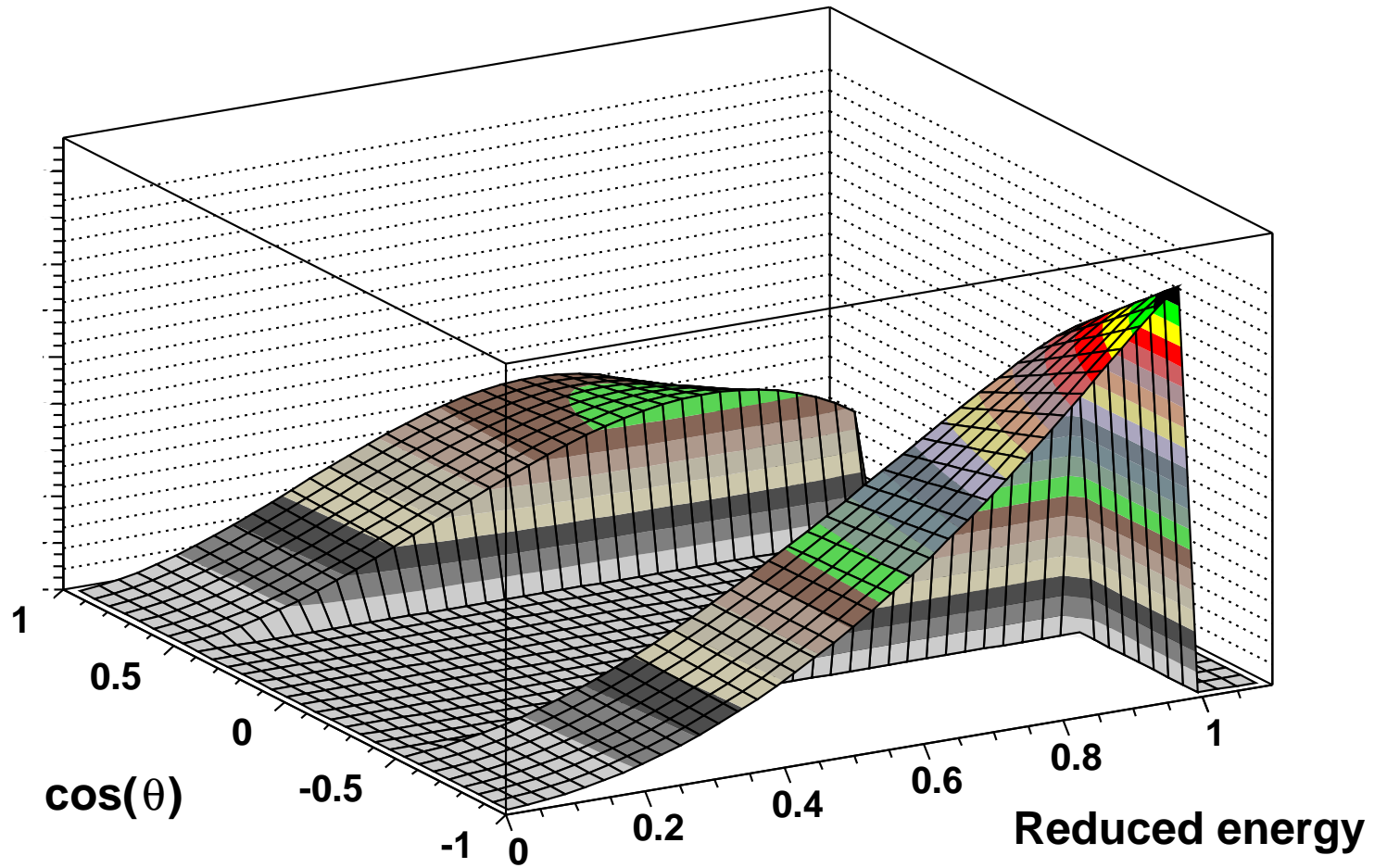
A. Gaponenko

on behalf of the *TWIST* collaboration.

- Energy scale in *TWIST*
- Sensitivities of Michel parameters
- Calibration method
- Implementation & results
- Conclusion

Michel spectrum.

The middle region can not
be seen by *TWIST*.



Energy scale in *TWIST*

- Energy scale β : defines a distortion of the reconstructed spectrum in the form

$$E \longrightarrow (1 + \beta) E$$

- Upstream and downstream parts of the detector will be calibrated separately $\Rightarrow \beta_{up}$ and β_{dn} .
- Systematic error on a Michel parameter due to energy scale:

