Estimating $\mathbf{P}_\mu$ for the $\mathcal{TWIST}$ Measurement of $\mathbf{P}_{\mu\xi}$

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Overview

- What is $P_\mu\xi$?
- Some Physics motivation for $P_\mu\xi$
- Initial P_μ and Depolarization Effects
- Statement of the problem
- Review of Spin
- Spin propogation in Magnetic Fields
- Overall $\mathcal{TWIST}$ Muon Depolarization estimate

What is $P_\mu \xi$?

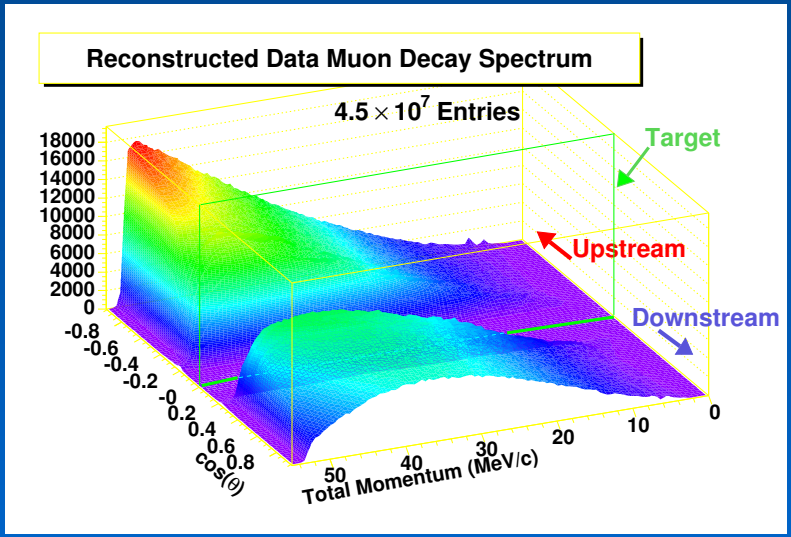
- P_μ is the polarization of the muon, ξ is asymmetry in angle of decay positrons from normal μ decay
- Standard Model (V-A) predicts $\xi = 1$ and $P_\mu = 1$

$$\frac{d^2\Gamma}{dx\, d\cos\theta} \propto x^2 - x^3 + \frac{2}{9}\rho(4x^3 - 3x^2) + \eta x_0(x - x^2) + \frac{1}{3}P_\mu \xi \cos\theta(x^2 - x^3 + \frac{2}{3}\delta(4x^3 - 3x^2)) \quad (1)$$

