Final Results on Muon Decay from TWIST

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Outline

• Introduction to muon decay
• TWIST experiment
• Previous TWIST results
• Final TWIST results
Muon decay spectrum

• The energy and angle distributions of positrons following polarized muon decay obey the Michel spectrum:

\[
\frac{d^2\Gamma}{x^2 dx d(\cos \theta)} \propto (3 - 3x) + \frac{2}{3} \rho (4x - 3) + 3\eta \frac{r_0}{x} (1 - x) + P_\mu \xi \cos \theta \left[(1 - x) + \frac{2}{3} \delta (4x - 3)\right] \quad (+ \text{rad. corr.})
\]

where \( x = \frac{E_e}{E_{e,\text{max}}} \)

• Pre-TWIST accepted values for the muon decay (Michel) parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.7518 ( \pm ) 0.0026</td>
</tr>
<tr>
<td>( \eta )</td>
<td>-0.007 ( \pm ) 0.013</td>
</tr>
<tr>
<td>( P_\mu \xi )</td>
<td>1.0027 ( \pm ) 0.0079 ( \pm ) 0.0030</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.7486 ( \pm ) 0.0026 ( \pm ) 0.0028</td>
</tr>
<tr>
<td>( P_\mu (\xi \delta / \rho) )</td>
<td>&gt; 0.99682 (90% c.l.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3/4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Muon decay matrix element

- Most general Lorentz-invariant, local, lepton-number conserving muon decay matrix element:

\[ M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T, \epsilon, \mu=R,L} g_{\epsilon\mu}^\gamma \langle \overline{e}_\epsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle (\overline{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle \]

- The muon decay parameters are bi-linear combinations of the \( g_{\epsilon\mu}^\gamma \).
- In the Standard Model, \( g_{VLL}^V = 1 \), all others are zero.
- Pre-TWIST global fit results (all 90% c.l.):

| \(|g_{RR}^S| < 0.066\) | \(|g_{RR}^V| < 0.033\) | \(|g_{RR}^T| \equiv 0\) |
| --- | --- | --- |
| \(|g_{LR}^S| < 0.125\) | \(|g_{LR}^V| < 0.060\) | \(|g_{LR}^T| < 0.036\) |
| \(|g_{RL}^S| < 0.424\) | \(|g_{RL}^V| < 0.110\) | \(|g_{RL}^T| < 0.122\) |
| \(|g_{LL}^S| < 0.550\) | \(|g_{LL}^V| > 0.960\) | \(|g_{LL}^T| \equiv 0\) |
Goal of TWIST

• Search for new physics that can be revealed by order-of-magnitude improvements in our knowledge of $\rho$, $\delta$, and $P_{\mu} \xi$

Two examples

• Model-independent limit on muon handedness

\[ Q_R^\mu = \frac{1}{2} \left[ 1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right] \]

• Left-right symmetric model: $SU(2)_L \times SU(2)_R \times U(1)$

\[ W_L = W_1 \cos \zeta + W_2 \sin \zeta \]
\[ W_R = e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta) \]
\[ \frac{3}{4} - \rho = \frac{3}{2} \zeta^2 \]
\[ 1 - P_{\mu} \xi = 4 \left( \zeta^2 + \zeta \left( \frac{M_1}{M_2} \right)^2 + \left( \frac{M_1}{M_2} \right)^4 \right) \]

• …..
What is required?

Must:

- Understand sources of muon depolarization
  -- $P_\mu$ and $\xi$ come as a product
- Determine positron yield vs. momentum and angle
  -- All three parameters
to within a few parts in $10^4$
Surface muon beam

500 MeV proton beam

Time Expansion Chamber (TEC)

π^+ + μ^+ + ν \rightarrow 500 \text{ MeV} \text{ proton beam}

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Detector array

Variable density gas degrader

PC1-4
DC1-8
DC9-22
PC5-6
PC7-8
DC23-36
DC37-44
PC9-12
Event topologies

- Many events are simple
  - A muon enters and stops
  - The decay positron leaves
- Other events aren’t simple

- We must be able to handle all kinds
2-d momentum-angle spectrum

Acceptance of the **TWIST** spectrometer
Fitting the data distributions

\[ n_i(\alpha_{\text{data}}) = n_i(\alpha_{\text{MC}}) + \frac{\partial n_i}{\partial \alpha} \Delta \alpha, \]

\[ \alpha = [\rho, \eta, \xi, \xi\delta] \]

- Fit data to sum of a MC base spectrum plus MC-generated derivative distributions.
- Decay distribution is linear in the muon decay parameters, so this is exact, no matter what values (\(\alpha_{\text{MC}}\)) are used in the MC base spectrum.

