Measurement of the Muon Decay Parameter δ

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The muon decay parameter δ has been measured by the TWIST collaboration. We find $\delta = 0.74964 \pm 0.00066(\text{stat.}) \pm 0.00112(\text{syst.})$, consistent with the Standard Model value of 3/4. This result implies that the product $P_\mu \xi$ of the muon polarization in pion decay, P_μ, and the muon decay parameter ξ falls within the 90% confidence interval $0.9960 < P_\mu \xi \leq \xi < 1.0040$. It also has implications for left-right-symmetric and other extensions of the Standard Model.

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The TWIST spectrometer [1] was designed to measure a broad range of the normal muon decay spectrum, $\mu^+ \to e^+ \nu_e \overline{\nu}_\mu$, allowing the simultaneous extraction of the spectrum shape parameters. Assuming the weak interaction is local and invariant under the Lorentz group, the effective four fermion muon decay matrix element can be written in terms of helicity-preserving amplitudes:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T;\epsilon,\mu=R,L} g^{\gamma}_{\epsilon\mu} \langle \bar{e}_\epsilon | \Gamma^\gamma | \nu \rangle \langle \overline{\nu} | \Gamma_\gamma | \mu_\mu \rangle, \quad (1)$$

where the $g^{\gamma}_{\epsilon\mu}$ specify the scalar, vector, and tensor couplings between μ-handed muons and ϵ-handed electrons [2]. In this form, the Standard Model implies $g^V_{LL} = 1$ and all other coupling constants are zero.

The differential decay spectrum [3] of the e^+ emitted in the decay of polarized μ^+ is provided in terms of four parameters, ρ, δ, η, and ξ, commonly referred to as the Michel parameters, which are bilinear combinations of the coupling constants. In the limit where the electron and neutrino masses as well as radiative corrections are neglected, this spectrum is given by:

$$\begin{aligned} \frac{d^2\Gamma}{x^2 dx d(\cos\theta)} \;&\propto\; 3(1-x) + \frac{2}{3}\rho(4x-3) \\ &+\; P_\mu \xi \cos\theta [1 - x + \frac{2}{3}\delta(4x-3)], \quad (2) \end{aligned}$$

where θ is the angle between the muon polarization and the outgoing electron direction, $x = E_e/E_{max}$, and P_μ is the muon polarization. The fourth parameter, η, appears in the isotropic term when the electron mass is included in the analysis. In the Standard Model, the Michel parameters take on precise values.

The parameter ξ expresses the level of parity violation in muon decay, while δ parametrizes its momentum dependence. Recently, TWIST reported a new measurement of ρ [4]. In this Letter we report a new measurement of δ. The currently accepted value of $\delta = 0.7486 \pm 0.0026 \pm 0.0028$ [5] agrees with the Standard Model expectation of 3/4. Some Standard Model extensions require deviations from pure $V - A$ coupling that can alter δ. Some of these models involve right-handed interactions. The positive definite quantity,

$$\begin{aligned} Q^\mu_R \;&=\; \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RR}|^2 + |g^V_{LR}|^2 + |g^V_{RR}|^2 + 3|g^T_{LR}|^2 \\ &=\; \frac{1}{2}[1 + \frac{1}{3}\xi - \frac{16}{9}\xi\delta], \quad (3) \end{aligned}$$

can serve to set a model independent limit on any muon right-handed couplings [2, 6]. A recent review of muon decay is presented in [7].

TWIST measures highly polarized muons from the M13 beam line at TRIUMF that are stopped in a 125 μm Mylar foil, coated with $10^{+10}_{-5}\mu$m graphite layers on both sides, located at the center of the TWIST spectrometer. Decay positrons spiral in a 2 T magnetic field, passing through up to 28 high-precision wire chamber planes. A detailed description of the TWIST detector is given in

[1]. The precision of the detector and our knowledge of its parameters is demonstrated by the fact that fits to determine the endpoint momemtum indicate the absolute magnetic field matches the nominal value to within 2G. The analysis of the decay positron helical tracks to provide the momentum-angle spectra from which this extraction of δ is made used the same procedures as our ρ determination [4].