$x = E_e/W_{e\mu}$

$W_{e\mu} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$

$x_0 = \frac{m_e}{W_{e\mu}}$



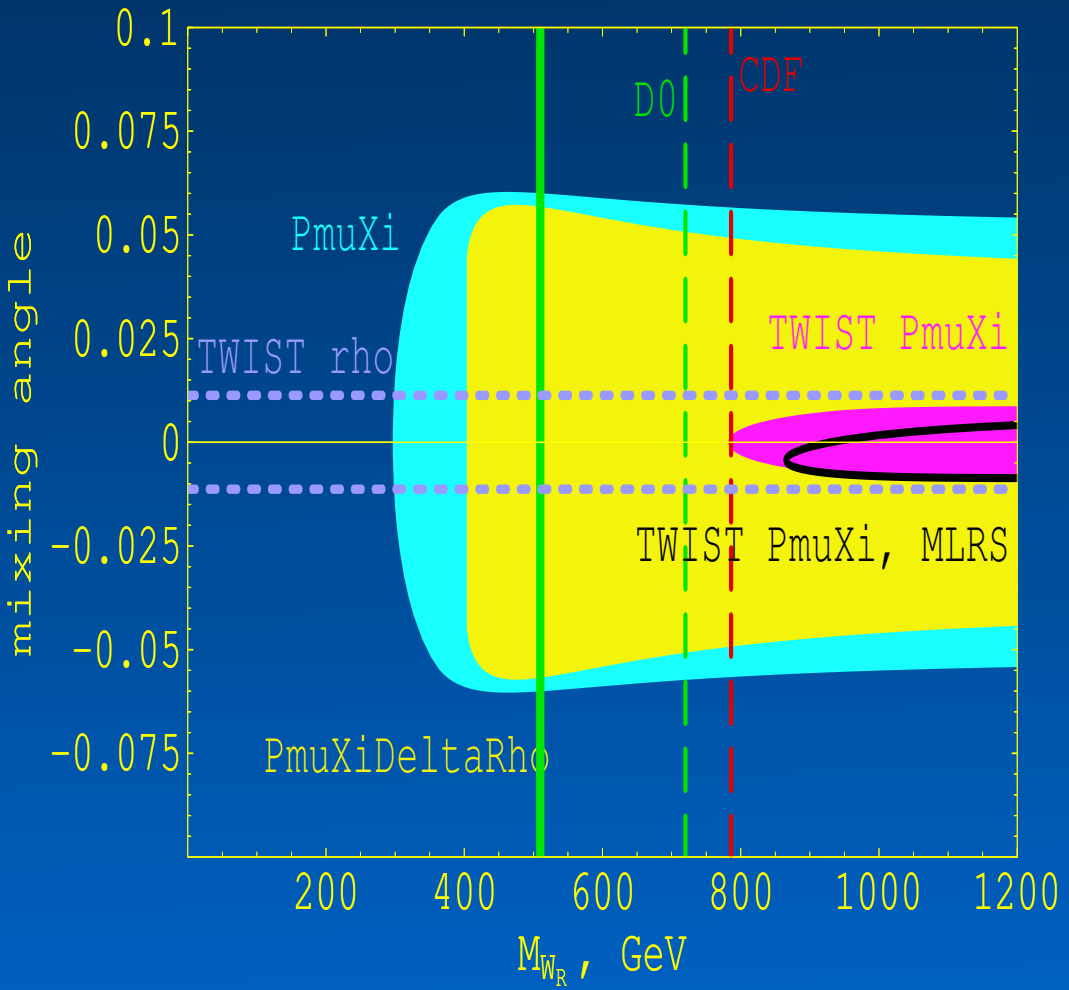
Physics and Motivation for $P_\mu\xi$

- Best Measurements:
  - $P_\mu\xi = 1.0027 \pm 0.0079 \pm 0.0030$ (Beltrami et. al., PL **B194** 326)
  - $P_\mu\xi\delta/\rho > 0.99682$, 90% conf. level (Jodidio et.al., PR **D34** 1967, PR **D37** 237)
- ξ and δ together give limit on probability of right-handed muon decaying into any handed positron:

$$Q_R^\mu = \frac{1}{2}(1 + \frac{1}{3}\xi - \frac{16}{9}\xi\delta) \tag{2}$$

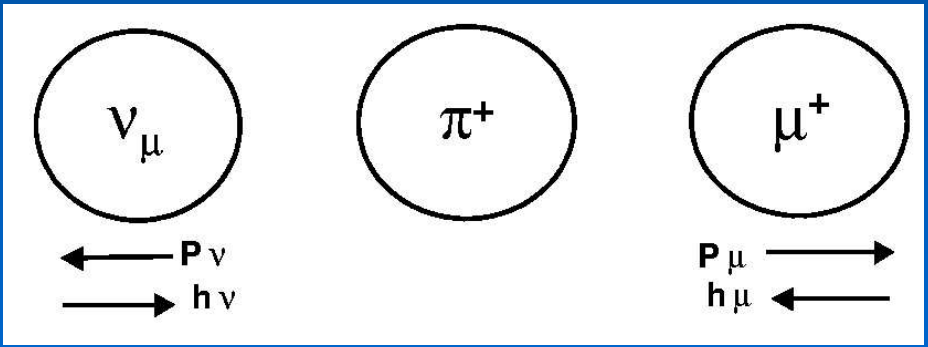
- In Left-right symmetric model, $P_\mu\xi$ sets limit on W_R mass (ϵ) and left/right mixing parameter (ζ):

$$P_\mu\xi = 1 - 2\epsilon^2 - 2\zeta^2 - 2\epsilon^2(\frac{V_{ud}^R}{V_{ud}^L})^2 - \epsilon\zeta\frac{V_{ud}^R}{V_{ud}^L} \tag{3}$$


