Positron energy calibration in the $\textit{TWIST}$ experiment.

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- Energy scale in $\textit{TWIST}$
- Sensitivities of Michel parameters
- Calibration method
- Implementation & results
- Conclusion
Michel spectrum.

The middle region can not be seen by TWIST.
**Energy scale in TWIST**

- **Energy scale** $\beta$: defines a distortion of the reconstructed spectrum in the form
  \[ E \rightarrow (1 + \beta) E \]

- **Upstream** and **downstream** parts of the detector will be calibrated separately $\Rightarrow \beta_{up}$ and $\beta_{dn}$.

- **Systematic error** on a Michel parameter due to energy scale:
  \[ \Delta \rho = \frac{\partial \rho}{\partial \beta} \Delta \beta \]

  ▶ **Sensitivity** $\partial \rho / \partial \beta = ?$

  ▶ **How well** can we measure $\beta$?
Sensitivity of Michel parameters to energy scale

- **Method:** generate a MC Michel spectrum. Fit it with a distorted function fixing different $\beta_{up}$ and $\beta_{dn}$
  \[ \frac{\partial \rho}{\partial \beta_{up}}, \frac{\partial \rho}{\partial \beta_{dn}}, \frac{\partial \eta}{\partial \beta_{up}}, \frac{\partial \eta}{\partial \beta_{dn}} \ldots \]

- **Realization:**
  - $10^9$ decays of 100% polarized muons simulated.
  - First order radiative correction included in the generation and in the fitting.
  - Log likelihood fit for 4 Michel parameters and normalization in the upstream and downstream regions simultaneously. Fit region is
    \[ [0.4 \leq x \leq 0.97] \times [0.5 \leq |\cos(\theta)| \leq 0.98] \]
  - Distortion: 5 steps in $\beta_{up}$ between $-30 \cdot 10^{-4}$ and $+30 \cdot 10^{-4}$.
    Same for the $\beta_{dn}$. 
Deviations in Michel parameters vs $\beta_{up}$

- $\rho$
  - $p_0 = -0.00013$
  - $p_1 = 0.049$

- $\xi$
  - $p_0 = 5.7e-05$
  - $p_1 = -2.8$

- $\eta$
  - $p_0 = -0.0089$
  - $p_1 = 11$

- $\delta$
  - $p_0 = -2.5e-05$
  - $p_1 = 1.9$
Sensitivities—the result

For the fit region

\[ 0.4 \leq x \leq 0.97 \times 0.5 \leq |\cos(\theta)| \leq 0.98 \]:

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \frac{\partial}{\partial \beta_{up}} )</th>
<th>( \frac{\partial}{\partial \beta_{dn}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.05</td>
<td>1.</td>
</tr>
<tr>
<td>( \eta )</td>
<td>11.</td>
<td>-1.9</td>
</tr>
<tr>
<td>( \xi )</td>
<td>-2.8</td>
<td>-0.9</td>
</tr>
<tr>
<td>( \delta )</td>
<td>1.9</td>
<td>-1.</td>
</tr>
</tbody>
</table>
The energy calibration method

- The sharp edge of the Michel spectrum at the upper kinematic limit provides a natural calibration point.

- Positron energy loss affects the position of the reconstructed spectrum edge along with the energy scale.

- The planar detector geometry is essential. For that geometry the following equation for the edge of reconstructed spectrum is rigorously valid:

\[ E_{edge} = (1 + \beta) \left( E_{\text{max}} - \frac{\alpha}{|\cos(\theta)|} \right) \]

- Having determined \( E_{edge} \) for different angles we can find the constants \( \alpha \) and \( \beta \) by fitting the equation.
Fitting the end point—1

- **Shape** of the reconstructed spectrum edge is mainly defined by the resolution function.

- **Approximation:** theoretical Michel spectrum convoluted with a Gaussian.
  - No analytical expression available.
Fitting the end point—2

- **Implementation:**
  - Log likelihood fits with 3 free parameters: $E_{\text{edge}}$, $\sigma$ and a normalization.
  - $\cos(\theta)$ for an angular bin $[\theta_1, \theta_2]$ is fixed at the mean value $(\cos(\theta_1) + \cos(\theta_2))/2$

- **Testing:**
  - Data sample corresponding to $10^9$ total decays of 100% polarized muons.
  - Generated energy smeared with a Gaussian, $\sigma = 0.005$ (0.26 MeV)
  - Energy distributions in 4° angular bins between 10° and 58° and symmetrically upstream were produced and fitted with the convolution.
End point fit examples