TWIST determines the Michel parameters by fitting two-dimensional histograms of reconstructed experimental decay positron momenta and angles with histograms of reconstructed Monte Carlo (MC) data. This approach has several advantages. First, spectrum distortions introduced by the event reconstruction largely cancel because MC and experimental data are analyzed identically. Second, because the MC simulates the detector response well, no explicit corrections of the result are required. Third, a blind analysis of the result is straightforward. It is implemented by utilizing hidden Michel parameters ρ_H, δ_H, and ξ_H to generate the theoretical decays. The decay rate can be written as

$$\left.\frac{d^2\Gamma}{dx d(\cos\theta)}\right|_{\rho_H,\delta_H,\xi_H} + \sum_{\alpha=\rho,P_\mu\xi,P_\mu\xi\delta} \frac{\partial}{\partial\alpha}\left[\frac{d^2\Gamma}{dx d(\cos\theta)}\right]\Delta\alpha$$

since the decay spectrum is linear in the shape parameters. The sum of MC spectra is fit to the data spectrum by adjusting the $\Delta\alpha$. The Monte Carlo spectra were generated including full $O(\alpha)$ radiative corrections with exact electron mass dependence, leading and next-to-leading logarithmic terms of $O(\alpha^2)$, leading logarithmic terms of $O(\alpha^3)$, corrections for soft pairs, virtual pairs, and an ad-hoc exponentiation [8]. The η parameter was fixed to the current accepted value of -0.007 [6] for MC spectra production. The uncertainty of 0.013 on the accepted value of η gives a negligible uncertainty on the final value of δ. $\Delta\delta$ is extracted from $\Delta(P_\mu\xi)$ and $\Delta(P_\mu\xi\delta)$, then δ is calculated as $\delta_H + \Delta\delta$ after δ_H is revealed once the analysis is complete.

The TWIST simulation model is based on GEANT 3.21 [9] with the chamber response based on GARFIELD [10]. It contains virtually all the components of the spectrometer with which a muon or a decay positron could possibly interact. The simulation includes the frequency of additional beam particles in an event as well as the rate of muon polarization relaxation. The output exactly mimics the binary files generated by the data acquisition system.

Factors that influence the momentum and angle determination must be well simulated in the MC, so special runs were taken specifically to address the accuracy of the simulation of energy loss and multiple scattering. Muons were stopped in the extreme upstream wire chambers in both the experiment and in the MC simulation. The decay positrons, which have the full Michel spectrum momentum range, were tracked through the upstream($-z$) half and separately through the downstream($+z$) half of the spectrometer. Differences in momentum and angle were histogrammed on a track by track basis. Figure 1 presents, for both data and MC, the energy lost and the change in angle that occur primarily at the central stopping target. The Δp distribution mean(RMS) for the data and MC are -0.17(0.41) MeV/c and -0.17(0.39) MeV/c, respectively. The $\Delta\theta$ mean(RMS) for the data and MC are -0.95(17.0) and -0.37(18.0) milliradians respectively. The MC reproduces the data very well.

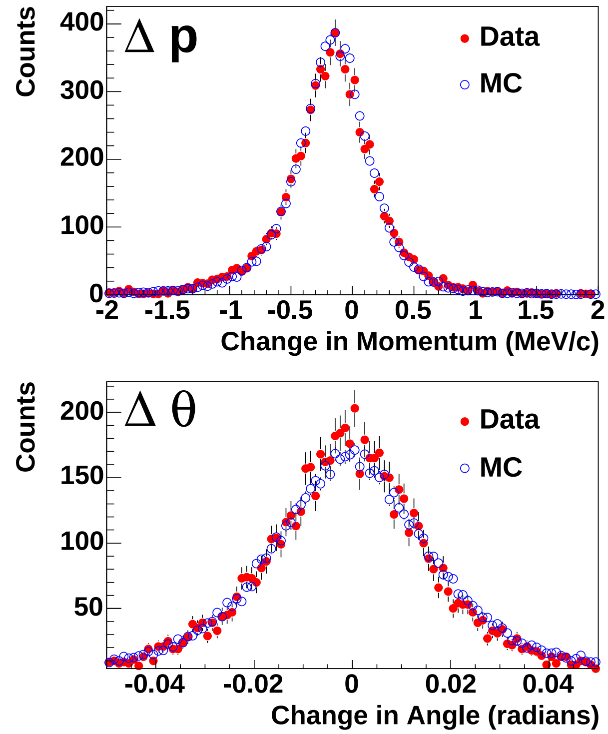

FIG. 1: (color online) The difference between downstream and upstream tracks, for both data and MC, resulting in: Top, the positron momentum change in the central stopping target, Bottom, $\Delta\theta$ for a positron that passed through the central stopping target. The MC results were normalized to those of the data for the purposes of this figure.

The result for δ presented here employed a substantial sample size consisting of 6×10^9 events recorded in Fall, 2002. This data sample is comprised of 16 data sets, many taken under conditions chosen to establish the sensitivity of the detector to systematic effects. Four of the data sets, sets A and B taken at 2.00 T six weeks apart and two other sets taken at 1.96 T and at 2.04 T, were analyzed and fit to their corresponding MC samples to derive the value of δ.