\(\alpha_{\text{MC}}\) hidden \(\Rightarrow\) blind analysis
Physics data sets

• **Fall 2002**
  – Test data-taking procedures and develop analysis techniques
  – First physics results – $\rho$ and $\delta$
  – Graphite-coated Mylar target not suitable for $P_\mu \xi$

• **Fall 2004**
  – Aluminum target and Time Expansion Chamber enabled first $P_\mu \xi$ measurement
  – Improved determinations of $\rho$ and $\delta$

• **2006-07**
  – Both silver (2006) and aluminum (2007) targets
  – Ultimate **TWIST** precision for $\rho$, $\delta$, and $P_\mu \xi$
  – Also measured negative muon decay-in-orbit when bound to Al
Data distributions from 2002 and 2004

Angle-integrated spectrum from the 2002 data

Asymmetry vs momentum from the 2004 data
TWIST results before now

A global analysis that combined the first TWIST $\rho$ and $\delta$ results and a concurrent measurement from PSI of the $e^+$ transverse polarization (PRL 94, 021802) together with all pre-TWIST muon decay parameter measurements found:

$$\eta = -0.0036 \pm 0.0069$$
How to do better?

- **TWIST** is a *systematics-dominated experiment*

- **Must have:**
  - Improved data-taking procedures
  - Better understanding of the detector
  - Improved analysis techniques

- **Leading systematics in our previous $P_{\mu\xi}$ measurement**
  - Muon depolarization while entering the solenoid
  - Time-dependent muon depolarization in the stopping target

- **Leading systematics in our previous $\rho$ and $\delta$ measurements**
  - Chamber response
  - Momentum calibration
  - Positron interactions
At target, $P_\mu = 0.9975$

Muons can be depolarized as they cross flux lines when entering the solenoid.
Validating the fringe field effects

- The average muon beam trajectory inside the detector is sensitive to the muon transverse momentum
- Identify changes in muon beam properties between TEC measurements
- Comparisons between nominal and mis-steered beams
  - Observed muon beam trajectories within the detector
  - Difference in the decay asymmetry in data vs that predicted by the Monte Carlo
Ensuring muons stop in the metal target

- Muons that stop in gas can depolarize through muonium formation
- Use muon energy depositions near the stopping target to reject those that stop in gas
Measuring depolarization after stopping

High purity (>99.999% purity) Al and Ag targets

Subsidiary $\mu^+$SR study: no “fast depolarisation” down to 5 ns

$P_\mu(t) = P_\mu(0) \exp(-\lambda t)$

Previous TWIST

New TWIST

$\lambda$ (ms$^{-1}$)

<table>
<thead>
<tr>
<th></th>
<th>Al</th>
<th>Ag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous TWIST</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New TWIST</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Equal-time contours

- Direct determination of the effective distance vs time relation
- Accounts for small plane-to-plane fabrication differences
- Compensates for small (few keV) biases in the helix fitter
- Improved momentum resolution (near the endpoint)
  - Was ~ 69 keV/sin(θ) in Monte Carlo and ~ 74 keV/sin(θ) in data
  - Now ~ 58 keV/sin(θ) in both
Momentum calibration

- The endpoint at 52.83 MeV/c provides a calibration point
- Improvements in the endpoint fitter reduced previous small biases
- Find a ~10 keV/c difference in the absolute calibration of the data vs Monte Carlo
- Propagate this through the entire spectrum twice
  - Shift by a constant
  - Rescale the momentum axis
Bremsstrahlung

- Leading systematic for $\rho$ and $\delta$
- Two separate measurements
  - “Upstream stops”
  - “Broken” decay tracks
- Consistent results validate bremsstrahlung simulation in our GEANT3 Monte Carlo at the 2.5% level
Monte Carlo reproduces the data very well, even very far outside the fiducial region
Reproducibility of the results

- 14 separate measurements under various experimental conditions
- All 14 data sets are used for $\rho$ and $\delta$, $\chi^2 = 14.0$ and 17.7
- 9 of the data sets are used for $P_\mu \xi$, $\chi^2 = 9.6$
## Total uncertainty budget – $\rho$ and $\delta$

