Measurement of the muon decay parameters with the TRIUMF Weak Interaction Symmetry Test (TWIST) Experiment

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The Physics Motivation

The muon decay is very interesting

- Only weak interaction involved
- Muons are easy to produce
- One decay mode dominant
  \( \approx 100\% \)

\[ \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \]
The muon decay is very interesting

- Only weak interaction involved
- Muons are easy to produce
- One decay mode dominant ($\approx 100\%$)

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$
4-fermion formalism

To study the muon decay we describe it by using a very general formalism. The interaction is described as a derivative-free, Lorentz-invariant and lepton-number conserving matrix:

\[ M = 4 \frac{G_F}{\sqrt{2}} \sum_{\gamma = S, V, T} g_{\epsilon\mu}^{\gamma} \langle \bar{e}_\epsilon | \Gamma_\gamma | \nu_e \rangle < \bar{\nu}_\mu | \Gamma_\gamma | \mu_\mu \rangle \]

\[ \gamma = S, V, T \]
\[ \epsilon, \mu = R, L \]

- \( g_{RR}^T = g_{LL}^T = 0 \)
- A common phase doesn’t matter

\[ \Rightarrow 19 \text{ real and independent parameters} \]

Standard Model, V-A interaction

\( g_{LL}^V \) is the only non zero coupling
The differential decay rate can be written using the Michel parametrization:

\[
\frac{d^2\Gamma}{dx d\cos \theta} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} (F_{IS}(x) + P_\mu \cos \theta F_{AS}(x)) + \text{RC}.
\]

\[
x = \frac{E_e}{W_{e\mu}}
\]
The Michel parametrization

The isotropic and anisotropic parts of the Michel parameters are:

\[
F_{IS}(x) = x(1 - x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1 - x)
\]

\[
F_{AS}(x) = \frac{1}{3} \xi \sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3} \delta (4x - 3 + (\sqrt{1 - x_0^2} - 1)) \right]
\]

Standard Model predictions

\[
\rho = \frac{3}{4}, \quad \eta = 0, \quad P_{\mu} \xi = 1, \quad \delta = \frac{3}{4}
\]
The Michel parametrization

The isotropic and anisotropic parts of the Michel parameters are:

\[ F_{IS}(x) = x(1 - x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1 - x) \]

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**Standard Model predictions**

\[ \rho = \frac{3}{4}, \quad \eta = 0, \quad P_{\mu \xi} = 1, \quad \delta = \frac{3}{4} \]
The TWIST spectrometer

TWIST Spectrometer
(cutaway view)
The spectrometer in the M13 beamline
The spectrometer in the M13 beamline

Proton Beam

Anthony Hillairet Measurement of the muon decay parameters with the TWIST experiment.
The spectrometer in the M13 beamline

Proton Beam

Muon Beam
The spectrometer in the M13 beamline

Measurement of the muon decay parameters with the TWIST experiment.

Anthony Hillairet
Typical event

Anthony Hillairet

Measurement of the muon decay parameters with the TWIST experiment.
Typical event

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Typical event

Measurement of the muon decay parameters with the TWIST experiment.
A Blind Analysis

The TWIST analysis is blind to avoid any human bias:
- Choices of data samples
- Looking for errors if disagreement with expectations
- Systematic error evaluation influenced by final result

![Graph showing the value of \( \rho \) over years from 1950 to 1970]
The software analysis

**Experimental data**

**Monte Carlo data**

**MOFIA**

**Energy calibration**

**Michel Spectrum Fit**

$\Delta \xi, \Delta \rho, \Delta \delta$

$\xi, \rho, \delta$

Black Box

$\xi_{MC}, \rho_{MC}, \delta_{MC}$

Measurement of the muon decay parameters with the TWIST experiment.
The software analysis

- Experimental data
- MOFIA
- Monte Carlo data

Black Box
\( \xi_{MC}, \rho_{MC}, \delta_{MC} \)

Energy calibration

Michel Spectrum Fit
\( \Delta\xi, \Delta\rho, \Delta\delta \)

\( \xi, \rho, \delta \)
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$\Delta \xi, \Delta \rho, \Delta \delta$

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$\Delta \xi, \Delta \rho, \Delta \delta$

$\xi, \rho, \delta$

Black Box

$\xi_{MC}, \rho_{MC}, \delta_{MC}$
The software analysis

- Detector response included in MC
- Reconstruction biases reduced because $\Delta \xi, \Delta \rho, \Delta \delta$ are small
- Most systematics are from the difference between the MC simulation and the reality
Published results

\[
\begin{align*}
\rho &= 0.75080 \pm 0.00044 \text{ (stat)} \pm 0.00093 \text{ (sys)} \\
& \quad \pm 0.00023 \text{ (\eta)} \\
\delta &= 0.74964 \pm 0.00066 \text{ (stat)} \pm 0.00112 \text{ (sys)} \\
P_{\mu \xi} &= 1.0003 \pm 0.0006 \text{ (stat)} \pm 0.0038 \text{ (sys)}
\end{align*}
\]

- 2002 data \(\Rightarrow\) \(\rho\) and \(\delta\)
- Improvements:
  - TEC construction and installation
  - DC chambers rearranged
- 2004 data \(\Rightarrow\) \(P_{\mu}\)
Data taken in 2006 and 2007 for the last measurement of the three parameters. Final results publication planned for 2009.

