Final Results for the Muon Decay Parameters from TWIST

Glen Marshall, TRIUMF (for the TWIST Collaboration)
Physics of Fundamental Symmetries and Interactions, PSI, Oct 2010
Muon decay parameters $\rho, \eta, \mathcal{P}_\mu \xi, \delta$

muon differential decay rate vs. energy and angle:

$$\frac{d^2\Gamma}{dx \, d\cos \theta} = \frac{1}{4} m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot \{\mathcal{F}_{IS}(x, \rho, \eta) + P_\mu \cos \theta \cdot \mathcal{F}_{AS}(x, \xi, \delta)\} + R.C.$$  

where

$$\mathcal{F}_{IS}(x, \rho, \eta) = x(1 - x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1 - x)$$

$$\mathcal{F}_{AS}(x, \xi, \delta) = \frac{1}{3} \xi \sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3} \delta \left\{ 4x - 3 + \left( \sqrt{1 - x_0^2} - 1 \right) \right\} \right]$$

and $W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$, $x = \frac{E_e}{W_{\mu e}}$, $x_0 = \frac{m_e}{W_{\mu e}}$.

- Full $O(\alpha)$ radiative corrections with exact electron mass dependence.
- Leading and next-to-leading logarithmic terms of $O(\alpha^2 L^2)$ and $O(\alpha^2 L)$, $L = \ln((m_\mu/m_e)^2)$
- Leading logarithmic terms of $O(\alpha^3 L^3)$.
- Ignores $O(\alpha^2 L^0)$ (2007).

(\theta for TWIST is ($\pi - \theta$) in decay parameter definition)

Matrix elements

\[ M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} g_{e\mu}^\gamma \langle \bar{e}_e | \Gamma^{\gamma} | (\nu_e)_m \rangle \langle (\bar{\nu}_\mu)_m | \Gamma^{\gamma} | \mu_\mu \rangle \]

- Most general local, Lorentz-invariant, lepton-number conserving interaction determined by 19 real parameters.
- Includes scalar, vector, and tensor (\(\Gamma^S, \Gamma^V, \Gamma^T\)) interactions among left- and right-handed \(\mu, e\) (SM: \(g_{LL}^V = 1, \text{all others zero}\)).
- Decay parameters are bilinear combinations of \(g_{e\mu}^\gamma\)
- Probability for decay of \(\mu\)-handed muon to \(e\)-handed electron:

\[ Q_{\varepsilon\mu} = \frac{1}{4} |g_{e\mu}^S|^2 + |g_{e\mu}^V|^2 + 3(1 - \delta_{e\mu}) |g_{e\mu}^T|^2 \]

- For example, RH coupling in \(\mu\) decay in terms of decay parameters:

\[ Q_{\mu_R} = \frac{1}{2} \left[ 1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right] \]

Pre-TWIST decay parameters

From the Review of Particle Physics (SM values)

- $\rho = 0.7518 \pm 0.0026$ (S.E. Derenzo, Phys. Rev. 184 (1969) 1854) (0.75)
- $\delta = 0.7486 \pm 0.0026 \pm 0.0028$ (B. Balke et al., Phys. Rev. D37 (1988) 587) (0.75)
- $\mathcal{P}_\mu \xi = 1.0027 \pm 0.0079 \pm 0.0030$ (I. Beltrami et al., Phys. Lett. B194 (1987) 326) (1.00)
- $\mathcal{P}_\mu (\xi \delta/\rho) > 0.99682$ (90%CL) (A. Jodidio et al., Phys. Rev. D341(1986) 1967, and erratum) (1.00)
- $\eta = 0.011 \pm 0.085$ (H. Burkhardt et al., Phys. Lett. 160B (1985) 343) (now superseded) (0.00)

The goal of TWIST is to find any evidence for new physics that may become apparent by improving the precision of $\rho$, $\delta$, and $\mathcal{P}_\mu \xi$

by one order of magnitude compared to prior experimental results.

- measure yield vs. energy and angle, and understand depolarization, to a few parts in $10^4$. 
Uses highly polarized $\mu^+$ beam from M13.