<table>
<thead>
<tr>
<th>Uncertainties</th>
<th>$\rho \times 10^{-4}$</th>
<th>$\delta \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positron interactions</td>
<td>1.8</td>
<td>1.6</td>
</tr>
<tr>
<td>External uncertainties</td>
<td>1.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Momentum calibration</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Chamber response</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Beam stability</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Systematics added in quadrature</strong></td>
<td><strong>2.8</strong></td>
<td><strong>2.9</strong></td>
</tr>
<tr>
<td>Statistical uncertainty</td>
<td>0.9</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>Total uncertainty</strong></td>
<td><strong>3.0</strong></td>
<td><strong>3.3</strong></td>
</tr>
</tbody>
</table>

Comparison to 2004 leading systematics:
- Down factor of ~3

Similar magnitude:
- Down factor of ~3

Down factor of ~2
## Total uncertainty budget – $P_{\mu} \xi$

<table>
<thead>
<tr>
<th>Uncertainties</th>
<th>$P_{\mu} \xi$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depolarization in fringe field</td>
<td>+15.8, -4.0</td>
</tr>
<tr>
<td>Depolarization in stopping material</td>
<td>3.2</td>
</tr>
<tr>
<td>Background muons</td>
<td>1.0</td>
</tr>
<tr>
<td>Depolarization in production target</td>
<td>0.3</td>
</tr>
<tr>
<td>Chamber response</td>
<td>2.3</td>
</tr>
<tr>
<td>Resolution</td>
<td>1.5</td>
</tr>
<tr>
<td>Momentum calibration</td>
<td>1.5</td>
</tr>
<tr>
<td>External uncertainties</td>
<td>1.2</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>0.7</td>
</tr>
<tr>
<td>Beam stability</td>
<td>0.3</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Systematics added in quadrature</strong></td>
<td>+16.5, -6.2</td>
</tr>
<tr>
<td>Statistical uncertainty</td>
<td>3.5</td>
</tr>
<tr>
<td><strong>Total uncertainty</strong></td>
<td>+16.9, -7.2</td>
</tr>
</tbody>
</table>

Comparison to 2004 leading systematics

- Down factor of ~3
- Down factor of ~4

Systematics added in quadrature
Final **TWIST** results

- $\rho = 0.74991 \pm 0.00009$ (stat) $\pm 0.00028$ (syst)
- $\delta = 0.75072 \pm 0.00016$ (stat) $\pm 0.00029$ (syst)
- $P_\mu \xi = 1.00084 \pm 0.00035$ (stat) $+0.00165$ $-0.00063$ (syst)

- Correlations:
  - $\text{Corr}(\rho, \delta) = +0.69$
  - $\text{Corr}(\rho, P_\mu \xi) = -0.06$ (for $P_\mu \xi$ high) and $-0.14$ (for $P_\mu \xi$ low)
  - $\text{Corr}(\delta, P_\mu \xi) = -0.18$ (for $P_\mu \xi$ high) and $-0.43$ (for $P_\mu \xi$ low)

- Can combine the above to find $P_\mu \xi \delta/\rho = 1.00192^{+0.00167}_{-0.00063}$
  - This is asymmetry at endpoint $\rightarrow$ must be $\leq 1$
  - This $2.9\sigma$ surprise is currently under investigation
Implications for the muon decay matrix element

- The final **TWIST** results have been included in a new muon decay global analysis together with all previous muon decay parameter measurements.
- Find significantly tighter 90% c.l. upper limits on the coupling of right-handed muons to right- or left-handed electrons:
  - $|g^{S}_{RR}| < 0.031$\footnote{Factor of ~2 smaller than pre-TWIST values}
  - $|g^{V}_{RR}| < 0.015$
  - $|g^{S}_{LR}| < 0.041$
  - $|g^{V}_{LR}| < 0.018$\footnote{Factor of ~3 smaller than pre-TWIST values}
  - $|g^{T}_{LR}| < 0.012$
- New limit on right-handed muon couplings: $Q_{\mu R} < 5.8 \times 10^{-4}$ (90% c.l.)\footnote{Factor of ~9 smaller than pre-TWIST value}
- Uncertainty for $\eta$ reduced by 1/3 compared to 2005 global analysis
  - $\eta = -0.0033 \pm 0.0046$
  - Important for the determination of $G_{F}$
Implications for left-right symmetric models

\[ W_L = W_1 \cos \zeta + W_2 \sin \zeta \quad W_R = e^{i\omega}(-W_1 \sin \zeta + W_2 \cos \zeta) \]