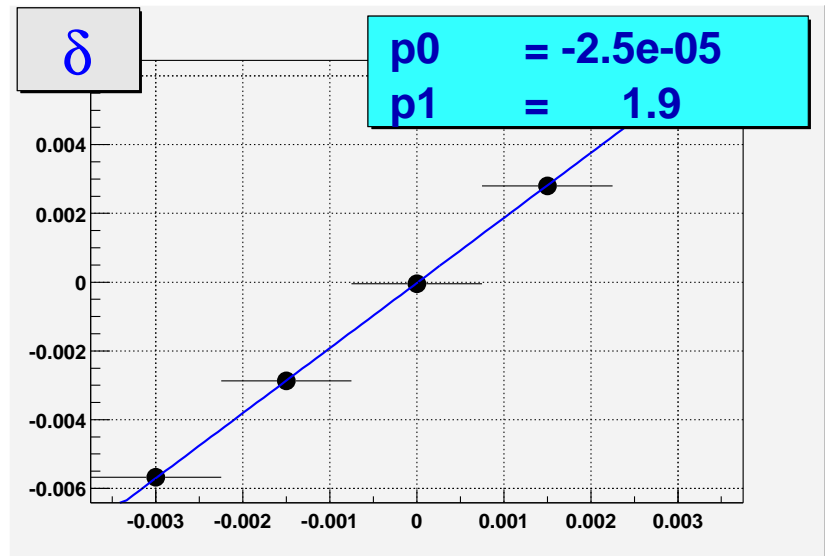
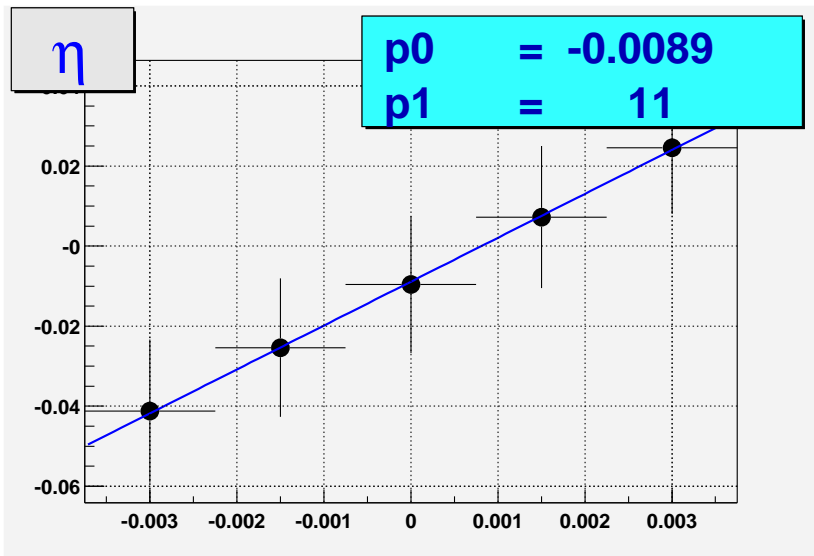
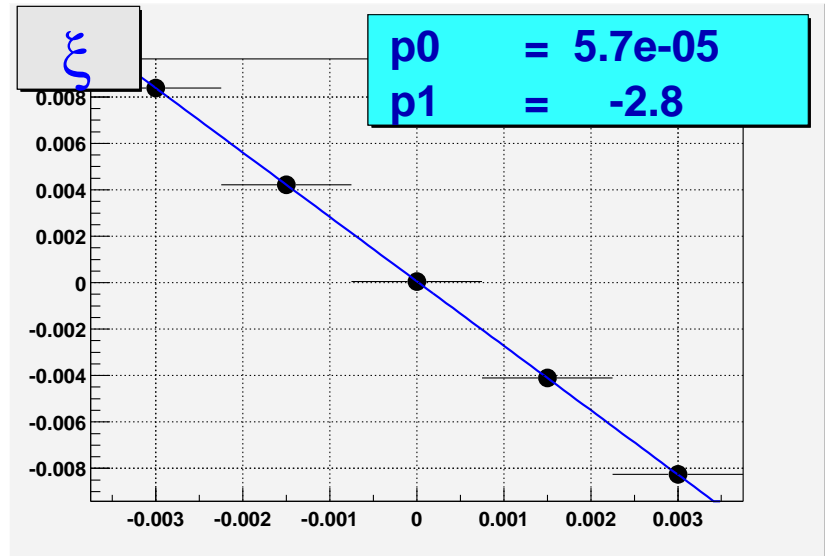
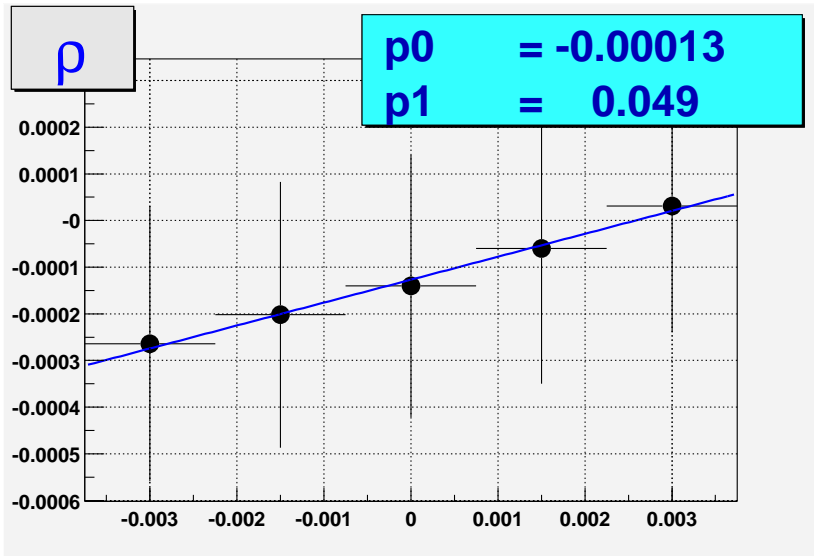
$$\Delta\rho = \frac{\partial\rho}{\partial\beta} \Delta\beta$$

- ▷ Sensitivity $\partial\rho/\partial\beta = ?$
- ▷ How well can we measure β ?

Sensitivity of Michel parameters to energy scale

- **Method:** generate a MC Michel spectrum. Fit it with a distorted function fixing different β_{up} and β_{dn}
→ $\partial\rho/\partial\beta_{up}$, $\partial\rho/\partial\beta_{dn}$, $\partial\eta/\partial\beta_{up}$, $\partial\eta/\partial\beta_{dn}$...
- **Realization:**
 - ▷ 10^9 decays of 100% polarized muons simulated.
 - ▷ First order radiative correction included in the generation and in the fitting.
 - ▷ Log likelihood fit for 4 Michel parameters and normalization in the upstream and downstream regions simultaneously. Fit region is
 $[0.4 \leq x \leq 0.97] \times [0.5 \leq |\cos(\theta)| \leq 0.98]$
 - ▷ Distortion: 5 steps in β_{up} between $-30 \cdot 10^{-4}$ and $+30 \cdot 10^{-4}$.
Same for the β_{dn} .

Deviations in Michel parameters vs β_{up}



Sensitivities—the result

For the fit region

$$[0.4 \leq x \leq 0.97] \times [0.5 \leq |\cos(\theta)| \leq 0.98]:$$

	$\frac{\partial}{\partial \beta_{up}}$	$\frac{\partial}{\partial \beta_{dn}}$
ρ	0.05	1.
η	11.	-1.9
ξ	-2.8	-0.9
δ	1.9	-1.