Table of the uncertainties $\times 10^{-4}$

<table>
<thead>
<tr>
<th></th>
<th>Published</th>
<th>Final Goal</th>
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<tbody>
<tr>
<td></td>
<td>Statistics</td>
<td>Systematics</td>
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<tr>
<td>$\rho$</td>
<td>4.4</td>
<td>9.3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>6.6</td>
<td>11.2</td>
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<tr>
<td>$P_{\mu}^\xi$</td>
<td>6.0</td>
<td>38</td>
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</table>
Improvements: Muon Beam Monitoring

- The individual muon tracks cannot be reconstructed
- The muon beam spots at each plane are used instead

**Improvements**

- Muon beam stability monitored
- Reduced $p_T \rightarrow$ lower depolarisation
Improvements: Better upstream stops data

The upstream stops data gives us information on the physics and the response of the detector:

- Test of the detector asymmetry
- Measure of the positron interaction with the target

...
## The TWIST Collaboration

<table>
<thead>
<tr>
<th>TRIUMF</th>
<th>Alberta</th>
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<tbody>
<tr>
<td>Ryan Bayes†</td>
<td>Andrei Gaponenko○</td>
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<tr>
<td>Yuri Davydov</td>
<td>Peter Kitching</td>
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<tr>
<td>Wayne Faszer</td>
<td>Robert MacDonald†</td>
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<td>Anthony Hillairet†</td>
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<td>Regina</td>
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<td>Ted Mathie</td>
<td>James Bueno†</td>
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<td>Roman Tacik</td>
<td>Mike Hasinoff</td>
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<td>Carl Gagliardi</td>
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<td>Jim Musser○</td>
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<td>Don Koetke</td>
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<td>Shirvel Stanislaus</td>
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○ Graduated

† Graduate student
Model-Independent search for right-handed interactions

\[
Q_{RR} = \frac{1}{4} |g_{RR}^S|^2 + |g_{RR}^V|^2
\]

\[
Q_{LR} = \frac{1}{4} |g_{LR}^S|^2 + |g_{LR}^V|^2 + 3|g_{LR}^T|^2
\]

- Right-handed interaction contribution in the muon decay:
  \[
  Q^\mu_R = Q_{RR} + Q_{LR}
  \]
- Also defined as:
  \[
  Q^\mu_R = \frac{1}{2} \left( 1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right)
  \]

Standard Model, V-A interaction

\[
Q^\mu_R = 0
\]
The Time Expansion Chamber (TEC)

- is low mass
- operates in the beamline vacuum
- is removed during data taking
Michel Parameters Derivatives

\[
\frac{d^2 \Gamma}{dx d(\cos \theta)} \bigg|_{\rho_{MC}, \delta_{MC}, \xi_{MC}} + \sum_{\alpha = \rho, \xi, \xi \delta} \frac{\partial}{\partial \alpha} \left[ \frac{d^2 \Gamma}{dx d(\cos \theta)} \right] \Delta \alpha
\]

MC spectrum

Derivatives fitted

\[\rho\text{ derivative}\]

\[\eta\text{ derivative}\]

\[\xi, \xi \delta\text{ derivative}\]

\[\xi \delta\text{ derivative}\]
TABLE II. Contributions to the systematic uncertainty in $\rho$. Average values are given for those denoted (ave), which are considered set dependent when performing the weighted average of the data sets.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Uncertainty</th>
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</thead>
<tbody>
<tr>
<td>Chamber response (ave)</td>
<td>$\pm 0.00051$</td>
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<tr>
<td>Stopping target thickness</td>
<td>$\pm 0.00049$</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>$\pm 0.00046$</td>
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<tr>
<td>Spectrometer alignment</td>
<td>$\pm 0.00022$</td>
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<tr>
<td>Momentum calibration (ave)</td>
<td>$\pm 0.00020$</td>
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<tr>
<td>Theoretical radiative corrections [12]</td>
<td>$\pm 0.00020$</td>
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<tr>
<td>Track selection algorithm</td>
<td>$\pm 0.00011$</td>
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<tr>
<td>Muon beam stability (ave)</td>
<td>$\pm 0.00004$</td>
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<tr>
<td>Total in quadrature</td>
<td>$\pm 0.00093$</td>
</tr>
<tr>
<td>Scaled total</td>
<td>$\pm 0.00097$</td>
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</tbody>
</table>

TABLE II. Contributions to the systematic uncertainty for $P_{\mu}^+$. Average values are denoted by (ave), which are considered set dependent when performing the weighted average of data sets.

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<tr>
<th>Effect</th>
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</tr>
</thead>
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<td>Spectrometer alignment</td>
<td>$\pm 0.00061$</td>
</tr>
<tr>
<td>Chamber response (ave)</td>
<td>$\pm 0.00056$</td>
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<tr>
<td>Positron interactions</td>
<td>$\pm 0.00055$</td>
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<tr>
<td>Stopping target thickness</td>
<td>$\pm 0.00037$</td>
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<tr>
<td>Momentum calibration (ave)</td>
<td>$\pm 0.00029$</td>
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<tr>
<td>Muon beam stability (ave)</td>
<td>$\pm 0.00010$</td>
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<tr>
<td>Theoretical radiative corrections [9]</td>
<td>$\pm 0.00010$</td>
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<tr>
<td>Upstream/downstream efficiencies</td>
<td>$\pm 0.00004$</td>
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