The **TWIST** Experiment: Testing the Standard Model with Muon Decay

Glen Marshall, for the **TWIST** Collaboration

µCP-07, Dubna, 18-21 June 2007
The TRIUMF Weak Interaction Symmetry Test

- Tests Standard Model predictions for muon decay.
- Uses highly polarized $\mu^+$ beam ($\mu^-$ don’t work!).
- Stops $\mu^+$ in a very symmetric detector.
- Tracks $e^+$ through uniform, well-known field.
- Extracts decay parameters by comparison to detailed and verified simulation.
Michel parameter description

Muon decay (Michel) parameters $\rho$, $\eta$, $P_\mu \xi$, $\delta$

- muon differential decay rate vs. energy and angle:

$$
\frac{d^2 \Gamma}{dx \ d \cos \theta} = \frac{1}{4} m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot \\
\{ F_{IS}(x, \rho, \eta) + P_\mu \cos \theta \cdot F_{AS}(x, \xi, \delta) \} + R.C.
$$

where

- $F_{IS}(x, \rho, \eta) = x(1-x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1-x)$

- $F_{AS}(x, \xi, \delta) = \frac{1}{3} \sqrt{x^2 - x_0^2} \left[ \xi \{1-x\} + \frac{2}{3} \xi \delta \left\{ 4x - 3 + \left( \sqrt{1-x_0^2} - 1 \right) \right\} \right]$}

- and $W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$, $x = \frac{E_e}{W_{\mu e}}$, $x_0 = \frac{m_e}{W_{\mu e}}$. 

$\mu$CF-07, Dubna, June 18-21, 2007

G.M. Marshall, The TWIST Experiment
Pre-TWIST decay parameters

- From the Review of Particle Physics (SM values in parentheses):
  - $\rho = 0.7518 \pm 0.0026$ (Derenzo, 1969) (0.75)
  - $\eta = -0.007 \pm 0.013$ (Burkard et al., 1985) (0.00)
  - $\delta = 0.7486 \pm 0.0026 \pm 0.0028$ (Balke et al., 1988) (0.75)
  - $P_{\mu \xi} = 1.0027 \pm 0.0079 \pm 0.0030$ (Beltrami et al., 1987) (1.00)
  - $P_{\mu}(\xi \delta/\rho) > 0.99682$ (Jodidio et al., 1986) (1.00)

The goal of TWIST is to find any new physics which may become apparent by improving the precision of each of $\rho$, $\delta$, and $P_{\mu \xi}$ by at least one order of magnitude compared to prior experimental results.
• Full $O(\alpha)$ radiative corrections with exact electron mass dependence.
• Leading and next-to-leading logarithmic terms of $O(\alpha^2)$.
• Leading logarithmic terms of $O(\alpha^3)$.
• Corrections for soft pairs, virtual pairs and an ad-hoc exponentiation.

Anastasiou et al., hep-ph/0505069 to $O(\alpha^2)$. 
Michel parameters and coupling constants

- Fetscher and Gerber coupling constants (see PDG):

\[
M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T}^{\epsilon,\mu=R,L} g_{\epsilon\mu}^{\gamma} \langle \bar{e}_\epsilon | \Gamma^\gamma | (\nu_\epsilon)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle
\]

\[
\rho = \frac{3}{4} - \frac{3}{4} \left[ |g_{RL}^V|^2 + |g_{LR}^V|^2 + 2 |g_{RL}^T|^2 + 2 |g_{LR}^T|^2 \right.
\]
\[
+ \text{Re} \left( g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*} \right) \]

\[
\eta = \frac{1}{2} \text{Re} \left[ g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} + g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) \right]
\]

\[
\xi = 1 - \frac{1}{2} |g_{LR}^S|^2 - \frac{1}{2} |g_{RR}^S|^2 - 4 |g_{RL}^V|^2 + 2 |g_{RR}^V|^2 - 2 |g_{LR}^V|^2
\]
\[
+ 2 |g_{RL}^T|^2 - 8 |g_{LR}^T|^2 + 4 \text{Re} \left( g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*} \right)
\]

\[
\xi \delta = \frac{3}{4} - \frac{3}{8} |g_{RR}^S|^2 - \frac{3}{8} |g_{LR}^S|^2 - \frac{3}{2} |g_{RR}^V|^2 - \frac{3}{4} |g_{RR}^V|^2 - \frac{3}{4} |g_{LR}^V|^2
\]
\[
- \frac{3}{2} |g_{RL}^T|^2 - 3 |g_{LR}^T|^2 + \frac{3}{4} \text{Re} \left( g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*} \right)
\]
Coupling constants