- Stops $\mu^+$ in a symmetric detector.
- Tracks $e^+$ through uniform, well-known field.
- Completed data taking in 2007.
- Extracts decay parameters by comparison to detailed GEANT3 simulation.
Two-dimensional spectrum fit

- fit data to normalized GEANT3 simulation
- use linearity in $P_{\mu\xi}, P_{\mu\xi\delta}, \rho, \eta$
- measure differences from hidden parameters $\lambda_{MC}$

$$d\Gamma_{\text{data}}(\lambda) - d\Gamma_{\text{MC}}(\lambda_{MC}) = \frac{d\Gamma}{dP_{\mu\xi\delta}} \Delta P_{\mu\xi\delta} + \frac{d\Gamma}{dP_{\mu\xi}} \Delta P_{\mu\xi} + \frac{d\Gamma}{d\rho} \Delta \rho + \frac{d\Gamma}{d\eta} \Delta \eta$$
Spectrum fit quality

- Fiducial region: $p < 52.0$ MeV/c, $0.54 < \cos \theta < 0.96$,
- $10.0$ MeV/c $< p_T < 38.0$ MeV/c, $|p_Z| > 14.0$ MeV/c
- All data sets: $11 \times 10^9$ events, $0.55 \times 10^9$ in $(p, \cos \theta)$ fiducial
- Simulation sets: 2.7 times data statistics
Set-to-set statistical consistency

Key:

- **Ag target sets**
  - 68- \( \mu \) stop slightly US
  - 70- \( B = 1.96T \)
  - 71- \( B = 2.04T \)
  - 72- TECs in
  - 74- production
  - 75- production
  - 76- \( \mu \) beam mis-steered

- **Al target sets**
  - 83- DS extra material
  - 84- production
  - 86- \( \mu \) beam mis-steered
  - 87- production
  - 91- low beam momentum
  - 92- low beam momentum
  - 93- low beam momentum

Differences (\( \Delta \)) are with respect to blind parameters. Set-dependent corrections are applied; error bars and weights for the means are statistical only.
The blind analysis results showed evidence of possible mistakes:

- set-to-set statistical consistency satisfactory for $\rho$, $\delta$, and $P_\mu^\pi\xi$, but $P_\mu^\pi\xi\delta/\rho$ different for Al and Ag targets by 3.9$sigma$.
- $P_\mu^\pi\xi\delta/\rho$ averaged over all sets was 2.9$sigma$ greater than 1.0.
  - unlikely in four-fermion formulation with massless neutrinos.

Search for mistakes identified two corrections and two procedural changes:

- radiative decay: small correction for Ag only
- mean stopping position differences (data vs. simulation): corrected set-by-set, based on better analysis of stop position
- separate systematic uncertainties for Ag and Al targets for bremsstrahlung, target thickness, and mean stopping position
- $\rho$ and $\delta$ correlations from all sets applied to $P_\mu^\pi\xi$

After the revisions, the Ag-Al $P_\mu^\pi\xi\delta/\rho$ difference becomes $<1$sigma$. 

### Uncertainties in $\rho$ and $\delta$

<table>
<thead>
<tr>
<th>COMMON</th>
<th>Uncertainty in $\rho$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum calibration</td>
<td>$\pm 1.2$</td>
</tr>
<tr>
<td>Chamber response</td>
<td>$\pm 1.0$</td>
</tr>
<tr>
<td>Radiative corrections, $\eta$</td>
<td>$\pm 1.3$</td>
</tr>
<tr>
<td>Resolution</td>
<td>$\pm 0.6$</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>$\pm 0.5$</td>
</tr>
<tr>
<td>Others</td>
<td>$\pm 0.3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ag TARGET</th>
<th>Uncertainty in $\rho$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bremsstrahlung rate</td>
<td>$\pm 1.8$</td>
</tr>
<tr>
<td>Ag thickness/stop position</td>
<td>$\pm 3.8$</td>
</tr>
<tr>
<td>Statistical</td>
<td>$\pm 1.2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Al TARGET</th>
<th>Uncertainty in $\rho$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bremsstrahlung rate</td>
<td>$\pm 0.7$</td>
</tr>
<tr>
<td>Al thickness/stop position</td>
<td>$\pm 0.2$</td>
</tr>
<tr>
<td>Statistical</td>
<td>$\pm 1.4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weighted total systematic</th>
<th>Uncertainty in $\rho$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 2.3$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weighted total statistical</th>
<th>Uncertainty in $\rho$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 1.2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TOTAL</th>
<th>Uncertainty in $\rho$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 2.6$</td>
<td></td>
</tr>
</tbody>
</table>

