Extraction of the Michel parameter, $\rho$, of normal muon decay

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Reconstruction code output

Physics variables

- Decay $e^+$ momentum, $|\vec{p}|$
- Decay $e^+$ cos $\theta$

Variables for cuts

- Particle time of flight
- $\mu^+$ stopping location

Fitter results

Histogram in bins of $|\vec{p}|$ and cos $\theta$
$\cos \theta$ vs. momentum

17% of standard data set
Fitting $\rho$

The energy and angular distribution of the Michel positron is given by:

$$\frac{d^2\Gamma}{dxd(\cos \theta)} \propto F_{IS}(x, \rho, \eta) + P_\mu \cos \theta F_{AS}(x, \xi, \delta)$$

Where

$$F_{IS} = x^2(1 - x) + \frac{2}{9}\rho x^2(4x - 3) + \eta\frac{m_e}{E_{e\text{max}}} x(1 - x)$$

$$F_{AS} = \frac{1}{3}\xi x[1 - x + \frac{2}{3}\delta(4x - 3)]$$
Fitting $\rho$

Integrating over $\cos \theta$ or computing Forward + Backward yields:

\[
\frac{d\Gamma}{dx} \propto F_{IS}(x, \rho, \eta) + \epsilon P_{\mu} F_{AS}(x, \xi, \delta)
\]

Where $\epsilon$ represents an unknown reconstruction asymmetry.

Note that the effect of $F_{AS}(x, \xi, \delta)$ can be reduced by minimizing $\epsilon$ and/or $P_{\mu}$. 
Fitting $\Delta \rho$ and $\Delta \eta$

\[
\begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Data} = \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std}
\]

\[+
\frac{\partial}{\partial \rho} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} \Delta \rho + \frac{\partial}{\partial \eta} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} \Delta \eta + \frac{\partial}{\partial \xi} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} \Delta \xi + \frac{\partial}{\partial \delta} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} \Delta \delta
\]

Where

\[
\frac{\partial}{\partial \rho} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} = k \frac{\partial}{\partial \rho} \begin{bmatrix}
F_{IS}(x, \rho, \eta)
\end{bmatrix}_{Std}
\]

\[
\frac{\partial}{\partial \eta} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} = k \frac{\partial}{\partial \eta} \begin{bmatrix}
F_{IS}(x, \rho, \eta)
\end{bmatrix}_{Std}
\]

\[
\frac{\partial}{\partial \xi} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} = \epsilon \mu k \frac{\partial}{\partial \xi} \begin{bmatrix}
F_{AS}(x, \xi, \delta)
\end{bmatrix}_{Std}
\]

\[
\frac{\partial}{\partial \delta} \begin{bmatrix}
\frac{d\Gamma}{dx}
\end{bmatrix}_{Std} = \epsilon \mu k \frac{\partial}{\partial \delta} \begin{bmatrix}
F_{AS}(x, \xi, \delta)
\end{bmatrix}_{Std}
\]
Michel distribution in $x$ (reduced energy)

Standard Model (top), $\Delta \rho$ (bottom, red), $\Delta \eta$ (bottom, blue)
Fitting $\rho$ with a blind analysis
(Ignoring $\cos \theta$ term)

Consider

$$\rho = \rho_{Std} + \Delta \rho \quad \rightarrow \quad \rho = \rho_o + \Delta \rho'$$

$$\eta = \eta_{Std} + \Delta \eta \quad \rightarrow \quad \eta = \eta_o + \Delta \eta'$$

Fitting $\Delta \rho'$ and $\Delta \eta'$

$$\left[ \frac{d\Gamma}{dx} \right]_{Data} = \left[ \frac{d\Gamma}{dx} \right]_{\rho_o, \eta_o} + \frac{\partial}{\partial \rho} \left[ \frac{d\Gamma}{dx} \right]_{\rho_o, \eta_o} \Delta \rho' + \frac{\partial}{\partial \eta} \left[ \frac{d\Gamma}{dx} \right]_{\rho_o, \eta_o} \Delta \eta'$$

Where

$$\frac{\partial}{\partial \rho} \left[ \frac{d\Gamma}{dx} \right]_{\rho_o, \eta_o} = k \frac{\partial}{\partial \rho} \left[ FIS(x, \rho, \eta) \right]_{\rho_o, \eta_o}$$

$$\frac{\partial}{\partial \eta} \left[ \frac{d\Gamma}{dx} \right]_{\rho_o, \eta_o} = k \frac{\partial}{\partial \eta} \left[ FIS(x, \rho, \eta) \right]_{\rho_o, \eta_o}$$
Systematic errors

Reconstruction efficiency as a function of $\cos \theta$, $|\vec{p}|$
and . . .

- DC HV (track fitting)
- PC HV (pattern recognition)
- Chamber gas density
- $\mu^+$ rate
- Beam $e^+$ rate
- Upstream/downstream asymmetry*

Philosophy: exaggerate a possible source of error to put limits on its effect.
Surface (17% of standard data set), DC HV 1850V (19%), PC HV 1950V (14%)
Surface (17% of standard data set), Cloud (6%)
Summary

Data has been taken for a measurement of $\rho$ with a statistical precision of a part in $10^3$.

Data with comparable statistics has been taken to study a variety of possible systematic effects.

*TWIST* is setting limits on systematic effects by using data to data comparisons.

*TWIST* is employing an effective blind analysis technique.