![Graphs showing fitting results for different data bins with chi-squared and degrees of freedom values.](image-url)
Downstream energy calibration

\[ x_{\text{edge}} \text{ on } 1/\cos(\theta) \]

\[ \text{Chi2 / ndf } = 6.526 / 10 \]

\[ \beta = -2.521 \times 10^{-5} \pm 9.634 \times 10^{-5} \]

\[ \alpha = -1.364 \times 10^{-5} \pm 6.454 \times 10^{-5} \]

\[ \sigma \text{ on } 1/\cos(\theta) \]

\[ \chi^2/\text{ndf of the end point fits} \]

Horizontal scale is \( 1/\cos(\theta) \).
Upstream energy calibration

\[ x_{\text{edge}} \text{ on } 1/\cos(\theta) \]

\[ \begin{align*}
\beta &= 3.157 \times 10^{-5} \pm 2.466 \times 10^{-5} \\
\alpha &= -1.629 \times 10^{-5} \pm 1.792 \times 10^{-5}
\end{align*} \]

\[ \text{Chi2 / ndf = 5.024 / 10} \]

\[ \sigma \text{ on } 1/\cos(\theta) \]

\[ \chi^2/\text{ndf of the end point fits} \]

Horizontal scale is \(1/\cos(\theta)\).
Energy calibration summary

- **Precision** of the upstream energy scale fit $\beta_{up}$
is $0.25 \cdot 10^{-4}$, for $\beta_{dn}$ it’s $0.96 \cdot 10^{-4}$. The
numbers were obtained assuming that the polarization of the decaying muons is perfect.

- **Actual data set** in addition to the surface muons contains also cloud muons. Not doing
the RF cut brings in the analysis 5% of muons with the opposite polarization.

- **For that subsample**

$$\beta_{dn} = \sqrt{\frac{1}{0.05}} \times 0.25 \cdot 10^{-4} \approx 1.1 \cdot 10^{-4}.$$ 

- **Intrinsic** to the data set energy calibration which combines the surface and the cloud
muons gives precision for $\beta_{up} \pm 0.25 \cdot 10^{-4}$ and for $\beta_{dn} \pm 0.72 \cdot 10^{-4}$. 
**Conclusion**

- **Sharp edge** of the Michel spectrum at the upper kinematic limit provides a natural calibration point.

- **Planar detector design** makes possible an **exact** account of the edge shift due to the positron energy loss.

- **A calibration** can be obtained which is **intrinsic** to the physics data sample. Systematic errors due to the e-scale uncertainty with that calibration are (in $10^{-4}$ units):
  \[
  \Delta \rho = \pm 0.7 \quad \Delta \eta = \pm 3.
  \]
  \[
  \Delta \xi = \pm 1. \quad \Delta \delta = \pm 0.9
  \]

- **In addition**, a **better** calibration can be obtained by taking data using a beam with low muon polarization.
Additional slides.
Downstream calibration with fixed $\alpha$

Amount of material is fixed: $\alpha_{up} + \alpha_{down} = \text{const}$

- Make a measurement of the sum in a dedicated run with low muon polarization.

- For physics data:
  - Fit $\alpha_{up}$, $\beta_{up}$.
  - Fix $\alpha_{down}$ at a known value and do a single parameter fit for $\beta_{dn}$ $\Rightarrow$ much smaller error.
Deviations in Michel parameters vs $\beta_{dn}$

\[ \rho \]
- $p_0 = -0.00015$
- $p_1 = 0.99$

\[ \xi \]
- $p_0 = 7.8\times10^{-5}$
- $p_1 = -0.88$

\[ \eta \]
- $p_0 = -0.0096$
- $p_1 = -1.9$

\[ \delta \]
- $p_0 = -5.3\times10^{-5}$
- $p_1 = -1$
Dependence of sensitivities to $\beta_{up}$ on fit region

- $d\rho/d\beta$
- $0.1 \times d\eta/d\beta$
- $d\xi/d\beta$
- $d\delta/d\beta$

Graphs showing the dependence of sensitivities on fit regions.
Dependence of sensitivities to $\beta_{dn}$ on fit region

- $\frac{d\rho}{d\beta}$
- $0.1 \times \frac{d\eta}{d\beta}$
- $\frac{d\xi}{d\beta}$
- $\frac{d\delta}{d\beta}$