0.1
0.075
0.05
0.025
0
-0.025
-0.05
-0.075
mixing angle
D0
CDF
PmuXi
TWIST rho
TWIST PmuXi
TWIST PmuXi, MLRS
PmuXiDeltaRho
200
400
600
800
1000
1200
M_{W_R}, GeV

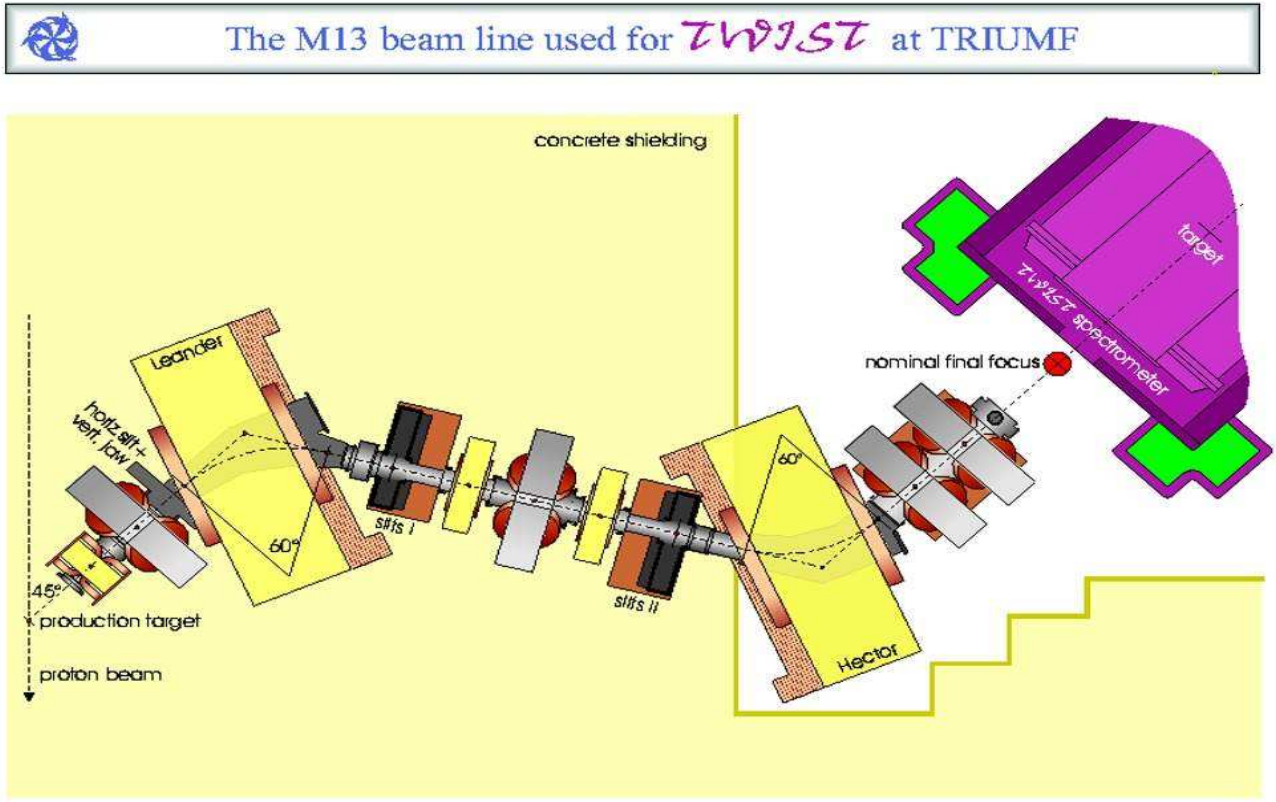
Initial P_μ and Depolarization Effects

- Muon from π decay at rest has spin opposite direction from momentum since:
  - Standard Model ν is left handed
  - Conservation of Angular Momentum
- Depolarization Effects:
  - Precession of Spin in Magnetic Fields
    - Beam Divergence
    - Radial Fringe Fields
  - Muonium Formation in Non-metals