- Coupling constants \( g'_{\epsilon\mu} \) can be related to handedness, e.g., total muon right-handed coupling:
  \[
  Q^\mu_R \equiv Q_{RR} + Q_{LR} = \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RR}|^2 + |g^V_{LR}|^2 + |g^V_{RR}|^2 + 3|g^T_{LR}|^2
  \]

- Global analysis of \( \mu \) decay (Gagliardi et al., PRD 72 2005)
  - no existing similar analysis for other weak decays.

\[
\begin{align*}
|g^S_{RR}| &< 0.066(0.067) & |g^V_{RR}| &< 0.033(0.034) & |g^T_{RR}| &\equiv 0 \\
|g^S_{LR}| &< 0.125(0.088) & |g^V_{LR}| &< 0.060(0.036) & |g^T_{LR}| &< 0.036(0.025) \\
|g^S_{RL}| &< 0.424(0.417) & |g^V_{RL}| &< 0.110(0.104) & |g^T_{RL}| &< 0.122(0.104) \\
|g^S_{LL}| &< 0.550(0.550) & |g^V_{LL}| &> 0.960(0.960) & |g^T_{LL}| &\equiv 0
\end{align*}
\]

- Neutrino mass implications at \( 10^{-7}-10^{-4} \) for LR/RL:
Fitting the data distributions

- Decay distribution is linear in $\rho$, $\eta$, $P_\mu \xi$, and $P_\mu \xi \delta$, so a fit to first order expansion is exact.

- Fit data to simulated (MC) base distribution with hidden assumed parameters,
  $\lambda_{MC} = (\rho, \eta, P_\mu \xi | P_\mu \xi \delta, P_\mu \xi \delta)$
  plus MC-generated distributions from analytic derivatives, times fitting parameters ($\Delta \lambda$) representing deviations from base MC.

(graphic thanks to Blair Jamieson)
**Evaluation of Systematic Uncertainties**

- **TWIST** relies on a fit to simulation:
  - Simulation must be verified.
  - Reconstruction systematics eliminated if simulation is perfect.

- **General method:**
  - exaggerate a condition (in data or MC) which may cause error.
  - measure effect by fitting, using correlated sets where practical.
  - scale results according to variance in a data set.
  - Linearity? Double counting?

- Positron interactions:
  - Energy smearing
  - Multiple scattering
  - Hard interactions
  - Material in detector
  - Material outside

- Chamber response:
  - DC and PC efficiencies
  - Dead zone
  - Long drift times
  - HV variations
  - Temperature, pressure
  - Chamber foil bulges
  - Crosstalk
  - Variation of $t_0$

- Momentum calibration:
  - End point fits
  - Field reproduction

- Muon beam stability:
  - Stopping location
  - Beam intensity
  - Magnet stability

- Spectrometer alignment:
  - Translations
  - Rotations
  - Longitudinal
  - Field to detector axis
Fits to data distributions

Above: normalized residuals of fit, and fiducial region used for fit: \( p < 50 \text{ MeV/c}, 0.50 < |\cos \theta| < 0.84, |p_z| > 13.7 \text{ MeV/c}, p_T < 38.5 \text{ MeV/c}.\)

Left: comparison of data to fit (MC) vs. momentum, also showing (MC reconstructed)/(MC thrown) comparisons and normalized residuals.
Fits to data distributions (cont.)

Angular distributions for restricted momentum ranges. Dashed lines show fiducial region of two-dimensional fit.

Dependence of asymmetry on momentum, its two contributions, and comparison of data and fit (MC)distributions.
Summary of results: $\rho$ and $\delta$

- $\rho = 0.75080 \pm 0.00044 \text{(stat)} \pm 0.00093 \text{(syst)} \pm 0.00023 \text{(\eta)}$
  - 2.5 times better precision than PDG value.
  - Uncertainty scaled for $\chi^2$/dof = 7.5/4 (CL=0.11) for different data sets.

- $\delta = 0.74964 \pm 0.00066 \text{(stat)} \pm 0.00112 \text{(syst)}$
  - 2.9 times better precision than PDG value.