- Significantly improved limits on both \( W_R \) mass and \( L-R \) mixing angle \( \zeta \)
- Many other limits also apply if \( V_{ud}^R \) is near 1; for example:
  - Direct production at the Tevatron finds \( M(W_R) > 1 \) TeV
  - \( 0^+ \rightarrow 0^+ \) nuclear beta decay finds \( |\zeta| < 0.0005 \)
- **TWIST** limits in the general case make no assumption about \( V_{ud}^R \)
Conclusions

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The TWIST Collaboration (http://twist.triumf.ca)

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Coupling constants and Michel parameters

- The Michel parameters are bilinear combinations of the coupling constants:

\[
\rho = \frac{3}{4} - \frac{3}{4} \left| g_{RL}^V \right|^2 + \left| g_{LR}^V \right|^2 + 2 \left| g_{RL}^T \right|^2 + 2 \left| g_{LR}^T \right|^2 \\
+ \Re \left( g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*} \right)
\]

\[
\eta = \frac{1}{2} \Re \left( g_{RR}^V g_{LL}^S + g_{LL}^V g_{RR}^S + g_{RL}^V (g_{LR}^S + 6 g_{LR}^T) + g_{LR}^V (g_{RL}^S + 6 g_{RL}^T) \right)
\]

\[
\xi = 1 - \frac{1}{2} \left| g_{LR}^S \right|^2 - \frac{1}{2} \left| g_{RR}^S \right|^2 - 4 \left| g_{RL}^V \right|^2 + 2 \left| g_{LR}^V \right|^2 - 2 \left| g_{RR}^V \right|^2 \\
+ 2 \left| g_{LR}^T \right|^2 - 8 \left| g_{RL}^T \right|^2 + 4 \Re \left( g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*} \right)
\]

\[
\xi \delta = \frac{3}{4} - \frac{3}{8} \left| g_{RR}^S \right|^2 - \frac{3}{8} \left| g_{LR}^S \right|^2 - \frac{3}{2} \left| g_{RR}^V \right|^2 - \frac{3}{4} \left| g_{RL}^V \right|^2 - \frac{3}{4} \left| g_{LR}^V \right|^2 \\
- \frac{3}{2} \left| g_{RL}^T \right|^2 - 3 \left| g_{LR}^T \right|^2 + \frac{3}{4} \Re \left( g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*} \right)
\]
Precision detector construction

- Very low mass (~$10^{-4} X_0$ per U-V pair), high precision chambers
- Longitudinal alignment by engineering
- Transverse alignment using particle tracks
- >5000 wires, efficiency >99.8%

To a few parts in $10^5$
“Good muons” vs “bad muons”

- Positive muons can arise from pion decay at rest or in flight
  - At rest ("surface") muons have large (~1) negative polarization
  - In flight ("cloud") muons have small (~20%) positive polarization
- In TWIST, we (usually) want the former and not the latter
- Achieved by cutting on the muon arrival time at the spectrometer
Previous muon decay parameter results

- From Fall, 2002 run:
  - \( \rho = 0.75080 \pm 0.00032 \) (stat) \( \pm 0.00097 \) (syst) \( \pm 0.00023 \) (\( \eta \))
    PRL 94, 101805
  - \( \delta = 0.74964 \pm 0.00066 \) (stat) \( \pm 0.00112 \) (syst)
    PRD 71, 071101
- New global analysis (PRD 72, 073002) using the \( \rho \) and \( \delta \) results together with previous measurements, plus recent \( e^+ \) transverse polarization measurements from PSI (PRL 94, 021802):
  - Significant improvements in the limits for \( g^{S,V,T}_{LR} \)
  - \( \eta = -0.0036 \pm 0.0069 \)
- From Fall, 2004 run:
  - \( P_{\mu \xi} = 1.0003 \pm 0.0006 \) (stat) \( \pm 0.0038 \) (syst)
    PRD 74, 072007
  - \( \rho = 0.75014 \pm 0.00017 \) (stat) \( \pm 0.00044 \) (syst) \( \pm 0.00011 \) (\( \eta \))
  - \( \delta = 0.75067 \pm 0.00030 \) (stat) \( \pm 0.00067 \) (syst)
    PRD 78, 032010
- Factors of 2 (\( P_{\mu \xi} \)) to 5 (\( \rho, \delta \)) increased precision vs. pre-TWIST
### Table II. Summary of systematic uncertainties by category.