The energy calibration method

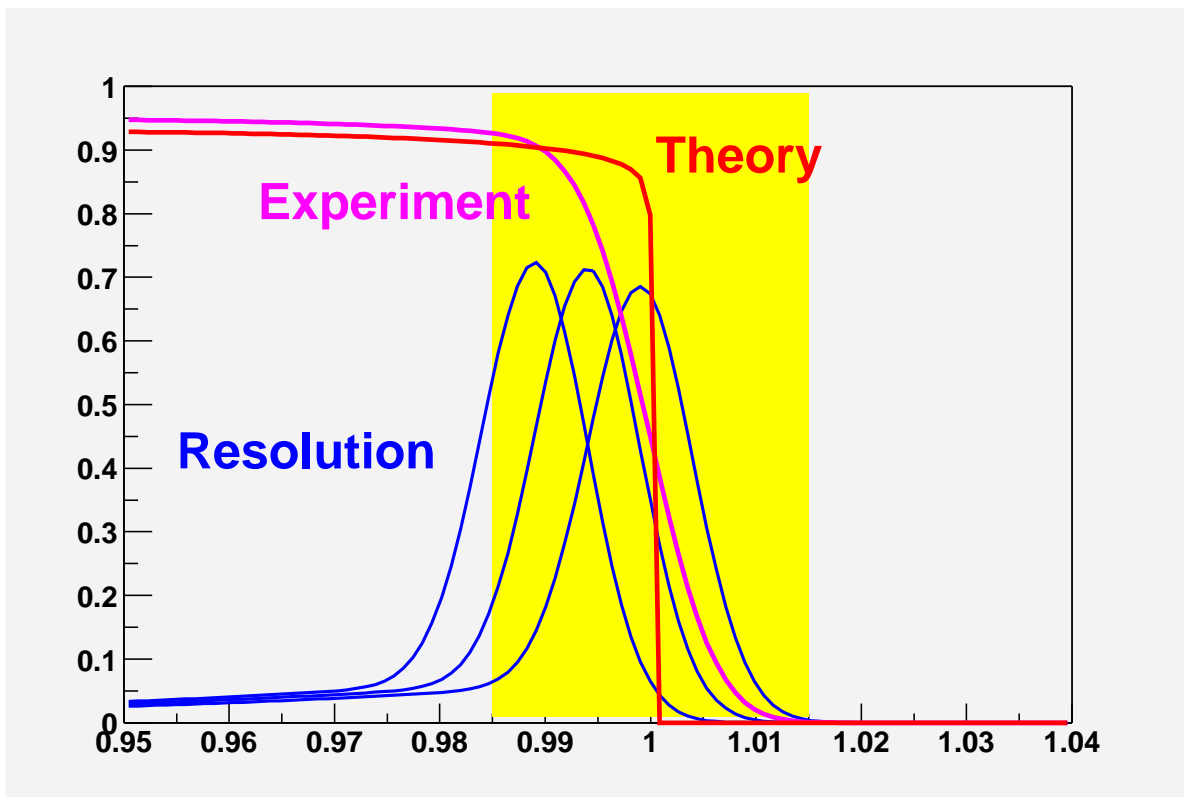
- The **sharp edge** of the Michel spectrum at the upper kinematic limit provides a **natural calibration point**.
- Positron **energy loss** affects the position of the reconstructed spectrum edge along with the **energy scale**.
- The **planar detector geometry** is essential. For that geometry the following equation for the edge of reconstructed spectrum is rigorously valid:

$$E_{edge} = (1 + \beta) \left(E_{max} - \frac{\alpha}{|\cos(\theta)|} \right)$$

- Having determined E_{edge} for different angles we can find the constants α and β by fitting the equation.