- Using the above values of $\rho$ and $\delta$, with $\mathcal{P}_\mu (\xi \delta/\rho) > 0.99682$ (PDG) and $Q_R^\mu \geq 0$, we get
  
  $0.9960 < \mathcal{P}_\mu \xi \leq \xi < 1.0040$ (90\% c.l.)
  
  - improves upon $\mathcal{P}_\mu \xi = 1.0027 \pm 0.0079 \pm 0.0030$. 
# Systematic uncertainties: $\rho$ and $\delta$

<table>
<thead>
<tr>
<th>Systematic uncertainties</th>
<th>$\rho$ ($\leq 10^4$)</th>
<th>$\delta$ ($\leq 10^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>published</td>
<td>current</td>
</tr>
<tr>
<td>Chamber response (ave)</td>
<td>5.1</td>
<td>3.2</td>
</tr>
<tr>
<td>Stopping target thickness</td>
<td>4.9</td>
<td>-</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>4.6</td>
<td>3.8</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>2.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Momentum calibration (ave)</td>
<td>2.0</td>
<td>1.1</td>
</tr>
<tr>
<td>Theoretical radiative correction</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Muon beam stability (ave)</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Track selection algorithm</td>
<td>1.1</td>
<td>-</td>
</tr>
<tr>
<td>Asymmetric efficiencies</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total in quadrature</strong></td>
<td>9.3</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Summary of results: $P_{\mu \xi}$

- $P_{\mu \xi} = 1.0003 \pm 0.0006^{\text{stat}} \pm 0.0038^{\text{syst}}$
  - 2.2 times better precision than PDG value (Beltrami et al.).
  - still not as precise as TWIST indirect result from $\rho$ and $\delta$.

- Dominated by systematic uncertainty from spectrometer fringe field depolarization:
  - prospects for improvement are excellent.
  - data was taken in 2004; new data with improved muon beam from data taken in 2006-07.
**Systematic uncertainties: $P_{\mu \xi}$**

<table>
<thead>
<tr>
<th>Systematic uncertainties</th>
<th>$P_{\mu \xi} \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depolarization in fringe field (ave)</td>
<td>3.4</td>
</tr>
<tr>
<td>Depolarization in muon stopping material (ave)</td>
<td>1.2</td>
</tr>
<tr>
<td>Chamber response (ave)</td>
<td>1.0</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>0.3</td>
</tr>
<tr>
<td>Positron interactions (ave)</td>
<td>0.3</td>
</tr>
<tr>
<td>Depolarization in muon production target</td>
<td>0.2</td>
</tr>
<tr>
<td>Momentum calibration</td>
<td>0.2</td>
</tr>
<tr>
<td>Upstream-downstream efficiency</td>
<td>0.2</td>
</tr>
<tr>
<td>Background muon contamination (ave)</td>
<td>0.2</td>
</tr>
<tr>
<td>Beam intensity (ave)</td>
<td>0.2</td>
</tr>
<tr>
<td>Michel $\eta$ parameter</td>
<td>0.1</td>
</tr>
<tr>
<td>Theoretical radiative correction</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Total in quadrature</strong></td>
<td><strong>3.8</strong></td>
</tr>
</tbody>
</table>
## Improving the systematics

<table>
<thead>
<tr>
<th>Systematic</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>positron interactions</td>
<td>precision target geometry, improved chamber spacing, simulation tuning</td>
</tr>
<tr>
<td>momentum calibration</td>
<td>new techniques with reduced bias</td>
</tr>
<tr>
<td>chamber response</td>
<td>online monitoring, improved instrumentation, drift time measurements</td>
</tr>
<tr>
<td>fringe field depolarization</td>
<td>beam monitoring (TEC), beam alignment and steering</td>
</tr>
<tr>
<td>stopping target depolarization</td>
<td>aluminum and silver targets, depolarization studies with $\mu$SR.</td>
</tr>
</tbody>
</table>
The TECs (time expansion chambers) are transverse drift chambers operating at 0.08 bar, separated from beam vacuum by 6 μm Mylar windows. Two modules measure \( x \) and \( y \).
Left-right symmetric models

- Weak eigenstates in terms of mass eigenstates and mixing angle:
  \[ W_L = W_1 \cos \zeta + W_2 \sin \zeta, \quad W_R = e^{i\omega}(-W_1 \sin \zeta + W_2 \cos \zeta) \]

- Assume possible differences in left and right couplings and CKM character.
  Use notation:
  \[ t = \frac{g_R^2 m_1^2}{g_L^2 m_2^2}, \quad t_\theta = t \frac{|V_{ud}^R|}{|V_{ud}^L|}, \quad \zeta_g = \frac{g_R^2}{g_L^2} \]