### Uncertainties in $\delta$

<table>
<thead>
<tr>
<th>Uncertainty in $\delta$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 1.2$</td>
</tr>
<tr>
<td>$\pm 1.8$</td>
</tr>
<tr>
<td>$\pm 0.6$</td>
</tr>
<tr>
<td>$\pm 0.7$</td>
</tr>
<tr>
<td>$\pm 0.2$</td>
</tr>
<tr>
<td>$\pm 0.3$</td>
</tr>
<tr>
<td>$\pm 1.6$</td>
</tr>
<tr>
<td>$\pm 6.4$</td>
</tr>
<tr>
<td>$\pm 2.1$</td>
</tr>
<tr>
<td>$\pm 0.7$</td>
</tr>
<tr>
<td>$\pm 0.8$</td>
</tr>
<tr>
<td>$\pm 2.4$</td>
</tr>
<tr>
<td>$\pm 2.7$</td>
</tr>
<tr>
<td>$\pm 2.1$</td>
</tr>
<tr>
<td>$\pm 3.4$</td>
</tr>
</tbody>
</table>
Positron interactions systematic

- “Broken tracks” analysis:
  - $2 \, e^- , \, 1 \, e^+ \rightarrow \delta$ -electron
  - $2 \, e^+ \rightarrow$ Bremsstrahlung

- Agreement of data and sim:
  - $\delta$ -electrons < 1%
  - Bremsstrahlung differs by 2.4%
Uncertainties for all three parameters are from the revised analysis.

Differences to blind results are small:

- $\sigma(\rho)$ changed by $-0.3 \times 10^{-4}$
- $\sigma(\delta)$ changed by $+0.1 \times 10^{-4}$
- $\sigma(P_{\mu}^{\pi\xi}_{\text{avg}})$ changed by $-0.2 \times 10^{-4}$

Difference of $P_{\mu}^{\pi\xi\delta}/\rho$ for Ag and Al is reduced to $<1\sigma$ in the revised analysis.
Fringe field depolarization

Position

Angle

YOKKE 2 m

TEC
Asymmetric depolarization systematic

At target, $P_\mu = 0.9975 \pm 0.0004 \pm 0.0016$
Decay parameter results

\[ \rho = 0.74977 \pm 0.00012 \text{ (stat)} \pm 0.00023 \text{ (syst)} \]
<(1\sigma \text{ from SM, } -1.4 \times 10^{-4} \text{ from blind)}

\[ \delta = 0.75049 \pm 0.00021 \text{ (stat)} \pm 0.00027 \text{ (syst)} \]
(+1.4\sigma \text{ from SM, } -2.3 \times 10^{-4} \text{ from blind)}

\[ \mathcal{P}^{\pi\xi} = 1.00084 \pm 0.00029 \text{ (stat)} \pm 0.00165 \text{ (syst)} \]
(+1.2\sigma \text{ from SM, same as blind)}

\[ \mathcal{P}^{\pi\xi\delta/\rho} > 0.99909 \text{ (90\%CL)} \]
from global analysis
Left-right symmetric analysis

- Heavy $W_R$ that mixes with $W_L$ to restore parity at high energy
  \[ W_L = W_1 \cos \zeta + W_2 \sin \zeta, \quad W_R = e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta) \]

- P. Herczeg, PRD 34 (1986) 3499 uses general parameters:
  \[ t = \frac{g_R^2 m_1^2}{g_L^2 m_2^2}, \quad t_\theta = t \frac{|V_{ud}^R|}{|V_{ud}^L|} \sim t \frac{\cos \theta_R}{\cos \theta_{Cab}}, \quad \zeta_g^2 = \frac{g_R^2}{g_L^2} \zeta^2 \]

- $g_L$, $g_R$ and $V_{ud}^L$, $V_{ud}^R$ permit differences in left and right sectors, with possible CP violating phases $\omega$ and $\alpha$, and for muon decay:
  \[ \rho \simeq \frac{3}{4} (1 - 2\zeta_g^2), \quad \delta = \frac{3}{4}, \quad \xi \simeq 1 - 2(t^2 + \zeta_g^2), \]

  \[ \mathcal{P}_\mu^\pi \simeq 1 - 2t_\theta^2 - 2\zeta_g^2 - 4t_\theta \zeta_g \cos(\alpha + \omega) \]