Fitting the end point—1

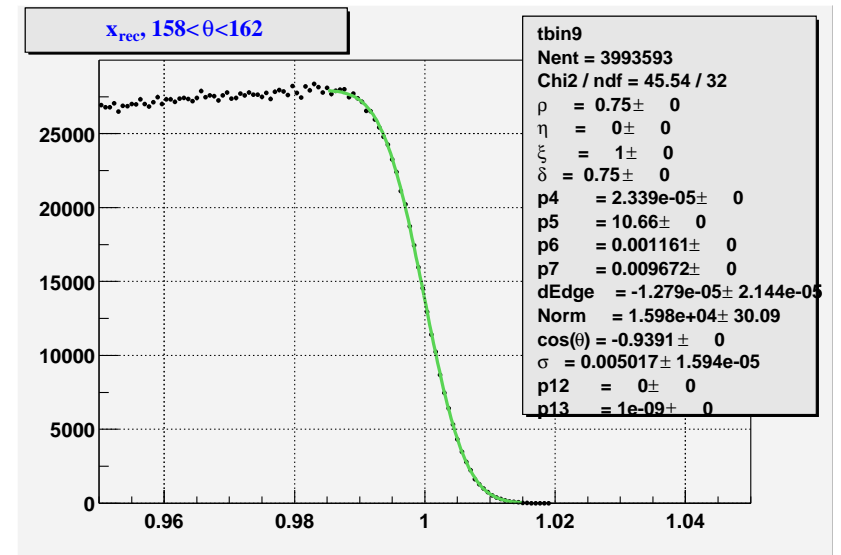
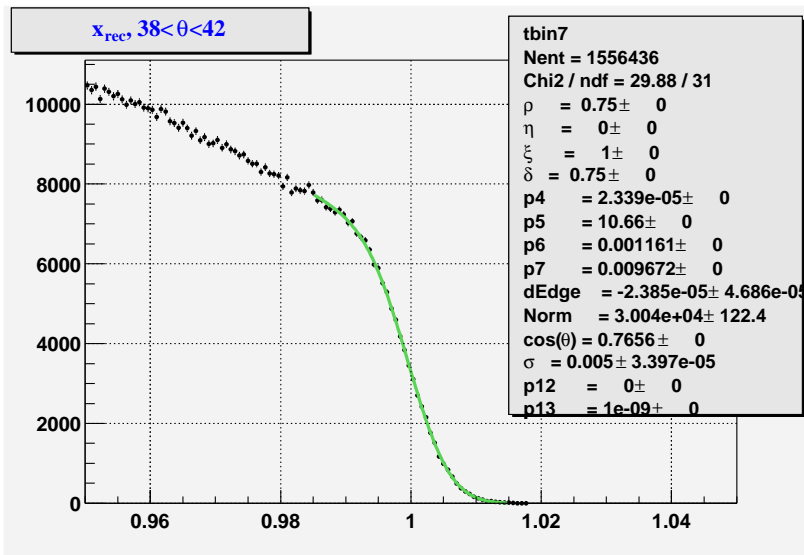
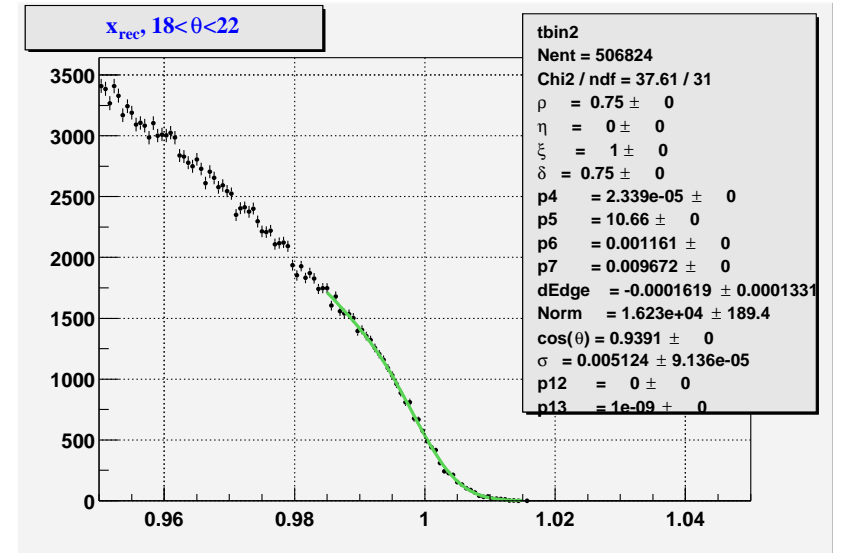
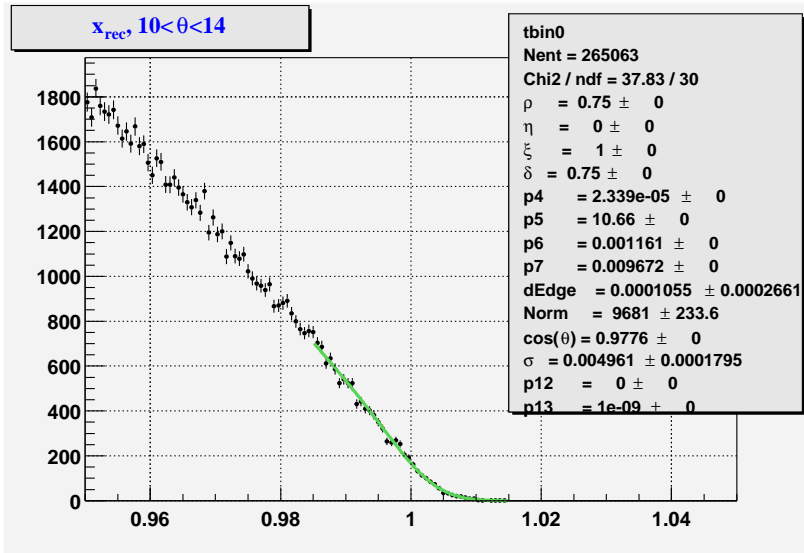
- **Shape** of the reconstructed spectrum edge is mainly **defined by the resolution** function.
- **Approximation: theoretical** Michel spectrum **convoluted with a Gaussian**.
 - ▷ No analytical expression available.