- Then, for muon decay, the Michel parameters are modified:
  \[ \rho = \frac{3}{4} (1 - 2\zeta_g^2), \quad \xi = 1 - 2(t_\theta^2 + \zeta_g^2), \]
  \[ \mathcal{P}_\mu = 1 - 2t_\theta^2 - 2\zeta_g^2 - 4t_\theta \zeta_g \cos(\alpha + \omega) \]
  - “manifest” LRS assumes \( g_R = g_L, \ V^R = V^L, \ \omega = 0 \) (no CP violation).
  - “pseudo-manifest” LRS allows CP violation, but \( V^R = (V^L)^* \) and \( g_R = g_L \).
  - RS “non-manifest” or generalized LRS makes no such assumptions.

- Most experiments must make assumptions about LRS models!
## Limits on LRS parameters: PDG06

| Observable                  | $m_2$ (GeV/c^2) | $|\zeta|$ | +         | -         |
|-----------------------------|-----------------|----------|-----------|-----------|
| $m(K_L - K_S)$              | $>1600$         |          | reach     | (P)MLRS   |
| Direct $W_R$ searches       | $>800$ (D0)     | $>786$ (CDF) | clear signal | (P)MLRS decay model |
| CKM unitarity               |                 | $<10^{-3}$ | sensitivity | (P)MLRS heavy $\nu_R$ |
| $\beta$ decay              | $>310$          | $<0.040$ | both parameters | (P)MLRS light $\nu_R$ |
| $\mu$ decay ($\textit{TWIST}$) | $>406$ ($>420$) | $<0.033$ ($<0.030$) | model independence | light $\nu_R$ |
Muon decay LRS limits

Exclusion (90% cl) plots for left-right symmetric model mixing angle and right partner boson $W_2$ mass $m_2$
Summary

- TWIST has produced its first direct measurement of $P_{\mu\xi}$, to add to previous results for $\rho$ and $\delta$.

- Analysis underway for second measurements for $\rho$ and $\delta$, representing further improvements by $\sim 2$.

- Reduction of depolarization systematics for $P_{\mu\xi}$ seems achievable, but it is not yet known by how much.

- In 2006-2008, TWIST will produce its final results:
  - goal remains the reduction of uncertainty by an order of magnitude compared to previous muon decay parameter experiments.
## TWIST Participants

<table>
<thead>
<tr>
<th>TRIUMF</th>
<th>Alberta</th>
<th>Kurchatov Institute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ryan Bayes *†</td>
<td>Andrei Gaponenko **</td>
<td>Vladimir Selivanov</td>
</tr>
<tr>
<td>Yuri Davydov</td>
<td>Peter Kitching</td>
<td>Vladimir Torokhov</td>
</tr>
<tr>
<td>Jaap Doornbos</td>
<td>Robert MacDonald *</td>
<td></td>
</tr>
<tr>
<td>Wayne Faszer</td>
<td>Maher Quraan †</td>
<td></td>
</tr>
<tr>
<td>Makoto Fujiwara</td>
<td>Nate Rodning §</td>
<td></td>
</tr>
<tr>
<td>David Gill</td>
<td>John Schaapman</td>
<td></td>
</tr>
<tr>
<td>Alex Grossheim</td>
<td>Glen Stinson</td>
<td></td>
</tr>
<tr>
<td>Peter Gumplinger</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anthony Hillairet *†</td>
<td>British Columbia</td>
<td></td>
</tr>
<tr>
<td>Robert Henderson</td>
<td>James Bueno *</td>
<td></td>
</tr>
<tr>
<td>Jingliang Hu</td>
<td>Mike Hasinoff</td>
<td></td>
</tr>
<tr>
<td>John A. Macdonald §</td>
<td>Blair Jamieson **</td>
<td></td>
</tr>
<tr>
<td>Glen Marshall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dick Mischke</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mina Nozar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Konstantin Olchanski</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Art Olin †</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robert Openshaw</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tracy Porcelli ‡</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jean-Michel Poutissou</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renée Poutissou</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grant Sheffer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bill Shin ‡‡</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Support under grants from NSERC (Canada) and DOE (USA).</td>
<td>- Computing facilities of WestGrid are gratefully acknowledged.</td>
<td></td>
</tr>
</tbody>
</table>