Figure 2 shows the decay positron angular distributions for representative momentum bins. Equation (2) indicates the angular distributions follow a $1 + A(p)\cos\theta$ shape, where by convention the asymmetry, $A(p)$, is positive when positrons are emitted preferentially along the

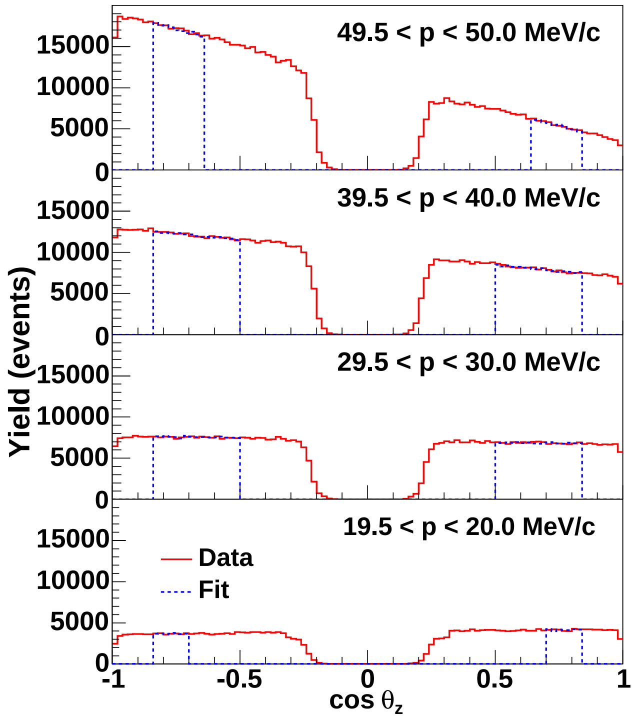

FIG. 2: (color online) Decay positron angular distributions from set B (solid curves) and the corresponding best fit distributions within the fiducial region (dashed curves) for selected momentum bins.

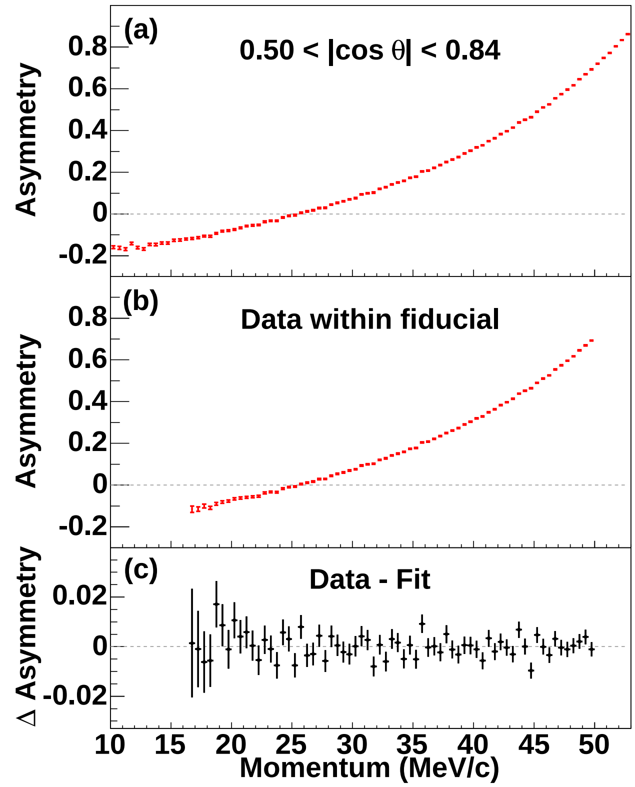

FIG. 3: (color online) (a) The observed muon decay asymmetry from set B for all events within $0.50 < |\cos\theta| < 0.84$. (b) The same quantity for those events that fall within the fiducial region. (c) The difference between the data in panel (b) and the best fit MC spectrum.

muon polarization axis ($-z$ direction, $\cos\theta_z < 0$). Figure 3(a) shows the observed muon decay asymmetry as a function of momentum for set B. The asymmetry provides a compact representation of the angular distributions, but we extract δ from a simultaneous fit of the full experimental momentum-angle distribution, as described above. The fiducial region adopted for this analysis requires $p < 50$ MeV/c, $|p_z| > 13.7$ MeV/c, $p_T < 38.5$ MeV/c, and $0.50 < |\cos\theta| < 0.84$. Figure 2 shows the results of the best fit to set B within the fiducial region for the selected momentum bins, while Fig. 3 shows the measured muon decay asymmetry and the difference between the measured asymmetry and the asymmetry calculated from the best fit MC spectrum for events within the fiducial region. The graphite coated mylar stopping target resulted in a time dependence of the muon polarization, P_μ, which prevented the simultaneous determination of a value for $P_\mu\xi$ from this data sample. $\langle P_\mu \rangle \sim -0.89$ at the time of decay for the data sets analyzed here.