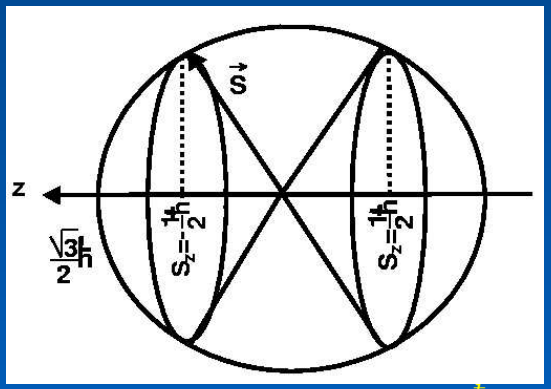
Statement of the Problem

- What is the average ΔP_μ as μ goes from production to stopping?



Review of Spin 1/2 Leptons

- Spin "angular momentum" is a fundamental property of a particle
- Magnetic dipole moment due to spin is:
  $\vec{M} = -\frac{ge\hbar}{2m}\vec{S} = -g\mu_B\frac{\vec{S}}{\hbar}$, $\mu_B = 5.788381749(43) \times 10^{-11} MeV/T$ i
  $g \approx 2$. due to relativistic kinematics, called Thomas Precession
- Torque ($\vec{\tau}$), and Force ($\vec{F}$) due to the intrinsic spin are:
  $\vec{\tau} = \vec{M} \times \vec{B}$
  $\vec{F} = \nabla(\vec{M} \cdot \vec{B})$



- Quantization of spin
- Spin must be 1/2 (ie 2s+1=2)
- Spin precesses about $\vec{B}$, along direction of B (z-axis): $S_z = \pm\frac{\hbar}{2}$
- Time average of Spin perpendicular to B is zero

Non-Relativistic Propogation of Spin in Uniform B

- The equation for propogation of spin in a uniform magnetic field is:

$$\frac{d\vec{S}}{dt'} = \frac{ge}{2mc}\vec{S} \times \vec{B}' \tag{4}$$

- Prime means defined in rest frame of the particle,$\vec{S}$ is the spin in that frame
- For perfect alignment of $\vec{S}$ and $\vec{B}$:

$$\begin{aligned} S_x &= \frac{\hbar}{\sqrt{2}} \sin \gamma_z t \\ S_y &= \frac{\hbar}{\sqrt{2}} \cos \gamma_z t \\ S_z &= -\frac{\hbar}{2} \\ \gamma_z &= \frac{ge}{2mc} B_z \end{aligned} \tag{5}$$

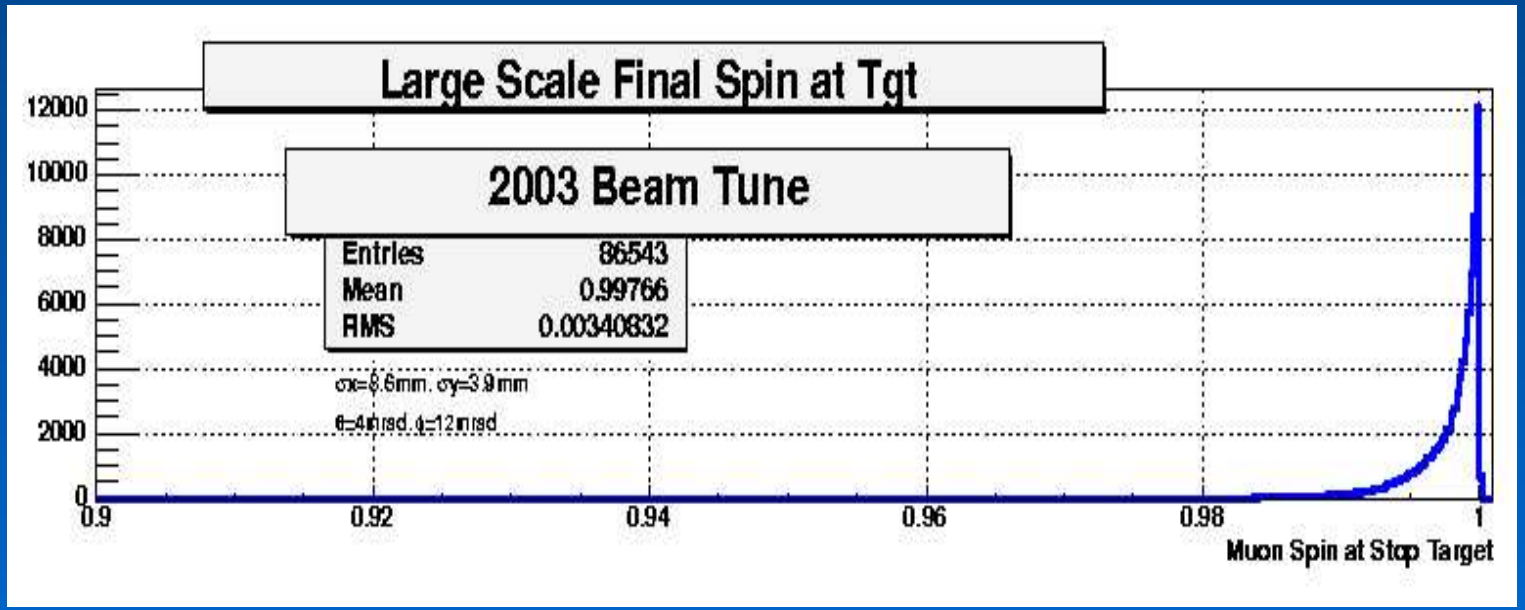
- Misalignment α between $\vec{S}$ and $\vec{B}$ results in depolarization: $\Delta P_\mu = 1 - |\cos \alpha|$