<table>
<thead>
<tr>
<th>Category</th>
<th>Δρ</th>
<th>Δδ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber response</td>
<td>0.00029</td>
<td>0.00052</td>
</tr>
<tr>
<td>Energy scale</td>
<td>0.00029</td>
<td>0.00041</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>0.00016</td>
<td>0.00009</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.00002</td>
<td>0.00003</td>
</tr>
<tr>
<td>Alignment and lengths</td>
<td>0.00003</td>
<td>0.00003</td>
</tr>
<tr>
<td>Beam intensity</td>
<td>0.00001</td>
<td>0.00002</td>
</tr>
<tr>
<td>Correlations with η</td>
<td>0.00011</td>
<td>0.00001</td>
</tr>
<tr>
<td>Theory</td>
<td>0.00003</td>
<td>0.00001</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.00046</strong></td>
<td><strong>0.00067</strong></td>
</tr>
</tbody>
</table>

### Table III. Contributions to the systematic uncertainty for $P_{\mu,\xi}$.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depolarization in fringe field (ave)</td>
<td>0.0034</td>
</tr>
<tr>
<td>Depolarization in stopping material (ave)</td>
<td>0.0012</td>
</tr>
<tr>
<td>Chamber response (ave)</td>
<td>0.0010</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>0.0003</td>
</tr>
<tr>
<td>Positron interactions (ave)</td>
<td>0.0003</td>
</tr>
<tr>
<td>Depolarization in production target</td>
<td>0.0002</td>
</tr>
<tr>
<td>Momentum calibration</td>
<td>0.0002</td>
</tr>
<tr>
<td>Upstream-downstream efficiency</td>
<td>0.0002</td>
</tr>
<tr>
<td>Background muon contamination (ave)</td>
<td>0.0002</td>
</tr>
<tr>
<td>Beam intensity (ave)</td>
<td>0.0002</td>
</tr>
<tr>
<td>Michel parameter $\eta$</td>
<td>0.0001</td>
</tr>
<tr>
<td>Theoretical radiative corrections</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Negative muon decay-in-orbit

- Future $\mu \rightarrow e$ conversion experiments plan to study negative muons bound to Al
- Most precise measurement ever of the muon decay-in-orbit spectrum
- Theoretical predictions include higher-order contributions from the muon+nucleus potential
- Need to include the $O(\alpha)$ radiative corrections that arise from the interaction between the muon and the outgoing electron
Tracking (in)efficiency

- Measure the tracking efficiency with “upstream stops” data
  - Reconstruct a track with a particular \((p, \cos(\theta))\) in one half
  - Do we reconstruct a track in the other half?
- Inefficiency in data is <0.5% throughout the fiducial region
- Inefficiency in data and Monte Carlo match to <0.05%
Fringe field and mis-steered beam

Move beam away from optimum position and/or angle to observe change in polarization:

- Comparison I: mis-steer y direction by 28 mrad
  Find: $\Delta P_\mu = -105 \pm 9 \times 10^{-4}$

- Comparison II: mis-steer x position by 10 mm and direction by 10 mrad
  Find: $\Delta P_\mu = -62 \pm 8 \times 10^{-4}$

- Comparison III: leave TEC in to introduce scattering
  Find: $\Delta P_\mu = -18 \pm 9 \times 10^{-4}$

Compare differences with simulation to check fringe field systematic
Sensitivity to fringe field transverse components

Estimate of error

Opera field

data minus simulation (number of $\sigma$)

$0.7 \quad 0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2 \quad 1.3 \quad 1.4 \quad 1.5$

$f$, scale factor applied to $B_x$ and $B_y$

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Bremsstrahlung

- Leading systematic for $\rho$ and $\delta$
- Two separate measurements
  - “Upstream stops”
  - “Broken tracks”
- Consistent results validate bremsstrahlung simulation in our GEANT3 Monte Carlo at the 2.5% level
Why are $\rho$ and $\delta$ systematics correlated?