Systematics were studied by employing the fitting technique described above to fit experimental data samples taken with a systematic parameter set at an exaggerated level to data taken under ideal conditions. This expresses the changes in the spectrum shape caused by the systematic effect in terms of changes in the Michel parameters. Other systematic sensitivities were determined by analyzing a data or MC sample with a systematic parameter offset from its nominal value and fitting to the same sample analyzed with this parameter at its nominal value. For the current analysis the largest uncertainties are for the detector alignment, for the simulation of positron interactions, and for the chamber response, in particular the time dependent effects due to gas density changes and to the variability of the cathode foil positions [1]. The latter parameters were monitored throughout the data accumulation periods and average values were used in the analysis. Uncertainties due to the detector alignment were established by analysis of data and generation of MC with purposely misaligned chambers. Upper limits for the positron interaction uncertainties were derived from studies of the data for muons stopped far upstream and from MC histograms that demonstrated the distortion of the momentum spectrum due to hard interactions. Other important systematic uncertainties for δ are the stopping target thickness and the momentum calibration. The target thickness issue was studied by varying the thickness of the graphite coating in MC. Less significant uncertainties result from the theoretical radiative corrections and upstream vs downstream efficiencies. The results of these studies for the parameter δ are presented in Tables I and II.

TABLE I: Results for δ. Each fit has 1887 degrees of freedom. Statistical and set-dependent systematic uncertainties are shown.

Data Set	δ	χ^2
Set A	$0.75087 \pm 0.00156 \pm 0.00073$	1924
Set B	$0.74979 \pm 0.00124 \pm 0.00055$	1880
1.96 T	$0.74918 \pm 0.00124 \pm 0.00069$	1987
2.04 T	$0.74908 \pm 0.00132 \pm 0.00065$	1947

TABLE II: Contributions to the systematic uncertainty for δ. Average values are denoted by (ave), which are considered set-dependent when performing the weighted average of data sets.

Effect	Uncertainty
Spectrometer alignment	± 0.00061
Chamber response(ave)	± 0.00056
Positron interactions	± 0.00055
Stopping target thickness	± 0.00037
Momentum calibration(ave)	± 0.00029
Muon beam stability(ave)	± 0.00010
Theoretical radiative corrections[8]	± 0.00010
Upstream/Downstream efficiencies	± 0.00004

The effects of chamber response, momentum calibration and muon beam stability, which have time dependent components, are treated as data set dependent effects with the average(ave) over the four sets used in the δ evaluation appearing in Table II.

We find $\delta = 0.74964 \pm 0.00066(\text{stat.}) \pm 0.00112(\text{syst.})$, consistent with the Standard Model expectation of 3/4. Using this result, our new value for ρ [4], the previous measurement of $P_\mu\xi\delta/\rho$ [11], and the constraint $Q_R^\mu > 0$, it is possible to establish new 90% confidence interval limits, $0.9960 < P_\mu\xi \leq \xi < 1.0040$, consistent with the Standard Model value of 1. This result is more restrictive than the current best measurements, $P_\mu\xi = 1.0027 \pm 0.0079 \pm 0.0030$ for muons from pion decay [12] and $P_\mu\xi = 1.0013 \pm 0.0030 \pm 0.0053$ for muons from kaon decay [13]. In addition, from these same results one finds that $Q_R^\mu < 0.00184$ with 90% confidence. This may be combined with Eq. (3) to find new 90% confidence limits on interactions that couple right-handed muons to left-handed electrons: $|g_{LR}^S| < 0.086$, $|g_{LR}^V| < 0.043$, and $|g_{LR}^T| < 0.025$. The lower limit, $0.9960 < P_\mu\xi$ can be used to determine a new limit on the mass of the possible right-handed boson, W_R, improving the existing lower limit of 406 GeV/c^2 (402 GeV/c^2 with modern $M_{W_L} = 80.423$ GeV/c^2) from [11] to 420 GeV/c^2 under the assumption of pseudomanifest left-right symmetry. For non-manifest left-right symmetric models the limit is $M_{W_R} g_L/g_R > 380\text{GeV/c}^2$, where g_L and g_R are the coupling constants [14]. The value of δ is sensitive to a proposed nonlocal interaction [15] that would be represented by a new parameter κ. A limit for κ may be estimated from our 90% confidence lower limit for δ using the relation $\delta = 3/4(1 - 6\kappa^2)$. This results in $\kappa \leq 0.024$, which compares with $\kappa = 0.013$ [15] hinted at by π decay experiments.

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