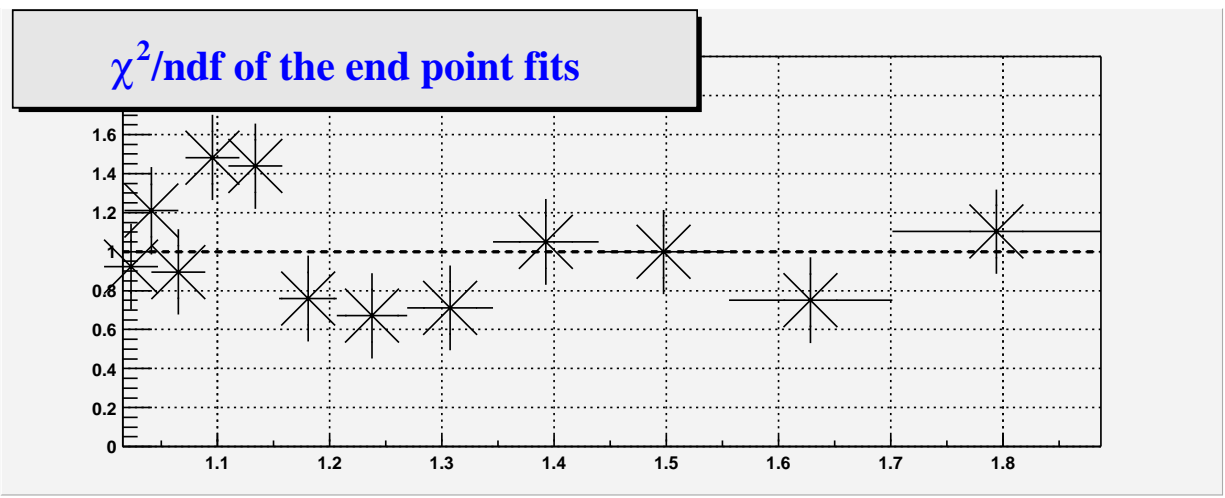
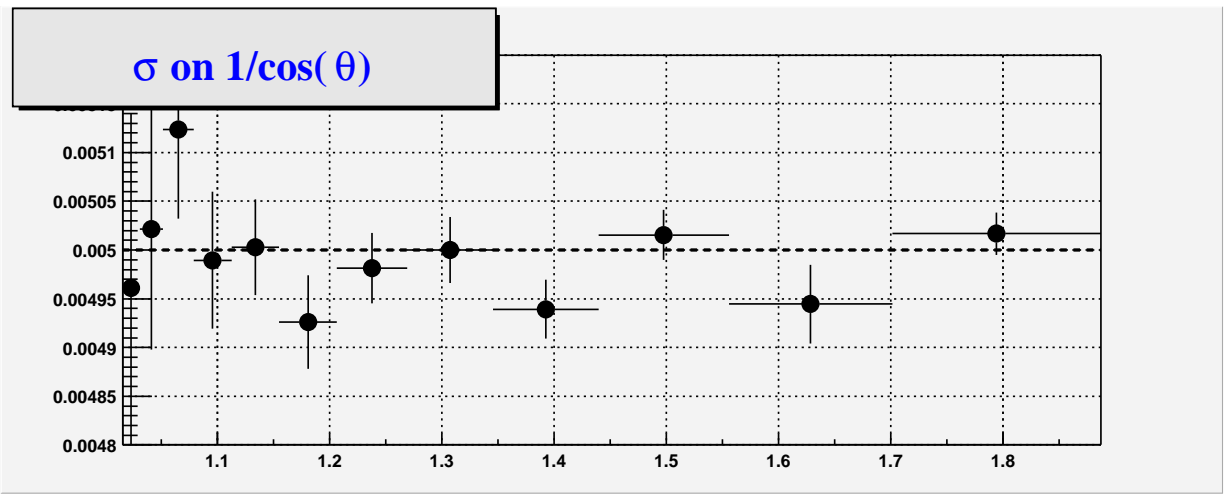
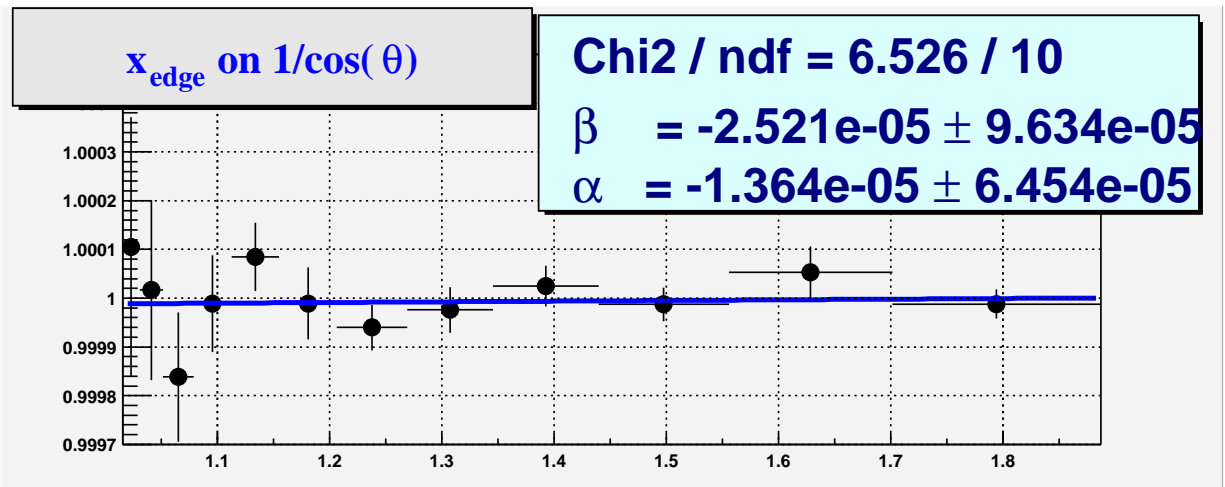
Fitting the end point—2

- Implementation:
 - ▷ Log likelihood fits with 3 free parameters: E_{edge} , σ and a normalization.
 - ▷ $\cos(\theta)$ for an angular bin $[\theta_1, \theta_2]$ is fixed at the mean value $(\cos(\theta_1) + \cos(\theta_2))/2$
- Testing:
 - ▷ Data sample corresponding to 10^9 total decays of 100% polarized muons.
 - ▷ Generated energy smeared with a Gaussian, $\sigma = 0.005$ (0.26 MeV)
 - ▷ Energy distributions in 4° angular bins between 10° and 58° and symmetrically upstream were produced and fitted with the convolution.

End point fit examples



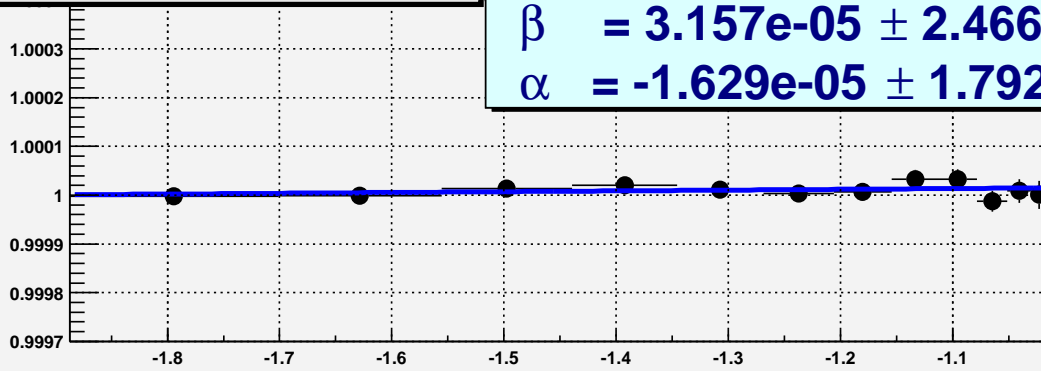
Downstream energy calibration



Horizontal scale is $1/\cos(\theta)$.