- $\rho$ and $\delta$ involve the momentum-dependence of the yield and asymmetry
- They have:
  - Same upstream shapes
  - Opposite downstream shapes
- Effects that
  - Distort the momentum, and
  - Couple to the yield
- Distort $\rho$ and $\delta$ similarly
- Example: bremsstrahlung

Derivatives at $\cos(\theta) = \pm 0.75$
Why are $\delta$ and $P_\mu \xi$ anti-correlated?

- Anti-correlation between statistical uncertainties for $\delta$ and $P_\mu \xi$
- Three types of systematics influence the asymmetry measurements
  - Distort $P_\mu$; only impact $P_\mu \xi$
  - Distort contribution of $P_\mu \xi \delta$ derivative; only impact $\delta$
  - Distort contribution of $P_\mu \xi$ derivative; impact BOTH $P_\mu \xi$ and $\delta$

In TWIST, the fit parameters are $P_\mu \xi$ and $P_\mu \xi \delta$

$$\delta = \frac{P_\mu \xi \delta}{P_\mu \xi}$$
Muon decay parameters in the global analysis

(Phys Rev D 72, 073002)

\[ Q_{RR} = \frac{1}{4} |g_{RR}^{S}|^2 + |g_{RR}^{V}|^2, \]
\[ Q_{LR} = \frac{1}{4} |g_{LR}^{S}|^2 + |g_{LR}^{V}|^2 + 3|g_{LR}^{T}|^2, \]
\[ Q_{RL} = \frac{1}{4} |g_{RL}^{S}|^2 + |g_{RL}^{V}|^2 + 3|g_{RL}^{T}|^2, \]
\[ Q_{LL} = \frac{1}{4} |g_{LL}^{S}|^2 + |g_{LL}^{V}|^2, \]
\[ B_{LR} = \frac{1}{16} |g_{LR}^{S}|^2 + 6g_{LR}^{T}|^2 + |g_{LR}^{V}|^2, \]
\[ B_{RL} = \frac{1}{16} |g_{RL}^{S}|^2 + 6g_{RL}^{T}|^2 + |g_{RL}^{V}|^2, \]
\[ I_\alpha = \frac{1}{4} [g_{LR}^{V}(g_{RL}^{S} + 6g_{RL}^{T})^* + (g_{RL}^{V})^*(g_{LR}^{S} + 6g_{LR}^{T})] \]
\[ = (\alpha + i\alpha')/2A, \]
\[ I_\beta = \frac{1}{2} [g_{LL}^{V}(g_{RR}^{S})^* + (g_{RR}^{V})^*g_{LL}^{S}] = -2(\beta + i\beta')/A. \]

0 \leq Q_{\epsilon\mu} \leq 1, \quad \text{where } \epsilon, \mu = R, L,

0 \leq B_{\epsilon\mu} \leq Q_{\epsilon\mu}, \quad \text{where } \epsilon\mu = RL, LR,

|I_\alpha|^2 \leq B_{LR}B_{RL}, \quad |I_\beta|^2 \leq Q_{LL}Q_{RR},

Q_{RR} + Q_{LR} + Q_{RL} + Q_{LL} = 1.

\[ \rho = \frac{3}{4} + \frac{1}{4}(Q_{LR} + Q_{RL}) - (B_{LR} + B_{RL}), \]
\[ \xi = 1 - 2Q_{RR} - \frac{10}{3}Q_{LR} + \frac{4}{3}Q_{RL} + \frac{16}{3}(B_{LR} - B_{RL}), \]
\[ \xi\delta = \frac{3}{4} - \frac{3}{2}Q_{RR} - \frac{7}{4}Q_{LR} + \frac{1}{4}Q_{RL} + (B_{LR} - B_{RL}), \]
\[ \xi' = 1 + 2Q_{RR} - 2Q_{RL}, \]
\[ \xi'' = 1 - \frac{10}{3}(Q_{LR} + Q_{RL}) + \frac{16}{3}(B_{LR} + B_{RL}), \]
\[ \eta = \frac{1}{3}(Q_{LR} + Q_{RL}) + \frac{2}{3}(B_{LR} + B_{RL}), \]
\[ \eta' = (\alpha - 2\beta)/A, \quad \eta'' = (3\alpha + 2\beta)/A. \]

• The global analysis combines the final TWIST results with all non-TWIST muon decay parameter measurements
• The fit parameters are $Q_{RR}$, $Q_{LR}$, $Q_{RL}$, $B_{LR}$, $B_{RL}$, $\alpha/A$, $\beta/A$, $\alpha'/A$, $\beta'/A$