Relativistic Propogation of Spin

- Spin propogation is given by Bargmann, Michel, Telegdi (BMT) equation:

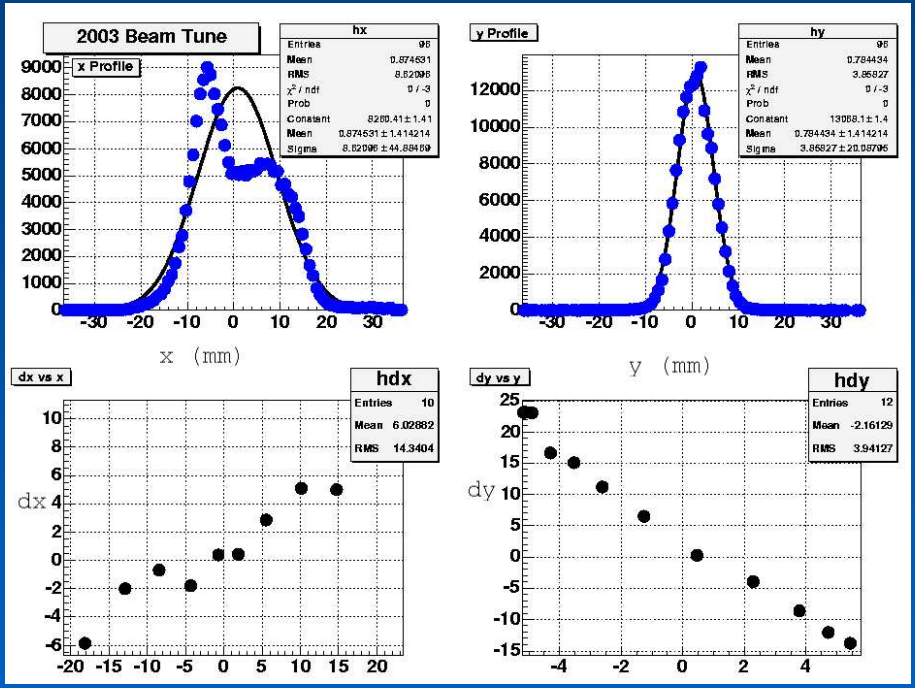
$$\frac{d\vec{s}}{dt} = \frac{e}{mc}\vec{s} \times [(\frac{g}{2} - 1 + \frac{1}{\gamma})\vec{B} - (\frac{g}{2} - 1)\frac{\gamma}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta}] \quad (6)$$