Upstream energy calibration

x_{edge} on $1/\cos(\theta)$

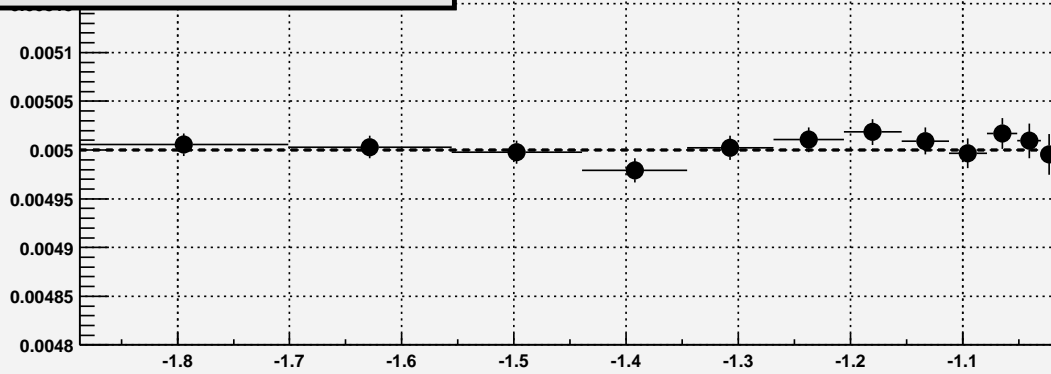


Chi2 / ndf = 5.024 / 10

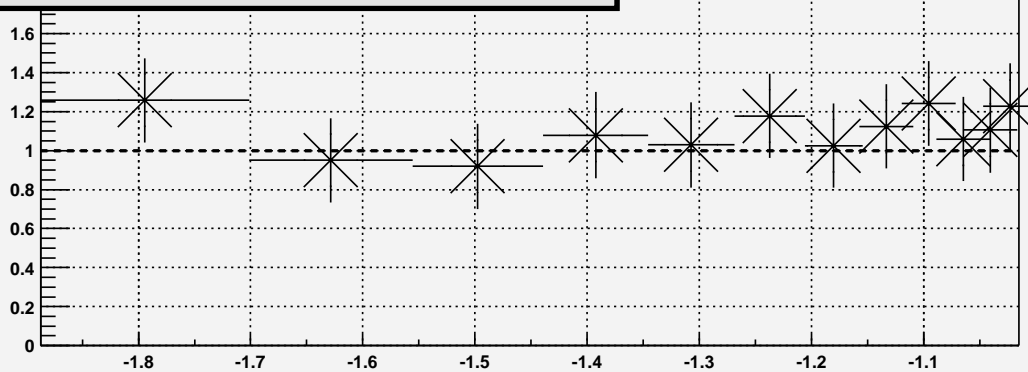
$\beta = 3.157\text{e-}05 \pm 2.466\text{e-}05$

$\alpha = -1.629\text{e-}05 \pm 1.792\text{e-}05$

σ on $1/\cos(\theta)$



χ^2/ndf of the end point fits



Horizontal scale is $1/\cos(\theta)$.

Energy calibration summary

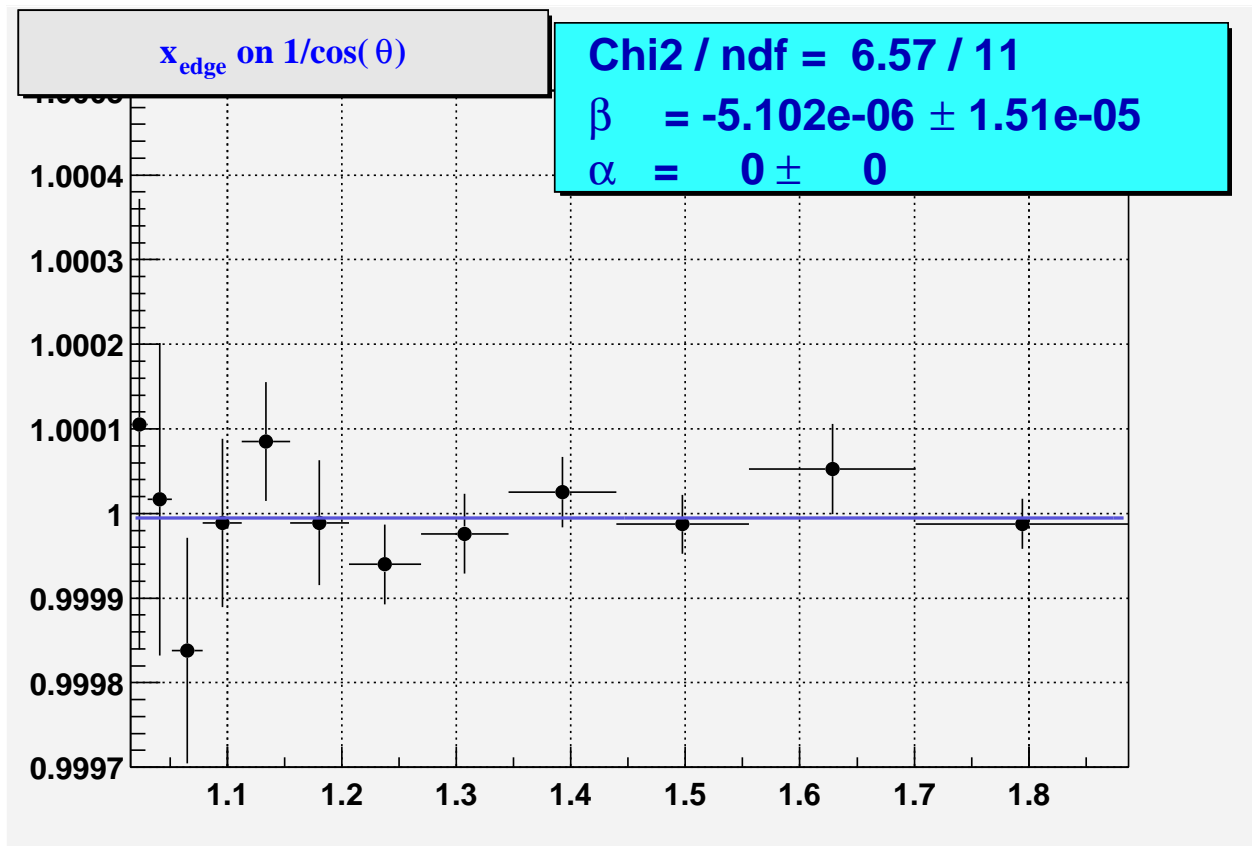
- Precision of the upstream energy scale fit β_{up} is $0.25 \cdot 10^{-4}$, for β_{dn} it's $0.96 \cdot 10^{-4}$. The numbers were obtained assuming that the polarization of the decaying muons is perfect.
- Actual data set in addition to the surface muons contains also cloud muons. Not doing the RF cut brings in the analysis 5% of muons with the opposite polarization.
- For that subsample
$$\beta_{dn} = \sqrt{\frac{1}{0.05}} \times 0.25 \cdot 10^{-4} \approx 1.1 \cdot 10^{-4}.$$
- Intrinsic to the data set energy calibration which combines the surface and the cloud muons gives precision for $\beta_{up} \pm 0.25 \cdot 10^{-4}$ and for $\beta_{dn} \pm 0.72 \cdot 10^{-4}$.