- allowing restrictions to be put on LRS mass $m_2$ and mixing $\zeta$, e.g.,
  \[ 1 - \frac{\mathcal{P}_\mu^\pi \zeta \delta}{\rho} \simeq 2t^2 (1 + \frac{\cos^2 \theta_R}{\cos^2 \theta_{Cab}}) + 2\zeta_g^2 + 4\zeta_g t \cos \theta_R \cos \theta_{Cab} \cos(\alpha + \omega) \]
Previous muon decay LRS parameter limits used individual limits for $\rho$, $P_\mu^{\pi \xi}$, or $P_\mu^{\pi \xi \delta}/\rho$.

*TWIST* has simultaneous measurements of three parameters; correlations contribute to the confidence interval.
LRS limit comparison

```
\[ m^2 > 582 \text{ GeV}/c^2 \]
\[-0.019 < \zeta < +0.014 \]

\[ (g_L/g_R)^{\pm} \zeta < 0.020 \]

D0 direct search
lower limit
Abazov et al.,
Phys. Rev. Lett. 100
(2008) 031804

\[ (g_L/g_R)m^2 > 578 \text{ GeV}/c^2 \]
\[-0.020 < (g_R/g_L)^{\pm} \zeta < +0.020 \]
```
Global analysis result

- Include new results with other muon decay observables to restrict coupling constants
  - influences mostly right-handed muon terms

\[ Q^\mu_R = \frac{1}{4} |g^S_{LR}|^2 + \frac{1}{4} |g^S_{RR}|^2 + |g^V_{LR}|^2 + |g^V_{RR}|^2 + 3|g^T_{LR}|^2 \]

\[ = \frac{1}{2} \left[ 1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right] \]

\[ < 8.2 \times 10^{-4} \quad (90\% \text{C.L.}) \]

- $\sim 6 \times$ reduction
Muon decay spectrum shape places limits on heavy neutrino mass and mixing in a mass region inaccessible with $\pi$ or $K$ decays.


Summary

- Systematic uncertainties in muon decay parameter measurements were substantially reduced in TWIST.
- Total uncertainties were reduced by factors of 10, 11, and 7 for $\rho$, $\delta$, and $P_\mu \pi \xi$ respectively, roughly achieving the goals of the experiment.
- Differences with Standard Model predictions are respectively $-0.9\sigma$, $+1.4\sigma$, and $+1.2\sigma$, after post-blind revisions.
- $P_\mu \pi \xi \delta / \rho$ deviates by $+2.3\sigma$ from the expected upper limit of 1.0.
**TWIST** participants, past and present

<table>
<thead>
<tr>
<th>TRIUMF</th>
<th>Alberta</th>
<th>Kurchatov Institute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ryan Bayes *†</td>
<td>Andrei Gaponenko **</td>
<td>Vladimir Selivanov</td>
</tr>
<tr>
<td>Yuri Davydov</td>
<td>Robert MacDonald **</td>
<td></td>
</tr>
<tr>
<td>Wayne Faszer</td>
<td>Maher Quraan</td>
<td>Texas A&amp;M</td>
</tr>
<tr>
<td>Makoto Fujiwara</td>
<td>Nate Rodning §</td>
<td>Carl Gagliardi</td>
</tr>
<tr>
<td>David Gill</td>
<td></td>
<td>Jim Musser **</td>
</tr>
<tr>
<td>Alexander Grossheim</td>
<td></td>
<td>Bob Tribble</td>
</tr>
<tr>
<td>Peter Gumplinger</td>
<td></td>
<td>Valparaiso</td>
</tr>
<tr>
<td>Anthony Hillairet *†</td>
<td></td>
<td>Don Koetke</td>
</tr>
<tr>
<td>Robert Henderson</td>
<td></td>
<td>Shirvel Stanislaus</td>
</tr>
<tr>
<td>Jingliang Hu</td>
<td>British Columbia</td>
<td></td>
</tr>
<tr>
<td>John A. Macdonald §</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glen Marshall</td>
<td>James Bueno *</td>
<td></td>
</tr>
<tr>
<td>Dick Mischke</td>
<td>Mike Hasinoff</td>
<td></td>
</tr>
<tr>
<td>Mina Nozar</td>
<td>Blair Jamieson **</td>
<td></td>
</tr>
<tr>
<td>Konstantin Olchanski</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Art Olin †</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robert Openshaw</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jean-Michel Poutissou</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renée Poutissou</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grant Sheffer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bill Shin ‡‡</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regina</td>
</tr>
<tr>
<td></td>
<td>Ted Mathie</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Roman Tacik</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kurchatov Institute</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vladimir Selivanov</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Recently graduated
** Graduated
† also U Vic
‡‡ also Saskatchewan
§ deceased
extra slides
Muon production and transport