- For non-uniform field solve by stepwise integration in Monte-Carlo

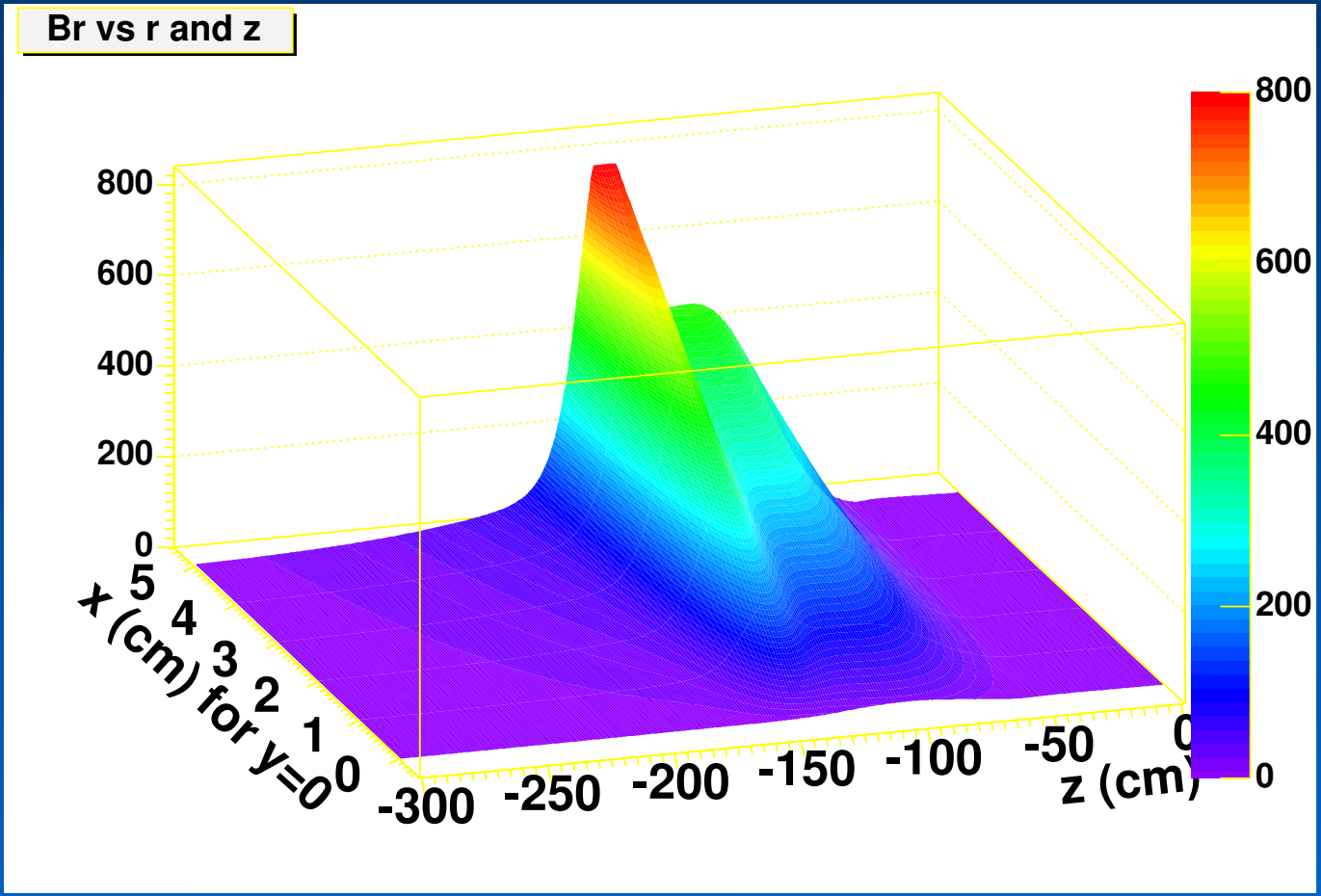


Inputs to Depolarization Calculation

- Field map
- Beam Tune



Radial Magnetic Field Map (Gauss)



Summary

- Estimated ΔP_μ for current tune is $\approx 3 \times 10^{-3}$
- Further reduction of beam size and divergence is desireable to reduce fringe field depolarization
- $\mathcal{TWIST}$ goal is for knowledge of ΔP_μ to better than 10^{-4}

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