Conclusion

- Sharp edge of the Michel spectrum at the upper kinematic limit provides a natural calibration point.
- Planar detector design makes possible an exact account of the edge shift due to the positron energy loss.
- A calibration can be obtained which is intrinsic to the physics data sample. Systematic errors due to the e-scale uncertainty with that calibration are (in 10^{-4} units):
$$\Delta\rho = \pm 0.7 \qquad \Delta\eta = \pm 3.$$
$$\Delta\xi = \pm 1. \qquad \Delta\delta = \pm 0.9$$
- In addition, a better calibration can be obtained by taking data using a beam with low muon polarization.

Additional slides.

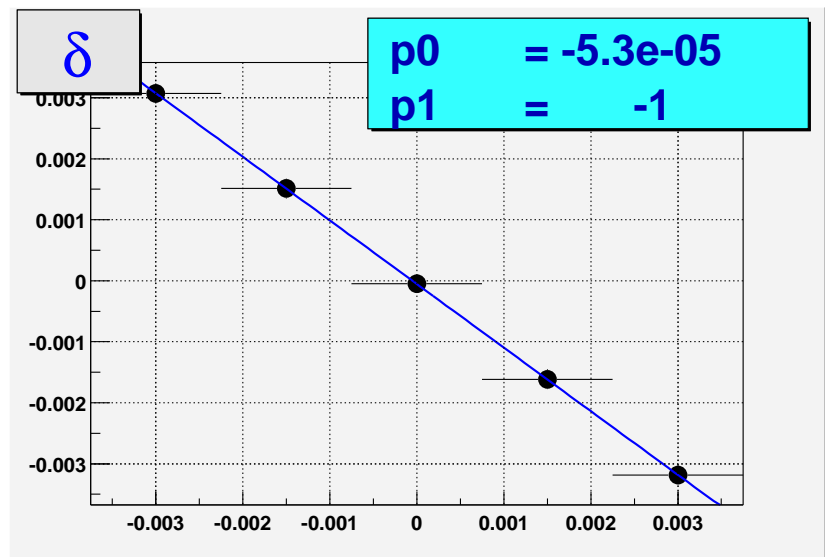
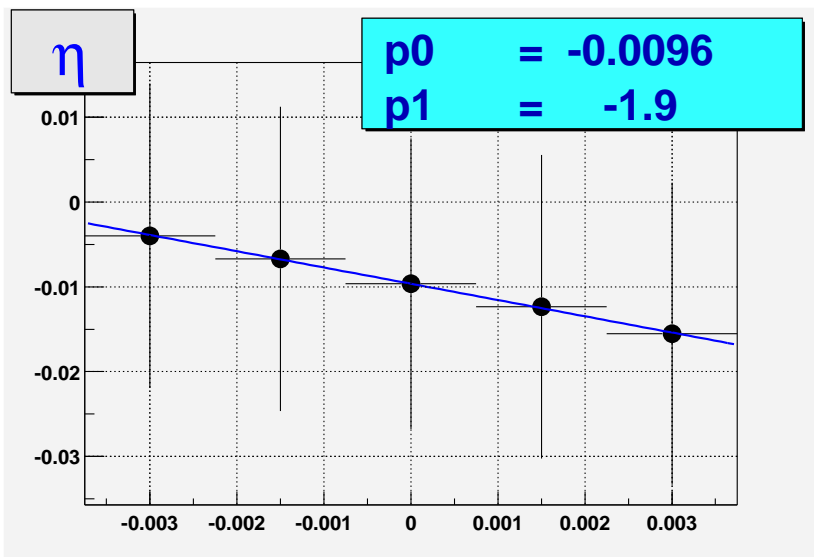
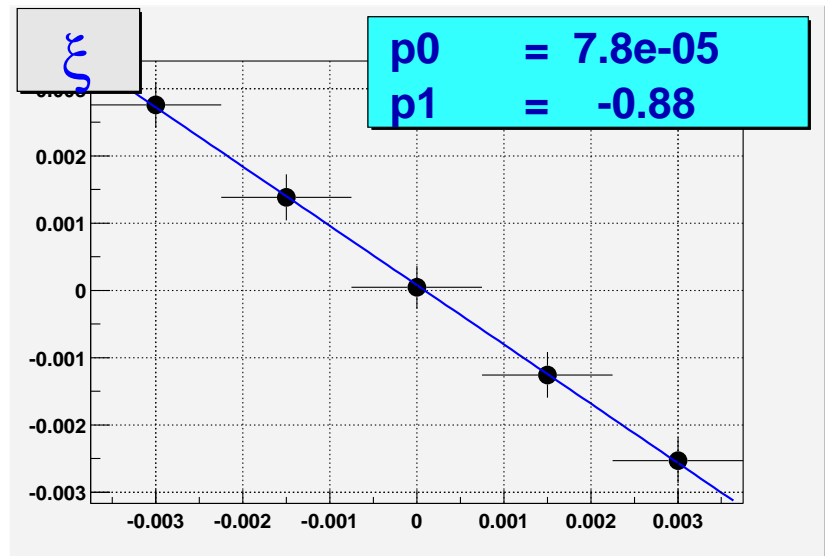
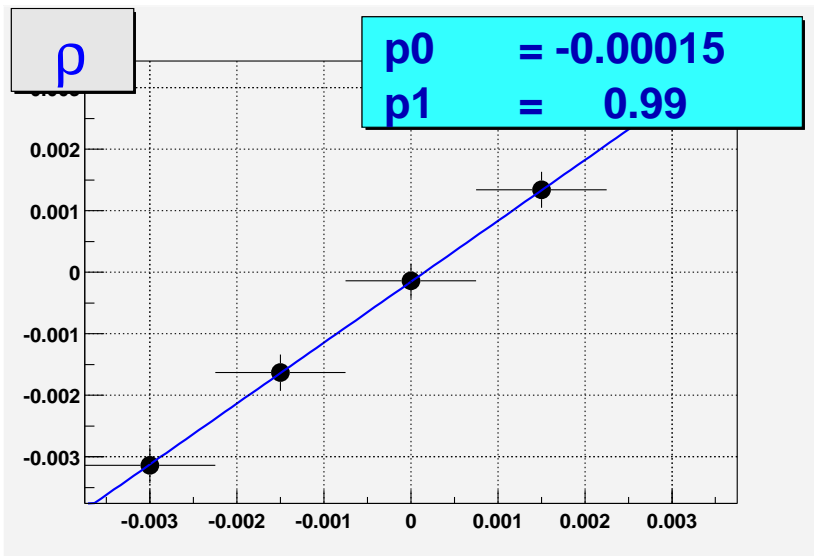
Downstream calibration with fixed α