- Improved engineering of TECs
- Muons selected from different depths
- Beam line upgraded: "quadrupole steering" added
Detector array


high precision ($\Delta l/l < 10^{-4}$)
low mass (2×0.1 g/cm²)
Detector array

Variable density gas degrader

select dE/dx for $\mu^+$ in target

Al and Ag targets

high precision ($\Delta l/l < 10^{-4}$)

low mass ($2 \times 0.1$ g/cm$^2$)

Downstream material to test backscatter

Trigger scintillator

TECs for beam characterization

- Need to know $x$, $y$, $\theta_x$, $\theta_y$, and correlations, for incident muon beam.
- Measure in two modules of low pressure (80 mbar) time expansion chambers (TEC).
- “Correct” for multiple scattering ($\sim 20$ mrad rms).
- Simulate by sampling corrected distributions.
- Decay parameters measured with TEC removed; multiple scattering reduces polarization.

J. Hu et al., NIM A566 (2006) 563-574
Surface muon polarization

- Pions decaying at rest produce muon beams with $P$ $\mu$ > 99%.
- Depolarization must be controlled using small beams near kinematic edge, 29.8 MeV/c.
- Use $\sim 4 \times 10^3$ $\mu^+$ s$^{-1}$.
- Muon total range at density $\sim 1$ only about 1.5 mm!
Momentum calibration

- Use kinematic edge at 52.8 MeV/c: energy loss and planar geometry lead to \( \cos \theta \) dependence.
- Difference of \(~10\) keV/c prior to calibration.
- Calibration at edge provides no guidance on how to propagate the difference to lower momenta in the spectrum.
Depolarization in muon target material

Estimate of relaxation is included in simulation; small correction is made to polarization parameter.

μSR experiment establishes no fast relaxation.

Statistical uncertainty in λ is included in decay parameter statistical uncertainty.

\[ P_\mu(t) = P_\mu(0) \exp(-\lambda t) \]
Selecting muons in metal target

- Place cut on 2-d distribution so that <0.5% of “stops in gas” contaminate “stops in target” region (zone 1).
Blind analysis

Hidden parameters; \( \rho_{MC}, \delta_{MC}, P_{\mu \pi \xi}^{MC} \)

Hidden parameter tolerances 0.01

Data from experiment

“Data” from simulation

Reconstruction software

Spectrum fit; \( \Delta \rho, \Delta \delta, \Delta P_{\mu \pi \xi} \)

Add to hidden parameters; \( \rho, \delta, P_{\mu \pi \xi} \)
Spectrum fit quality

Normalised residuals for nominal set (s87)

(data-simulation)/\sigma

momentum (MeV/c)

\cos^2

(data-simulation)/\sigma

\cos^2
Corrections to fit results

- **Depolarization from scattering in production target**
  - $+0.9 \times 10^{-4}$ for full momentum sets, $+5.6 \times 10^{-4}$ for reduced momentum sets, for $P_{\mu} \xi$ only.

- **Simulations generated with incorrect polarization relaxation rates**
  - $+2.9 \times 10^{-4}$ for Ag sets, $+2.4 \times 10^{-4}$ for Al sets

- **Statistical biases**
  - $\chi^2$ fitting of Poisson statistics with $1/N$ weight is biased
  - in fitting data to simulation, weight includes $1/N$ from both
    - for unequal statistics, this is biased by $\sim 0.5 \times 10^{-4}$
  - energy calibration fit bias of typically $(-1.1,-0.4,+1.9) \times 10^{-4}$ for $\rho, \delta, P_{\mu} \xi$, applied set-by-set