Amount of material is fixed: $\alpha_{\text{up}} + \alpha_{\text{down}} = \text{const}$

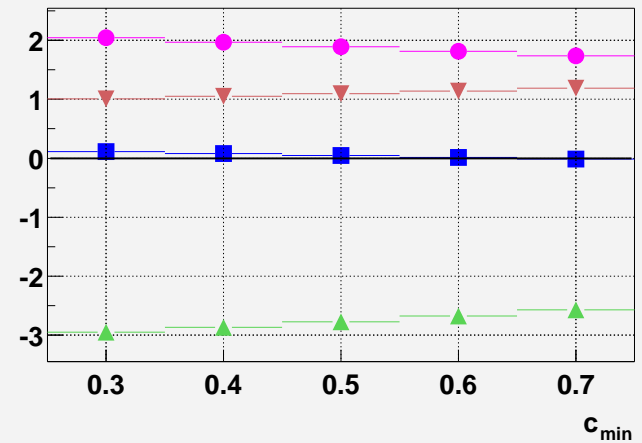
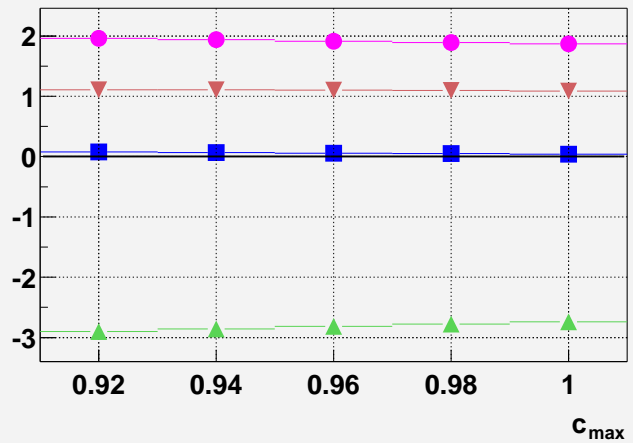
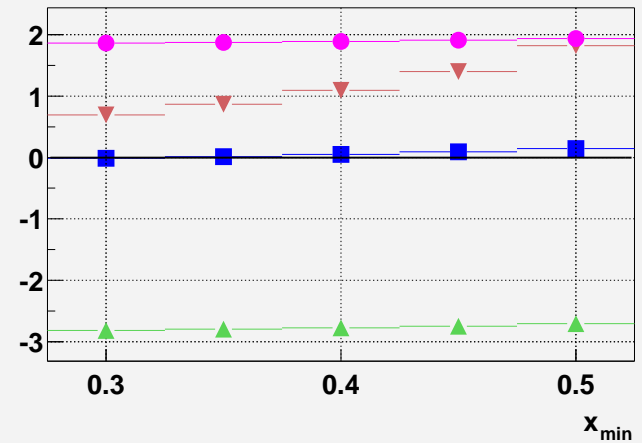
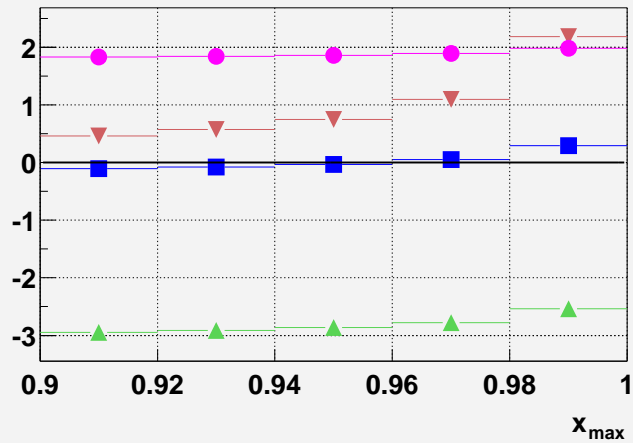
- Make a measurement of the **sum** in a **dedicated** run with **low** muon **polarization**.
- For physics data:
 - ▷ Fit $\alpha_{\text{up}}, \beta_{\text{up}}$.
 - ▷ Fix α_{down} at a known value and do a **single parameter fit** for $\beta_{\text{dn}} \Rightarrow$ much smaller error.

Deviations in Michel parameters vs β_{dn}



Dependence of sensitivities to β_{up} on fit region

■ $d\rho/d\beta$
 ▼ $0.1 \times d\eta/d\beta$
 ▲ $d\xi/d\beta$
 ● $d\delta/d\beta$



Dependence of sensitivities to β_{dn} on fit region

■ $d\rho/d\beta$ ▾ $0.1 \times d\eta/d\beta$ ▲ $d\xi/d\beta$ ● $d\delta/d\beta$

